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NATIONAL CENTRAL UNIVERSITY - COLLEGE OF EARTH SCIENCES
GRADUATE INSTITUTE OF APPLIED GEOLOGY

Probabilistic characterization of subsurface stratigraphic configuration with modified random field approach

Chao Zhao, Wenping Gong, Tianzheng Li, C. Hsein Juang, Huiming Tang, Hui Wang, 2021
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CONTENT

1. INTRODUCTION

In Geotechnical engineering, petroleum, and mining engineering works

- Site characterization and stratigraphic uncertainty is very important
- Geological model often limit the number of borehole due to budget
- The derived stratigraphic configurations and geo-material properties could be highly uncertain, especially at a complex deposits site

To solve this problem → The probabilistic modeling approaches, based on the coupled Markov chain (CMC) & stochastic Markov random field → The reliability of 2 models depends on the engineer's experience

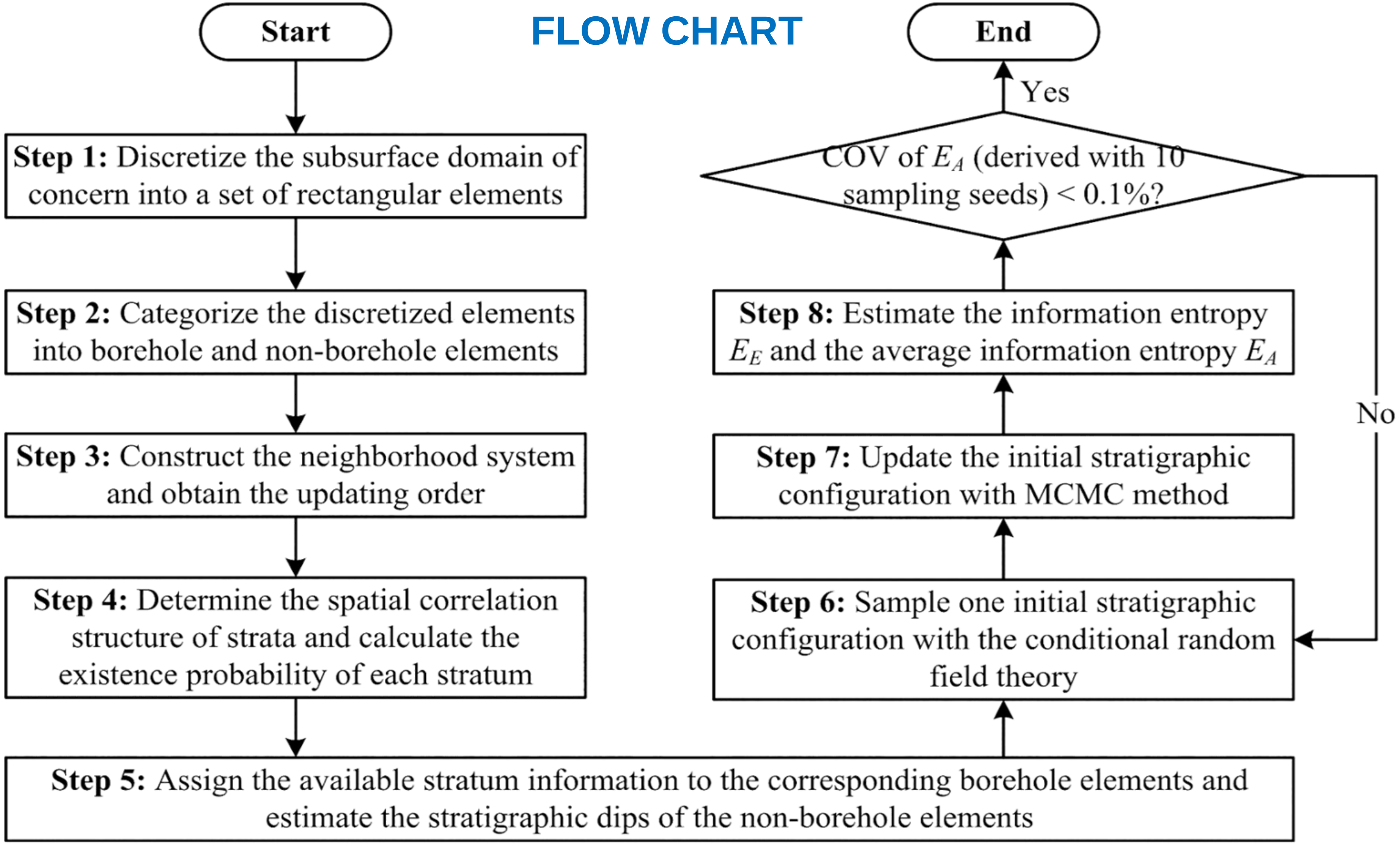
Limitation above ---> Proposed a modified random field-based approach for characterizing stratigraphic configuration and its uncertainty

CONTENT

2. METHODOLOGY

S1. Procedures for implementing the proposed method

FLOW CHART



Equations of Step 8

$$E_E(i) = -\sum_{k=1}^{k=m}[p_k(i)logp_k(i)]$$

$$E_A = \frac{1}{n_e - n_B}\sum_{i=1}^{i=n_e-n_B}[E_E(i)]$$

In which:

n_e number of elements discritized in the subsurface domain

n_B number of borehole elements

m number of strata

2. METHODOLOGY

S2. Sampling of initial stratigraphic configurations with conditional random field theory

$$\rho(i,j) = exp\left(-\frac{\pi d_H^2}{I_H{}^2} - \frac{\pi d_V^2}{I_V{}^2}\right)$$

