Elementary forms for land surface segmentation: The theoretical basis of terrain analysis and geomorphological mapping

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Outline

- Introduction
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Geomorphological regionalization and mapping remain fundamental research methods of geomorphology and provide many promising applications (see e.g. Cooke and Doornkamp, 1990; Evans, 1990; Voženílek, 2000; Lee, 2001).

However, the theoretical base for definition, delineation and interpretation of mapping units is not satisfactory. Only a few authors discuss the strict definition of landform segments and the minimisation of subjective factors in the segmentation process.

Yet the concept of a geomorphological information system (Dikau, 1993; Minár et al., 2005) requires strict definition of basic mapping units. Our aim is to provide this.
A large part of the work related explicitly to land surface segmentation deals with the definition of segment boundaries. Identification of a unit's boundary is the primary goal and the character of the interior may not influence the determination of its limits. This can be termed the graph-based approach (Brändli, 1996).

A theoretical application of the graph-based approach is the segmentation model of Lastoczkin (1987, 1991). He introduced the set of structural lines and characteristic points defining elementary surfaces.
The basic structural lines are singularities representing (on a profile) local extremes of altitude $z$ (ridge lines and valley lines); local extremes of slope gradient $z'$, the first derivative of altitude (inflection lines of maximum or minimum slope); and local extremes of profile curvature $z''$, the second derivative of altitude (convex and concave flexures involving breaks and changes of slope). Structural lines thus represent various kinds of discontinuities of morphometric properties. The basic structural lines are further classified on the basis of the linear, convex and concave shape of the profiles on either side.
<table>
<thead>
<tr>
<th>Lines of principal symmetry</th>
<th>Ridge and valley lines</th>
<th>Inflections</th>
<th>Convex and concave flexures</th>
</tr>
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<tbody>
<tr>
<td>Discontinuities of aspect</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>Discontinuities of curvature or its changes</td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>Lines of principal asymmetry</td>
<td>Discontinuities of aspect, slope and curvature</td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Fig. 2. Profiles across types of structural lines after Lastoczkin (1987), and their interpretation as lines of discontinuity.
A second major line of thought in land surface segmentation is the classification approach of Brändli (1996). This focuses on definition of the internal properties of elementary forms, from which the definition of boundaries follows. The simplest model is a 2×2 classification based on the signs of profile and plan curvature, giving four basic forms (Troeh, 1965).

| Table 1 |
|-----------------|-----------------|-----------------|-----------------|
| Local slope forms classified by curvature, profile first (varies down column), then plan (contour; varies along row), for the scheme of Richter (1962) [also Kecho (1983) and Dikau (1989)], that of Troeh (1965) and that proposed here in Fig. 5  |

<table>
<thead>
<tr>
<th>Troeh (1965)</th>
<th>Richter (1962) etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VX</td>
<td>VX</td>
</tr>
<tr>
<td>XX</td>
<td>SX</td>
</tr>
<tr>
<td>XX</td>
<td>XX</td>
</tr>
</tbody>
</table>

S=straight  
V=concave  
X=convex
Three types of relief unit can be distinguished on the basis of increasing complexity.

- Elementary forms represent the smallest and simplest units, which are indivisible at the resolution considered.
- Landforms that are composite forms create the second level of relief complexity.
- Characteristic patterns created by form associations provide a third level of complexity and are termed land systems.

Fig. 1. An example of elementary segment representation in a landform hierarchy.
We can identify three axioms, which create a theoretical base for land surface segmentation about continuity and discontinuity:

1) Land surface form can be analyzed as a continuum—the geometric field of altitude.

2) At a given scale, the land surface may, nevertheless, exhibit discontinuities; these may be recognized as natural boundaries of geomorphic objects.

3) These discontinuities and other characteristics of the land surface result from morphogenetic processes most of which are influenced by gravity.
Elementary forms

We can generalize the proposed model as follows. Land surface form consists of segments characterized by various types and degrees of homogeneity. These can ideally be expressed by constant values of altitude or its derived morphometric properties. Discontinuities of these properties provide logical boundaries to the segments.

