Analytical Solution for The Advection-Dispersion Transport Equation in Layered Media


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OUTLINE

- Introduction
- Methodology
- Results and Discussion
- Conclusions
INTRODUCTION
INTRODUCTION
$D_1, R_1, u_1, \mu_1, \varepsilon_1$

Layer 1

$D_2, R_2, u_2, \mu_2, \varepsilon_2$

Layer 2
<table>
<thead>
<tr>
<th></th>
<th>Dimension</th>
<th>Layer</th>
</tr>
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<tbody>
<tr>
<td>Al-Niami et al. (1979)</td>
<td>1</td>
<td>Single</td>
</tr>
<tr>
<td>Mikhailov et al. (1984)</td>
<td>1</td>
<td>Single</td>
</tr>
<tr>
<td>Leij et al. (1995)</td>
<td>1</td>
<td>Double</td>
</tr>
<tr>
<td>Liu et al. (1998)</td>
<td>1</td>
<td>Multiple</td>
</tr>
</tbody>
</table>
THE OBJECTIVE

Previous models were not efficient.

Most of previous researches considered single or double layers.

Present research developed a solution for multiple layers, and it is more efficient.
METHODOLOGY
CONCEPTUAL MODEL

\[ R_m \frac{\partial c_m}{\partial t} = D_m \frac{\partial^2 c_m}{\partial x^2} - u_m \frac{\partial c_m}{\partial x} - \mu_m c_m \quad x_{m-1} < x < x_m \]
\[ m = 1, 2, 3, \ldots, M \]

\[ c_m = c_{m+1}, m = 1, 2, \ldots, M - 1 \]
\[ k_m \frac{\partial c_m}{\partial x} = k_{m+1} \frac{\partial c_{m+1}}{\partial x} \]

Flow:
\[ u_1 c_1 - D_1 \frac{\partial c_1}{\partial x} = u_1 \bar{c}_0 \]

Layer 1

Layer 2

\[ x_0 \quad x_1 \quad x_2 \quad x_{M-1} \quad x_M \]

Layer M

\[ \frac{\partial c_M}{\partial x} = 0 \]
THE 1-D UNSTEADY A.D.E. IN FINITE COMPOSITE MEDIA OF M LAYERS

\[ R_m \frac{\partial c_m}{\partial t} = D_m \frac{\partial^2 c_m}{\partial x^2} - u_m \frac{\partial c_m}{\partial x} - \mu_m c_m \quad x_{m-1} < x < x_m \]

Absorption term Dispersion term Advection term Decay term

- \( c_m \): The concentration of solute in \( m \) layer \([ML^{-3}]\)
- \( R_m \): The retardation factor in \( m \) layer \([-]\)
- \( D_m \): The constant dispersion coefficient in \( m \) layer \([L^2T^{-1}]\)
- \( u_m \): The constant velocity in \( m \) layer \([LT^{-1}]\)
- \( \mu_m \): The decay constant in \( m \) layer \([T^{-1}]\)
RESULTS AND DISCUSSION
## VERIFICATION OF SOLUTION

<table>
<thead>
<tr>
<th>Case</th>
<th>Layer m</th>
<th>$u_m (cm/d)$</th>
<th>$D_m (cm^2/d)$</th>
<th>$\varepsilon_m$</th>
<th>$R_m$</th>
<th>$\mu_m (d^{-1})$</th>
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### CONVERGENCE TEST

(Liu et al., 1998)

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<th>( t = 0.2 ) day</th>
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</table>

<table>
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<tr>
<th>Case</th>
<th>x (cm)</th>
<th>( t = 0.2 ) day</th>
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<tbody>
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### Case 2

<table>
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<th>( n = 60 )</th>
<th>( n = 120 )</th>
<th>( L^{-1} )</th>
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<tbody>
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<td>0.000</td>
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</tbody>
</table>
TWO-LAYER MEDIA RESULTS

\[ D_1 = 50 \text{(cm}^2/\text{d}) \]
\[ R_1 = 3 \]
\[ u_1 = 25 \text{(cm/d)} \]
\[ \mu_1 = 3 \text{(d}^{-1}) \]
\[ \varepsilon_1 = 0.4 \]

\[ D_2 = 20 \text{(cm}^2/\text{d}) \]
\[ R_2 = 2 \]
\[ u_2 = 40 \text{(cm/d)} \]
\[ \mu_2 = 4 \text{(d}^{-1}) \]
\[ \varepsilon_2 = 0.25 \]

Symbol: Numerical
Line: Analytical

\[ t = 0.2d \]
\[ t = 0.4d \]
\[ t = 0.6d \]
\[ t = 0.8d \]
\[ t = \infty \]
FIVE-LAYER MEDIA

$Liu et al., 1998$

$D = 7\,(cm^2/d), R = 4.25, u = 10\,(cm/d), \mu = 0\,(d^{-1}), \varepsilon = 0.4$

$D = 18\,(cm^2/d), R = 14, u = 8\,(cm/d), \mu = 0\,(d^{-1}), \varepsilon = 0.5$
FIVE-LAYER MEDIA RESULTS

(Present research)

t = 2d  

\( t = 10d \)

\( t = 6d \)

(Liu et al., 1998)

\( t = 2d \)

\( t = 10d \)

\( t = 6d \)
CONCLUSIONS
CONCLUSIONS

1. Developed a closed-form analytical solution of the transient, one-dimensional advection–dispersion transport equation with first-order decay for multi-layered media.

2. The performance of the present analytical solution had faster convergence.
THANKS FOR YOUR ATTENTION!