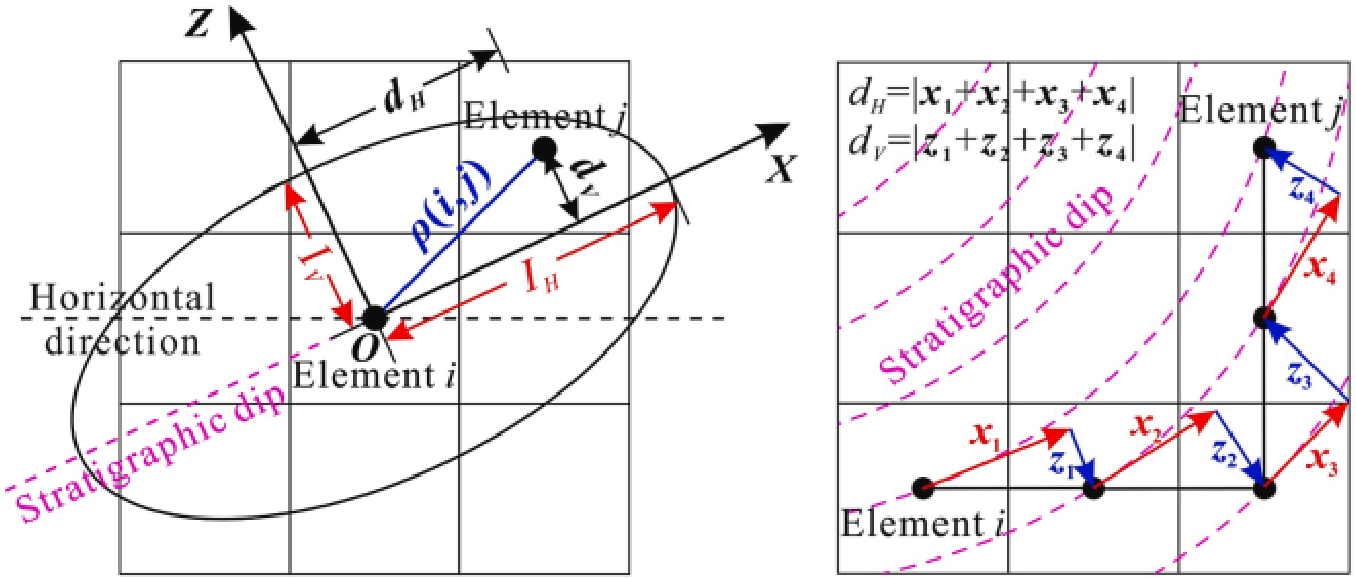
SQX autocorrelation function

$$P_k(i) = \frac{\rho_k(i)}{\rho(i)} = \frac{\sum_{l=1}^{l=n_B}[\rho_k(i,l)\cdot Index(l,k)]}{\sum_{h=1}^{h=m}\left\{\sum_{l=1}^{l=n_B}[\rho_h(i,l)\cdot Index(l,h)]\right\}}$$

The probability of the existence of a particular stratum k in an element i

$$u_c(i) = \left(\sum_{k=1}^{k=h-1} P_k(i) + \sum_{k=1}^{k=h} P_k(i)\right) \Big/ 2$$

The random number was generated with conditional random field



Where:

I_H and I_V are the scales of fluctuation that are parallel and perpendicular to the stratigraphic dip

d_H and d_V are the center-to-center distances between element i and element j

m the number of the strata

n_B the number of the boreholes

h the stratum label

2. METHODOLOGY

S3. Estimate of the spatial correlation of the strata with maximum likelihood principle

Find : $\boldsymbol{\theta}^{\mathrm{T}} = \{\boldsymbol{\phi}, I_H, I_V\}$

Subject to : $\mathbf{D}^{\mathrm{T}} = \{D_1, D_2, \ldots, D_{n_d}\}$

$$L(\mathbf{D}|\boldsymbol{\theta}) = \frac{1}{(2\pi)^{n_d/2}|\mathbf{C_D}|^{1/2}} exp\left[-\frac{1}{2}\left(\mathbf{D}-\overline{\mathbf{D}}\right)^{\mathrm{T}}\mathbf{C_D^{-1}}\left(\mathbf{D}-\overline{\mathbf{D}}\right)\right]$$

In which:

θ is the spatial correlation structure

ϕ is the type of autocorrelation function such as SQX, SNX, SMK

I_H and I_V are the related horizontal and vertical scales of fluctuation

D the stratum labels observed

D_s is the stratum label

nd the number of observations

L(D|θ) the likelihood of observing strata D given the spatial correlation structure θ

D represents the means of the observations

C_D represents the covariance matrix between observations D

2. METHODOLOGY

S4. Derivation of final stratigraphic configurations using the Markov Chain Monte Carlo method

To remove the potential local anomalies ---> using algorithm based on the MCMC method (the maximum-a-posteriori (MAP) estimates are derived based on the lowest posterior energy principle)

$$U\left(\boldsymbol{\omega}', \boldsymbol{\omega}^{0}\right) = \alpha \sum_{i \in \mathbf{E}} V_i\left(\omega_i' | \boldsymbol{\omega}^{0}\right) + \sum_{i \in \mathbf{E}} \sum_{j : j \in \mathbf{N}_i} V_c\left(\omega_i', \omega_j'\right)$$

$$P\left(\omega_i' | \boldsymbol{\omega}_{\mathbf{N}_i}', \boldsymbol{\omega}^{0}\right) = \frac{1}{Z_i} exp\left(\left(\alpha V_i\left(\omega_i' | \boldsymbol{\omega}^{0}\right) + \sum_{j : j \in \mathbf{N}_i} V_c\left(\omega_i', \omega_j'\right)\right) \Big/ T\right)$$

In which:

- Posterior energy $U(\omega', \omega^0)$
- Posterior conditional probabilities $P(\omega_i' | \omega_{Ni}', \omega^0)$
- $V_c(\omega i', \omega j')$ the prior potential function of the element pair (ωi', ωj')
- $V_i(\omega i' | \omega^0)$ represents the likelihood function
- c is a pair of the neighboring elements
- ω'_{Ni} the set of strata that are assigned to the elements of Ni
- Z_i local partition function
- α represents a smoothing factor