Then we can define ideal elementary forms as landform elements with a constant value of altitude, or of two or more readily interpretable morphometric variables, bounded by lines of discontinuity. The constant values which define the elementary form are termed the form-defining properties.
Methods

- A new system: the concept of elementary forms

  - The basic morphometric system introduced by Evans (1972) and Krcho (1973) can be extended formally in the form (Minár, 1999): where $M$ is the set of all *local morphometric variables* (definable at every point of the and — cf. Shary et al., 2002).

\[
M = \left\{ (0) M = \{ z \}, (1) M = \{ z_i \}, (2) M = \{ z_{ij} \}, (3) M = \{ z_{ijk} \}, \ldots \right\}
\]
**Methods**

**n**=normal direction (of slopelines)

**t**=tangential direction (of contour lines)

**const**=constant
• Equations for ideal elementary forms

• 1) The functional dependence of altitude \((z)\) of a point with map coordinates \((x, y)\) on distance from any start point with altitude \(H\) and coordinates \((I, J)\): where \(a\) and \(b\) are constants and \((H, I, J)\) can be coordinates of a single start point or a point on a start contour lying on the same slopeline as the point \((z, x, y)\), generally describes forms with parallel contours.

\[
z = H + \sqrt{a(x - I)^2 + b(y - J)^2}
\]
2) A polynomial function of i-th order represents a constant value of the i-th derivative of altitude in the normal direction. Then the function:
where $\xi(x,y)$ expresses the shape of contours, generally describes forms with constant normal change of down-slope curvature ($G_{nn}$) in models with parallel contours.

$$z = H + B\xi + C\xi^2 + D\xi^3$$
3) Forms with divergent contours can be defined by functional dependence of altitude on angular coefficient: where F is a constant and p and q can be either constants or functions p(x,y) and q(x,y).

\[ z = H + F \arctan \frac{y-q}{x-p} \]

Proposed (in Fig. 5)

<table>
<thead>
<tr>
<th>Contour shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>SS</td>
</tr>
<tr>
<td>VS</td>
</tr>
<tr>
<td>XS</td>
</tr>
<tr>
<td>vxS</td>
</tr>
<tr>
<td>xvS</td>
</tr>
</tbody>
</table>

S=straight  
V=concave  
X=convex
Methods

General fitted function: 
\[ z = a + b \xi + c \xi^2 + d \xi^3, \] 
where \( \xi = \sqrt{(gx - l)^2 + (hy - J)^2} + \arctan \frac{y - q}{x - p} \)

- \( b, c, d = 0 \) 
  - \( z = \text{const} \)
  - \( I = hy, J = gx \Rightarrow \xi = \sqrt{2(gx + hy)} \)
  - \( A_N = \text{const} \)
  - \( R > 0 \)

- \( I, J = \text{const and } g, h = 1 \Rightarrow \xi = \sqrt{(x - l)^2 + (y - J)^2} \)
  - \( R_n = \text{const, } A_{nn} = 0 \)
  - \( R < 0 \)

- \( I = y_c + e \sin \varphi, J = g = 1 \Rightarrow \xi = \arctan \frac{y - q}{x - p} \)
  - \( R_n = 0, R_{nn} = \text{const} \)
  - \( A_{nn} = 0, A_N = 0 \)

- \( G_n = \text{const} \)
  - \( G_n > 0 \)
  - \( G_n < 0 \)
  - \( *G_n = 0 \)

- \( G_{nn} = \text{const} \)
  - \( G_{nn} > 0 \)
  - \( G_{nn} < 0 \)
  - \( *G_{nn} = 0 \)

\( z \) – altitude
\( G \) – slope gradient
\( A_N \) – aspect
\( G_n \) – normal change of gradient \( G \)
\( G_{nn} \) – normal change of \( G_n \)
\( A_{nn} \) – normal change of aspect
\( A_{nnn} \) – normal change of \( A_{nn} \)
\( A_N \) – plan curvature (tangential change of aspect)
\( R \) – radius of plan curvature
\( (1/A_N) \)
\( R_n \) – normal change of radius
\( R_{nn} \) – normal change of tangential change of the radius

\( x, y \) – cartographic planar coordinates
\( a, b, c, d, e, g, h \) – specific constants
\( I, J \) – specific constants or variables
\( x_0, y_0, \varphi = f(x, y) \) – specification of the central curve (clothoid in this case)
\( S \) – straight, \( X \) – convex,
\( V \) – concave
\( vx \) – concave-convex
\( xv \) – convex-concave
The system of elementary forms is fully open; adding to or modifying it depends only on our ability to formally express and effectively interpret more complex (higher order) types of elementary forms. Forms characterized by homogeneous derivatives of altitude in non-standard directions provide an example.