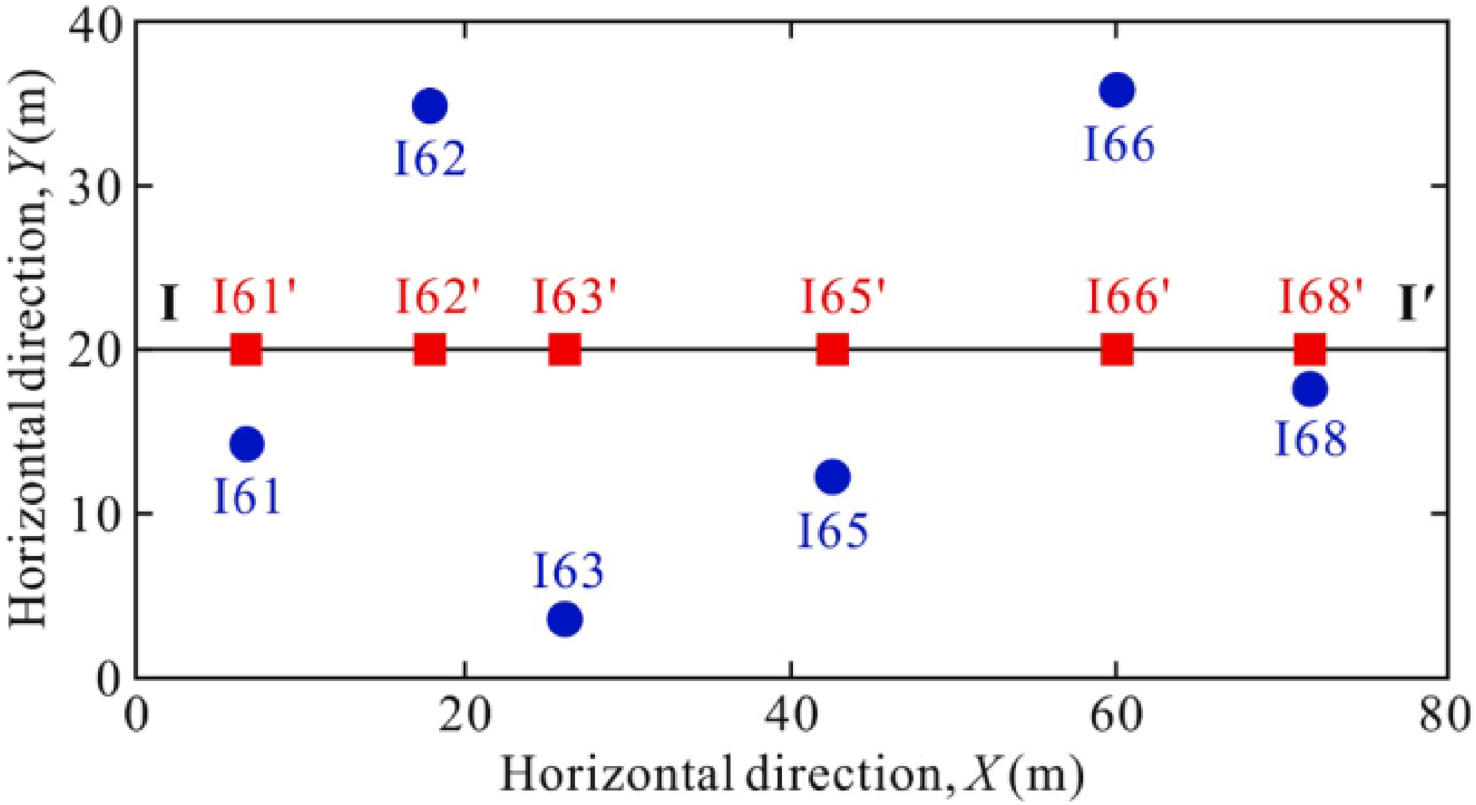
CONTENT

1. INTRODUCTION
2. METHODOLOGY
3. RESULTS AND DISCUSSION
4. CONCLUSIONS
5. FUTURE WORKS

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation

To demonstrate the applicability and effectiveness of the proposed method, we use the borehole data collected in the Central Business District, Perth, Western Australia.



Relative location of the boreholes

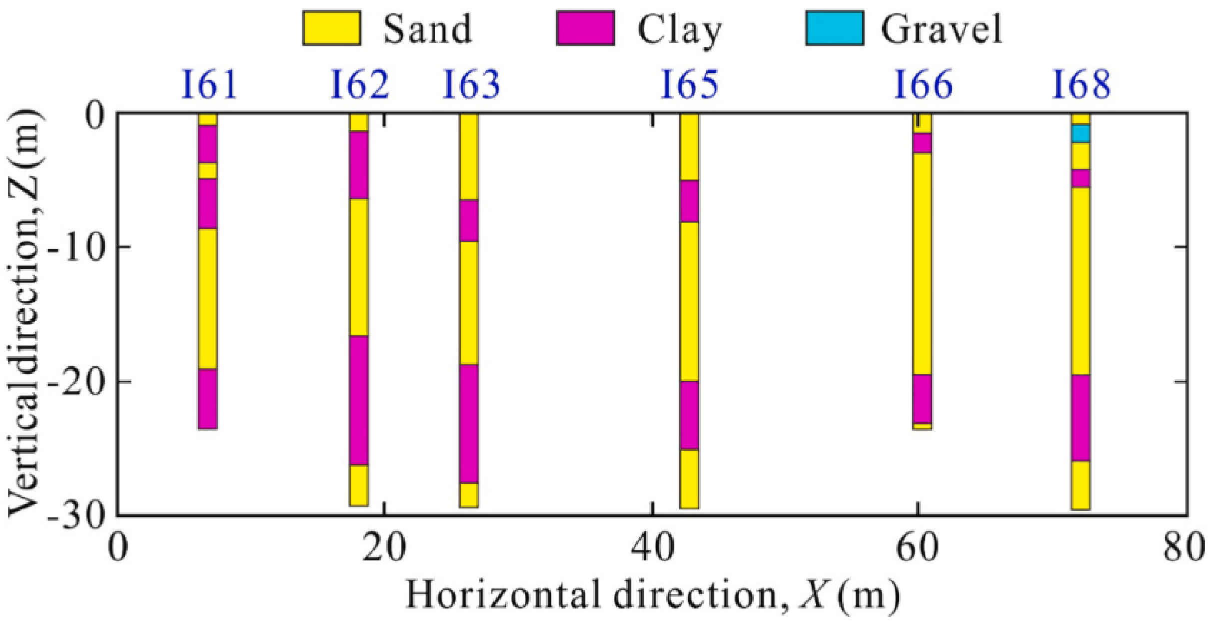
6 boreholes: I61, I62, I63, I65, I66, and I68 ---> projected to axis I-I′

To determine characterizing the subsurface stratigraphic configuration

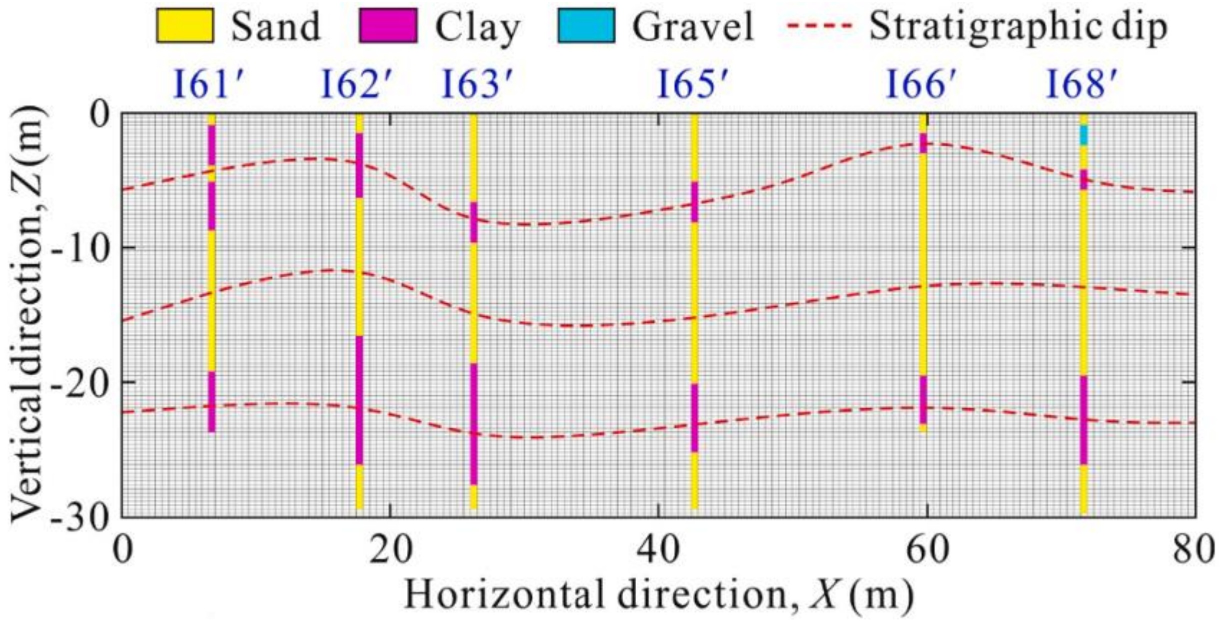
The projected boreholes are re-labeled as borehole I61′, I62′, I63′, I65′, I66′, and I68′

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation



Strata revealed at the borehole locations



Discretization of the subsurface domain (cross-section I-I′)

We create 16,000 elements, with size of 0.5m (horizontal) × 0.3m (vertical)

Non-borehole elements are obtained with the Kriging interpolation method (derived with the MLE)

The stratigraphic dips, represented by a set of red curves

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation

- To determine the strata's spatial correlation structure in this 2-D, we use three types functions: SQX, SNX & SMK

$$\rho(i,j) = exp\left(-\frac{\pi d_H^2}{{I_H}^2} - \frac{\pi d_V^2}{{I_V}^2}\right)$$

$$\rho_{\mathrm{SNX}}(i,j) = exp\left(-\frac{2d_H}{I_H} - \frac{2d_V}{I_V}\right)$$

$$\rho_{\mathrm{SMK}}(i,j) = \left(1+\frac{4d_H}{I_H}\right)\left(1+\frac{4d_V}{I_V}\right)exp\left(-\frac{4d_H}{I_H} - \frac{4d_V}{I_V}\right)$$

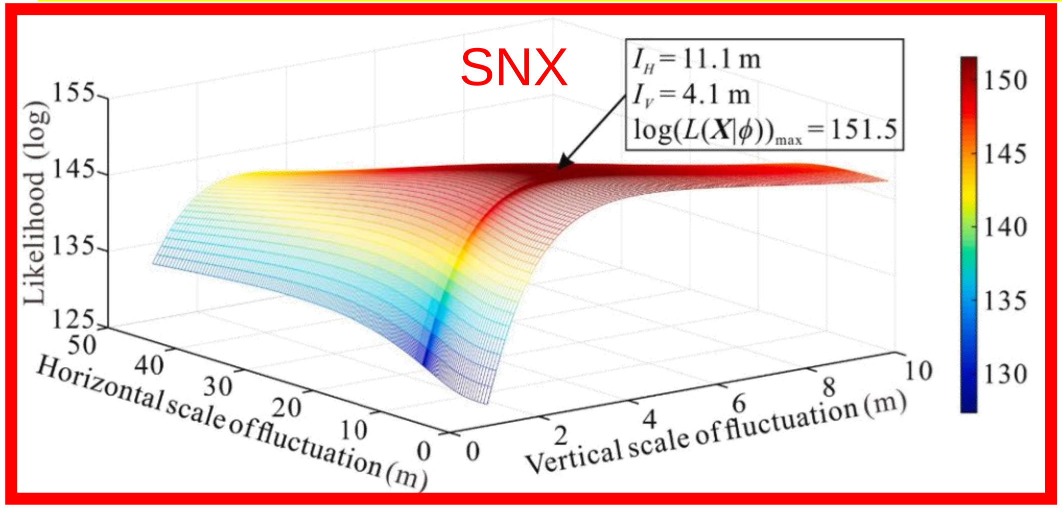
In which:

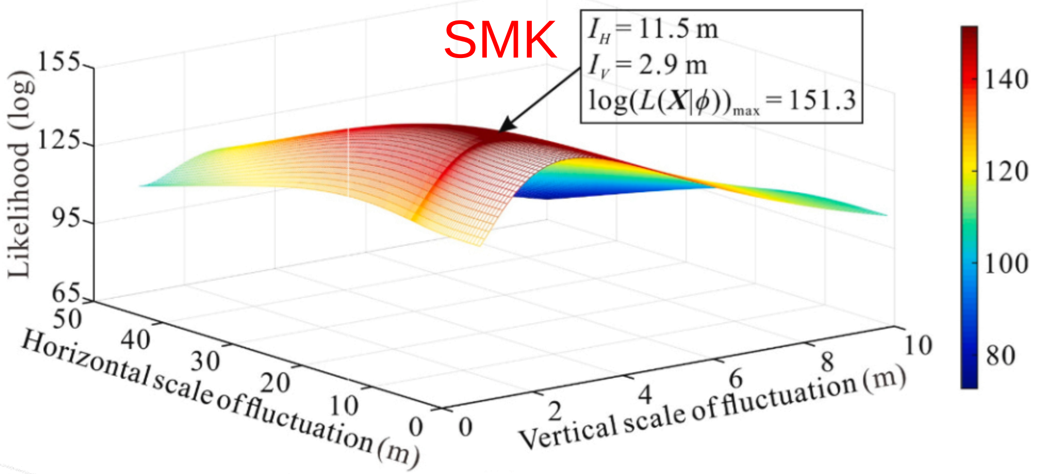
I_H and I_V are the related horizontal and vertical scales of fluctuation

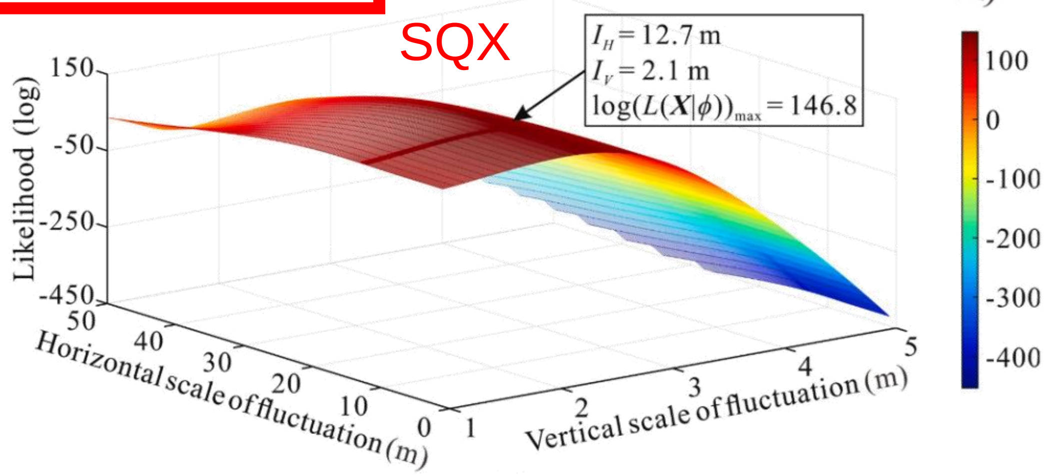
d_H and d_V are the center-to-center distances between element i and element j