\[
z = H + B(y \pm \sqrt{R^2 - (x - m)^2})
\]

- \(R > 0\) \(R = \text{const.}, z_y = \text{const}\) \(R < 0\)

\[
z = H + B(x^2 - y^2)
\]

- \(z_{xx} = \text{const.}, z_{yy} = \text{const}\)

\[
z = H + B(x^3 - y^3)
\]

- \(z_{xxx} = \text{const.}, z_{yyy} = \text{const}\)

<table>
<thead>
<tr>
<th>Ridge</th>
<th>Valley bottom</th>
<th>Saddle</th>
<th>Landslide</th>
</tr>
</thead>
<tbody>
<tr>
<td>z = H + B(y ± √(R² - (x - m)²))</td>
<td>z = H + B(x² - y²)</td>
<td>z = H + B(x³ - y³)</td>
<td></td>
</tr>
</tbody>
</table>
Boundaries of elementary forms

In the geometrically ideal case, the boundary of two elementary forms defined by different constant values of some morphometric properties must be a line of discontinuity—a sudden discrete change of value of some property. The property value which jumps at the boundary can be termed the boundary-defining property. If the line is characterized by discontinuity of only one property, we call it a simple discontinuity line; otherwise, it is a compound discontinuity line.

Methods

- Boundaries of elementary forms

Simple discontinuity lines of:

- Altitude
- Gradient
- Profile curvature
- Plan curvature
- Change of profile curvature
- Change of plan curvature
- Compound slope and aspect discontinuity line
Boundaries of elementary forms

1) Discontinuities

For example, the boundary between a horizontal plane \((z=\text{const})\) and a linear slope \((G=\text{const})\) can be an altitude or slope discontinuity line but, as curvature is zero on both sides, it cannot be a discontinuity of curvature or change of curvature. Identity of the boundary-defining property with a form-defining property of higher order \((^{(r)}zi=^{(t)}zi)\) is the most stable situation, which we term a form-determined boundary.
## Methods

<table>
<thead>
<tr>
<th>Condition</th>
<th>Diagram 1</th>
<th>Diagram 2</th>
<th>Diagram 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G, A_N = \text{const}$</td>
<td><img src="image1" alt="Diagram 1" /></td>
<td><img src="image2" alt="Diagram 2" /></td>
<td><img src="image3" alt="Diagram 3" /></td>
</tr>
<tr>
<td>$G_n, A_N = \text{const}$</td>
<td><img src="image4" alt="Diagram 4" /></td>
<td><img src="image5" alt="Diagram 5" /></td>
<td><img src="image6" alt="Diagram 6" /></td>
</tr>
<tr>
<td>$G_{nn}, A_N = \text{const}$</td>
<td><img src="image7" alt="Diagram 7" /></td>
<td><img src="image8" alt="Diagram 8" /></td>
<td><img src="image9" alt="Diagram 9" /></td>
</tr>
<tr>
<td>$z = \text{const}$</td>
<td>$A_Nr, G \text{ discontinuity}$</td>
<td>$A_Nr, G_n \text{ discontinuity}$</td>
<td>$A_Nr, G_{nn} \text{ discontinuity}$</td>
</tr>
<tr>
<td>$R = \text{const}$, $z = \text{const}$</td>
<td>$R, G_n \text{ discontinuity}$</td>
<td>$R, R_n \text{ discontinuity}$</td>
<td>$A_{NNs}, G_{ts}, G_{nn} \text{ discontinuity}$</td>
</tr>
<tr>
<td>$G_{nn}, R_n = \text{const}$</td>
<td>$A_Nr, G, R \text{ discontinuity}$</td>
<td>$G, A_Nr, R, R_n \text{ discontinuity}$</td>
<td>$A_{NNs}, A_{NNs}, G_{ts}, G_{nn} \text{ discontinuity}$</td>
</tr>
</tbody>
</table>
In figure, discontinuities are regular boundaries of elementary forms, so one equation can describe three elementary forms. But the inflection line is not a discontinuity and so it should not be the boundary of an elementary form; an homogeneous convex–concave element extends on both sides of the inflection.

\[ z = 350 \sqrt{x^2 + y^2} - 40(x^2 + y^2) + (x^2 + y^2)^{3/2} \]
Boundaries of elementary forms