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation



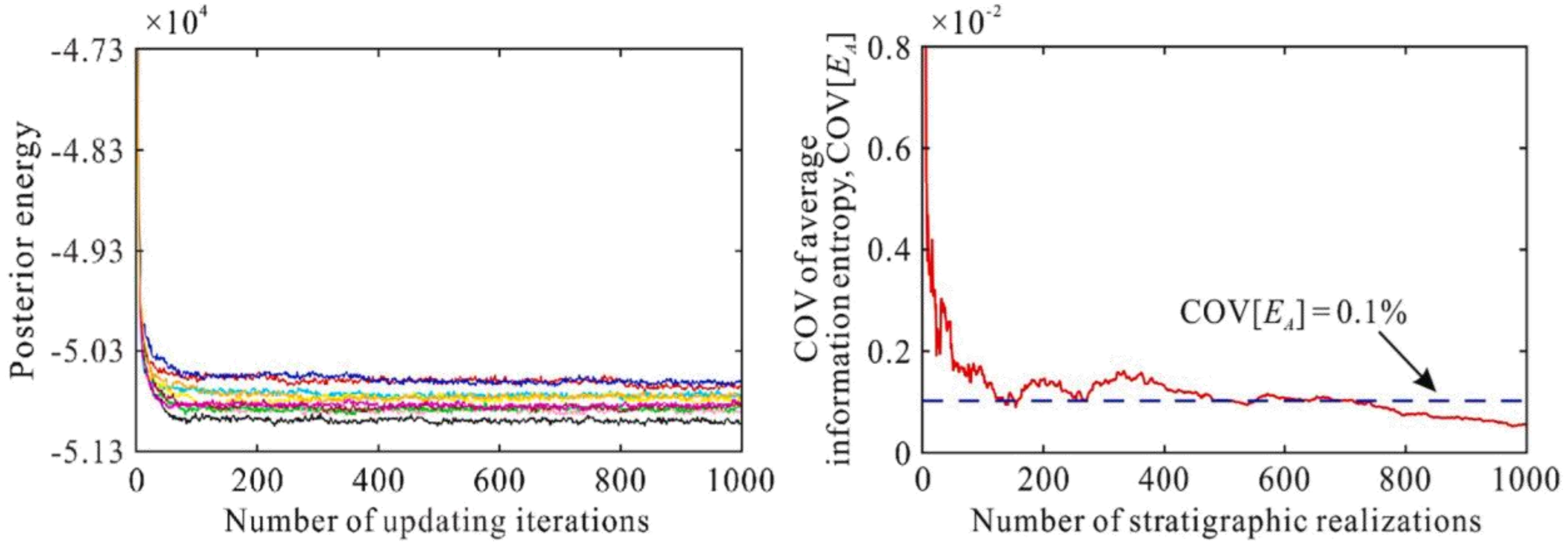




Influences of the type of autocorrelation function and scales of fluctuation on the estimated likelihood of the observations

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation

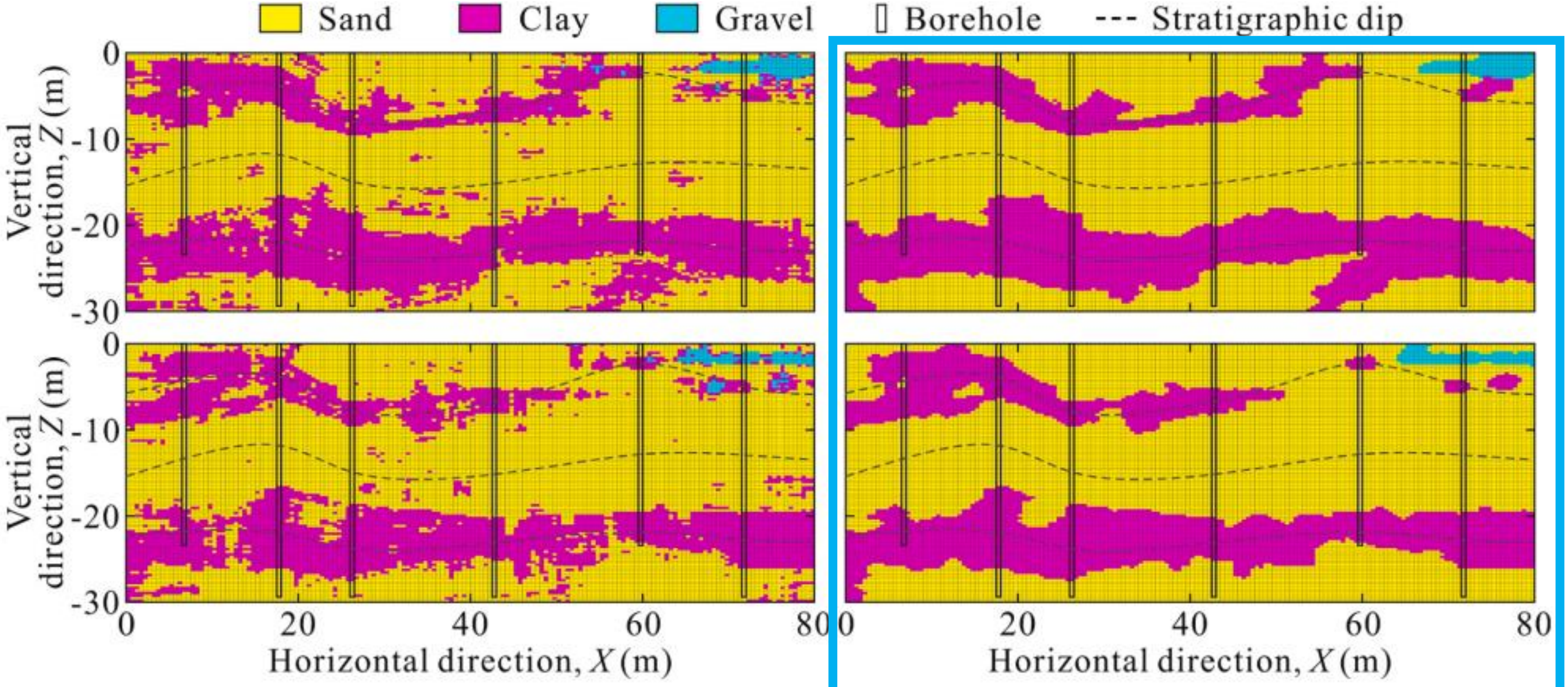


Determination of the **number of iterations** adopted in the MCMC updating and **the number of sampled stratigraphic realizations**

- We adopt the following simulation parameters: 200 iterations in the MCMC updating and 800 final stratigraphic configurations.