2) Contrast, smoothness and resolution

There are further relations between discontinuity lines and other singular lines. Discontinuity is an ideal geometric category and its relevance to real terrain depends on resolution. Greater resolution may transform a discontinuity of one property into an extreme line (maximum or minimum) of another property.
The number of lines has a tendency to rise with the order of the elementary form, complicating the situation. Another type of boundary transformation connected with change of resolution is where discontinuity lines change into elementary forms when viewed in greater detail.
Example

1. The most distinctive discontinuities

2. Subdivide protoform along a weaker (less sharp) discontinuity, if one exists

3. Acceptance of ‘substandard protoform’

4. Maximal affinity < Threshold limit of affinity
   
5. Maximal affinity > Threshold limit of affinity

6. ‘Protoform’ becomes the elementary form
DEM, grid mesh of 3.33m
11.111m² per cell

Contour interval 5 m

L:linear
C:circular
D:divergent
<table>
<thead>
<tr>
<th>Model</th>
<th>Slovínec (S1: 8089 m² = 28 x 26 = 728 cells)</th>
<th>Podhorské (P0: 12,000 m² = 36 x 30 = 1080 cells)</th>
<th>Sandberg (Sa: 7200 m = 27 x 24 = 648 cells)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μ</td>
<td>Mf</td>
<td>μ</td>
</tr>
<tr>
<td>0 — plain</td>
<td>1.02 m</td>
<td>0.59</td>
<td>2.30 m</td>
</tr>
<tr>
<td>L1</td>
<td>0.75 m</td>
<td>0.70</td>
<td>0.33 m</td>
</tr>
<tr>
<td>L2</td>
<td>0.66 m</td>
<td>0.76</td>
<td>0.33 m</td>
</tr>
<tr>
<td>L3</td>
<td>0.66 m</td>
<td>0.76</td>
<td>0.32 m</td>
</tr>
<tr>
<td>C1</td>
<td>0.43 m</td>
<td>0.83</td>
<td>0.28 m</td>
</tr>
<tr>
<td>C2</td>
<td>0.43 m</td>
<td>0.83</td>
<td>0.27 m</td>
</tr>
<tr>
<td>C3</td>
<td>0.40 m</td>
<td>0.84</td>
<td>0.26 m</td>
</tr>
<tr>
<td>D1</td>
<td>0.95 m</td>
<td>0.62</td>
<td>0.27 m</td>
</tr>
<tr>
<td>D2</td>
<td>0.82 m</td>
<td>0.67</td>
<td>0.26 m</td>
</tr>
</tbody>
</table>

Absolute mean deviation (μ) is computed from the volume difference between a ‘real segment’ (represented by DEM) and an ideal elementary form (computed by relations from Table 2), divided by segment area. Affinity of the real segment to an ideal elementary form model is expressed by the value of a membership function Mf defined by the relation $Mf = \frac{4\mu}{\bar{a} \cdot \tan \gamma_c}$, where $\bar{a}$ is mean length of the form in the direction of slopelines and $\gamma_c$ is a critical angle for distinguishing plain and slope (steeper segments must not be considered as plains; $\tan \gamma_c = 0.20$ in this case).
A major difference between this concept of elementary forms and some dynamically and genetically based approaches is the strict separation of elementary form delimitation and its subsequent dynamic and genetic characterization.

Most concepts of land units are not confined to purely morphometric variables but are based on visual interpretation from field survey or air photos. In land system analysis, aerial images are the main tools for landform recognition by numerous researchers.
The degree of affiliation of a real surface segment to an ideal elementary form can be expressed effectively by continuous (fuzzy) classification. This solves the classification problem of hierarchically inferior elementary forms; as a simpler elementary form is only a specific case of a more comprehensive higher-order form, it cannot approximate reality any better.

The membership function expresses the affinity of a segment to various geometric types of elementary form.
The concept of elementary forms exactly defines both area and boundary properties of basic landform segments that can serve as elementary geomorphic individuals.

The concept can be applied not only to geomorphological mapping, but also to synthetic landscape mapping, evaluation of natural hazards, carrying capacity, susceptibility and so on. It should be a basic part of a modern DEM-based geomorphological information system (cf. Minár et al., 2005), unifying these topics at a more synthetic level of scientific knowledge.
Thanks for your attention.