3. RESULTS AND DISCUSSION

3.1. The proposed method for 2-D stratigraphic configuration characterazation



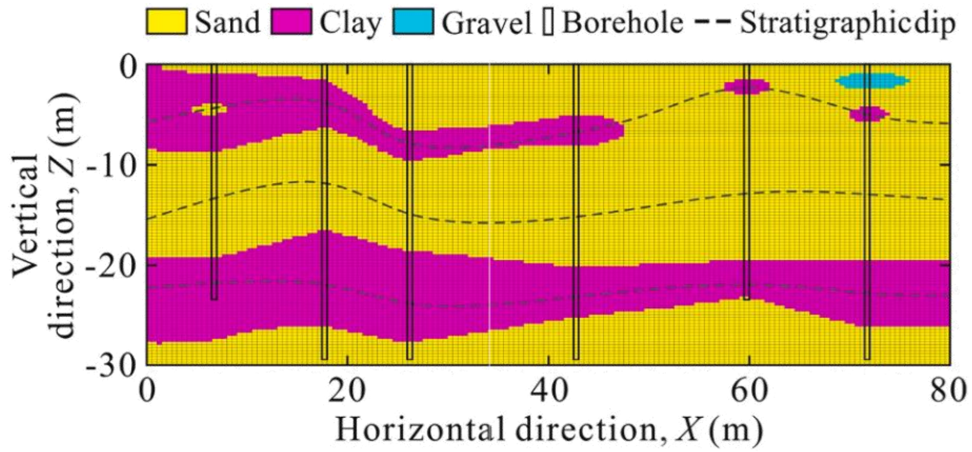
a) Initial stratigraphic configurations

b) Corresponding MAP estimates

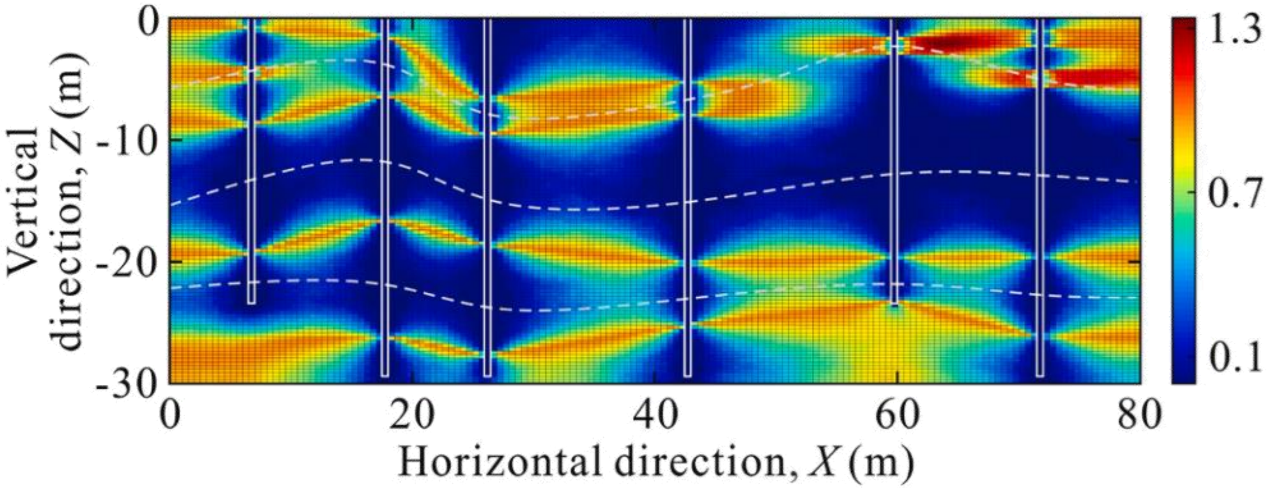
Two possible stratigraphic realizations generated with the new approach

3. RESULTS AND DISCUSSION

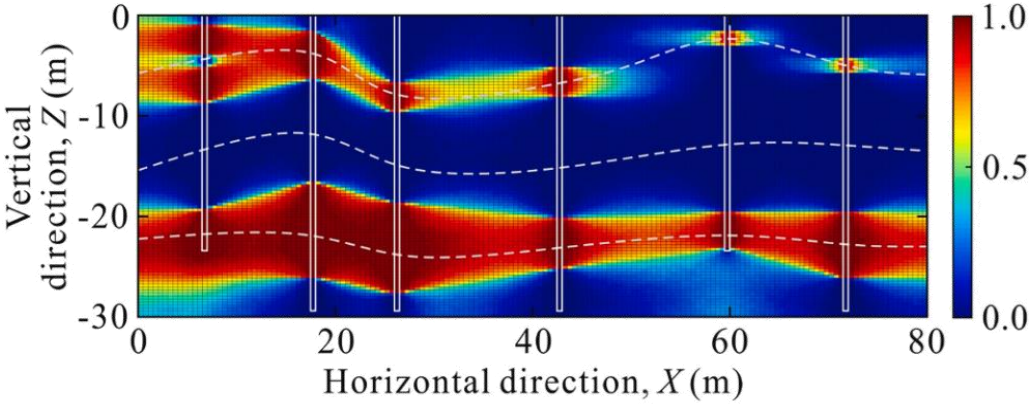
3.1. The proposed method for 2-D stratigraphic configuration characterazation



a) Most probable stratigraphic configuration



b) Spatial distribution of the modeled information entropy



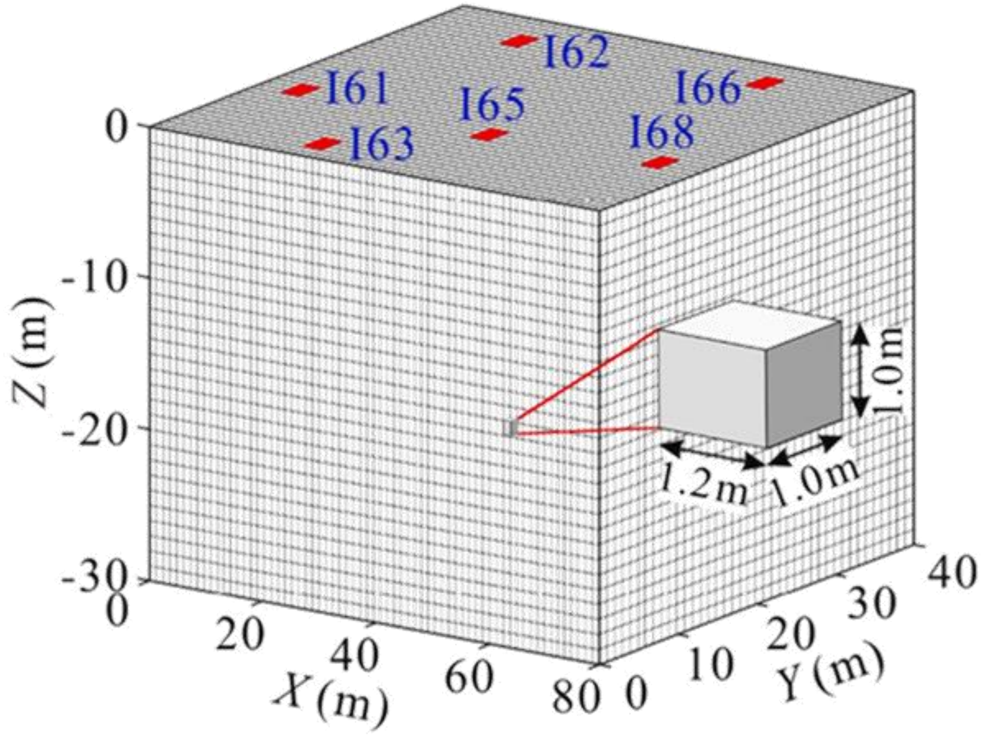
c) Spatial distribution of the probability of the existence of clay

The results of the stratigraphic and its uncertainty are consistent with state of knowledge, and the effectiveness of the proposed method was demonstrated.

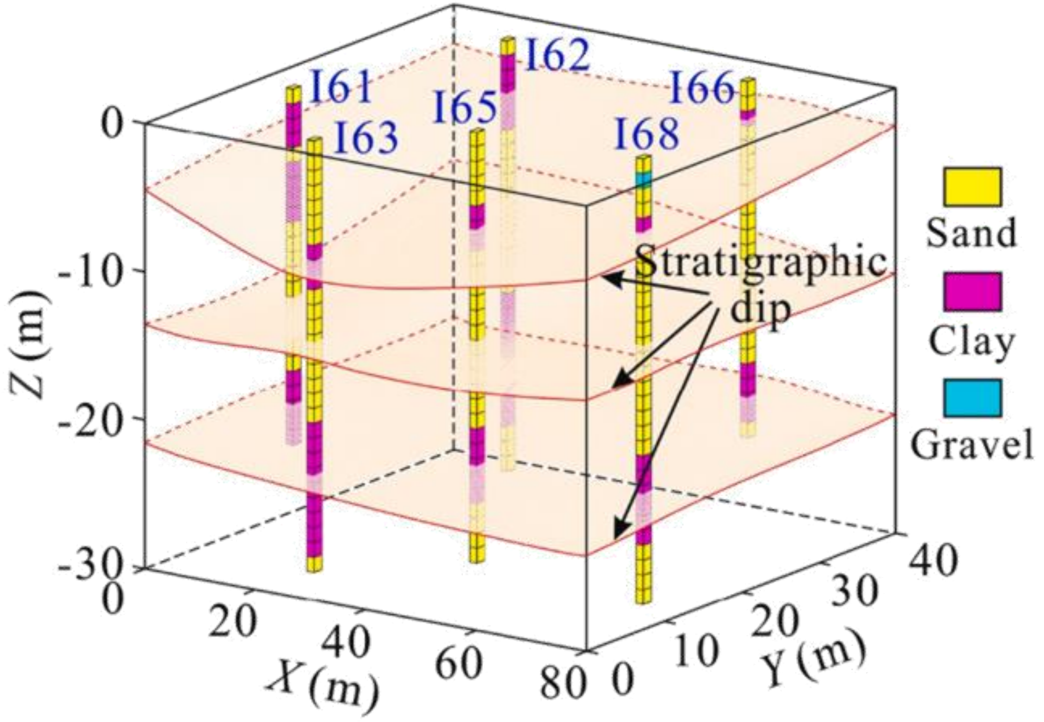
Stratigraphic uncertainty modeling results with the new approach

3. RESULTS AND DISCUSSION

3.2. The proposed method for 3-D stratigraphic configuration characterazation



(a)



(b)

a) 3-D dimension: 80m x 40 m x 30 m
79.200 cuboid elements

b) The borehole stratigraphies along with the assumed stratigraphic dip information

Borehole exploration program and borehole stratigraphies of the 3-D example site in Western Australia: Mesh scheme (left) Borehole exploration program (right)

3. RESULTS AND DISCUSSION

3.2. The proposed method for 3-D stratigraphic configuration characterazation

- To determine the strata's spatial correlation structure in this 3-D, we use three types functions: SQX, SNX & SMK (Vanmarcke, 1988; Li, 2017)

$$\rho_{SQX,3D}(i,j) = exp(- \frac{\pi\, d_{H1}^2}{I_{H1}^2} - \frac{\pi\, d_{H2}^2}{I_{H2}^2} - \frac{\pi\, d_{V}^2}{I_{V}^2})$$

$$\rho_{\mathrm{SNX,3\text{-}D}}(i,j) = exp\left(-\frac{2d_{H1}}{I_{H1}} - \frac{2d_{H2}}{I_{H2}} - \frac{2d_{V}}{I_{V}}\right)$$

$$\rho_{\mathrm{SMK,3\text{-}D}}(i,j) = \left(1+\frac{d_{H1}}{I_{H1}}\right)\left(1+\frac{d_{H2}}{I_{H2}}\right)\left(1+\frac{d_{V}}{I_{V}}\right) exp\left(-\frac{d_{H1}}{I_{H1}} - \frac{d_{H2}}{I_{H2}} - \frac{d_{V}}{I_{V}}\right)$$

In which:

I_H and I_V are the related horizontal and vertical scales of fluctuation
d_H and d_V are the center-to-center distances between element i and element j

3. RESULTS AND DISCUSSION

3.2. The proposed method for 3-D stratigraphic configuration characterazation

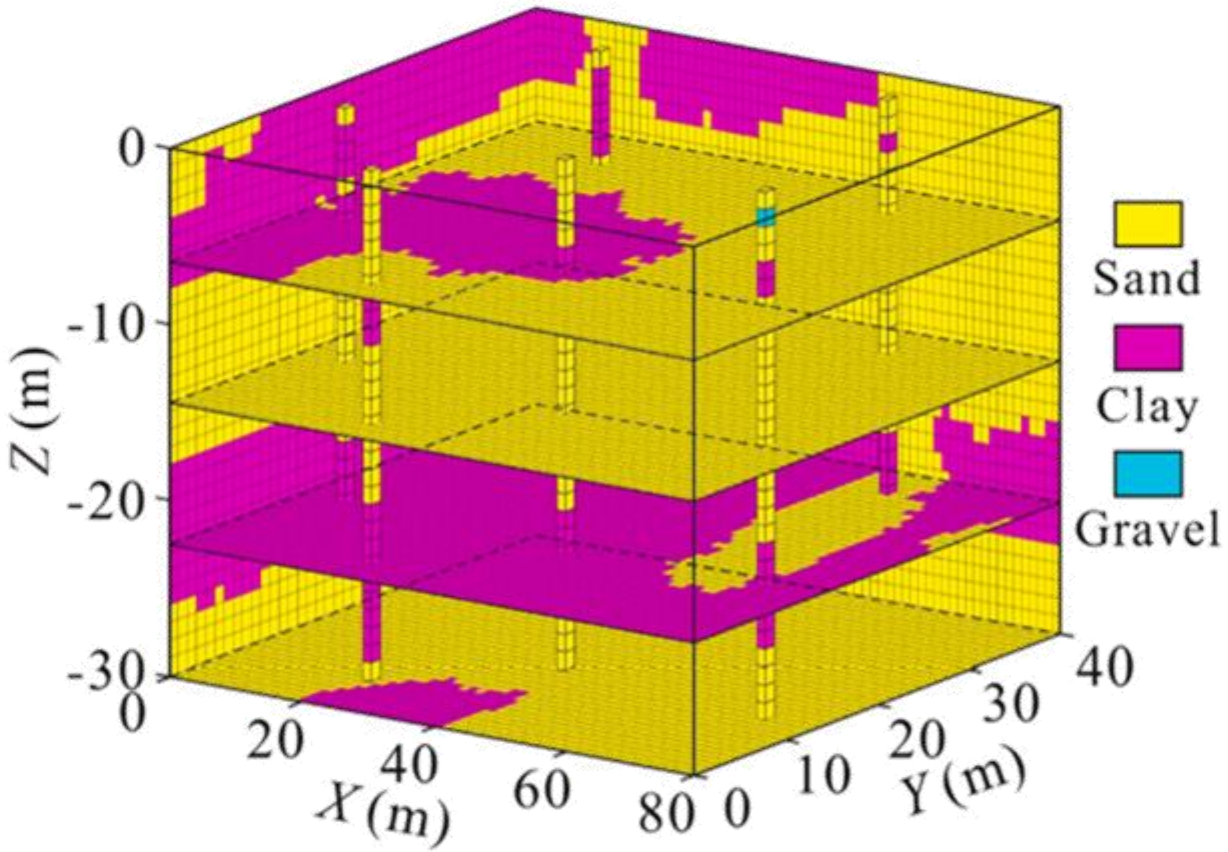
- The maximum likelihood analysis indicates that the spatial correlation structure of the strata at this 3-D site is well captured by the SNX autocorrelation function.

- The estimated scales of fluctuation are l_{H1} = 41.8 m, l_{H2} = 28.4 m, and l_{V} = 3.8 m

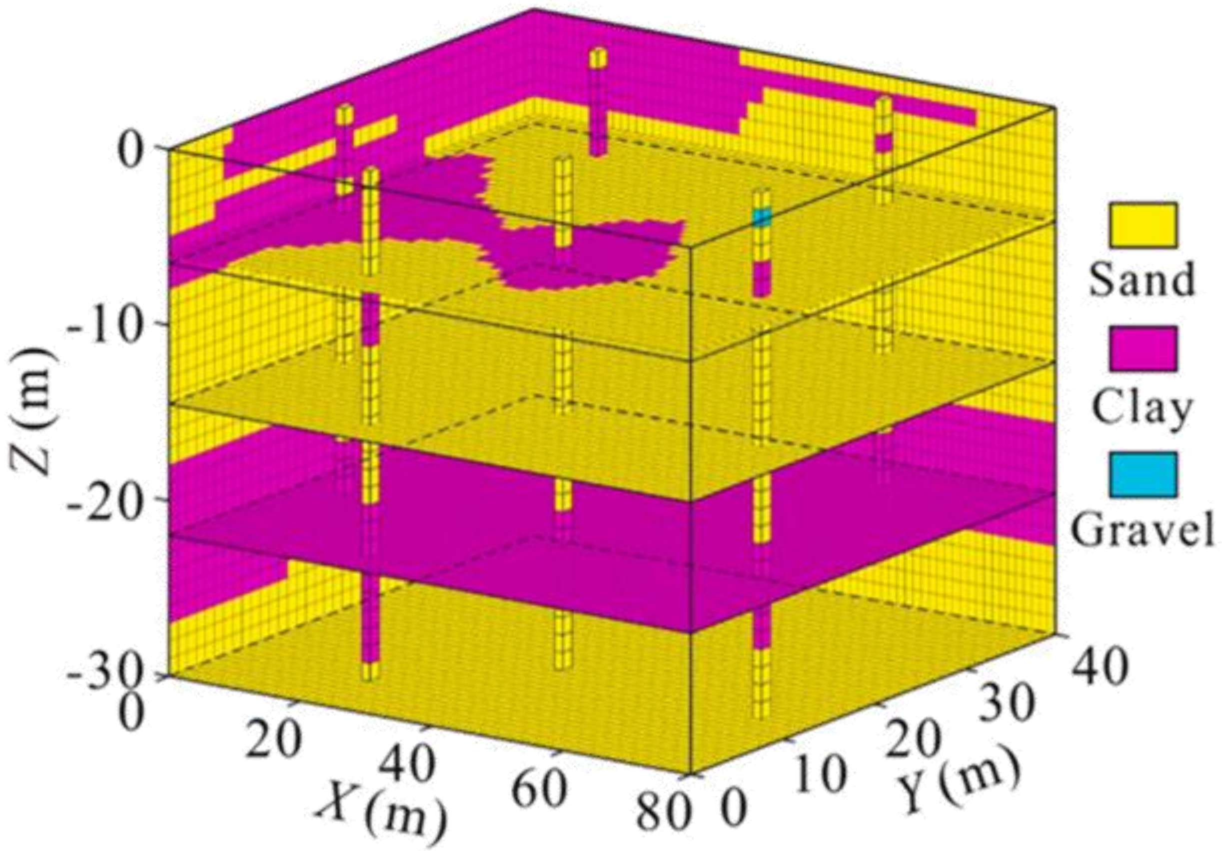
- We adopt the following simulation parameters: 200 iterations in the MCMC updating and 1200 final stratigraphic configurations.

3. RESULTS AND DISCUSSION

3.2. The proposed method for 3-D stratigraphic configuration characterazation



a) One possible stratigraphic realization

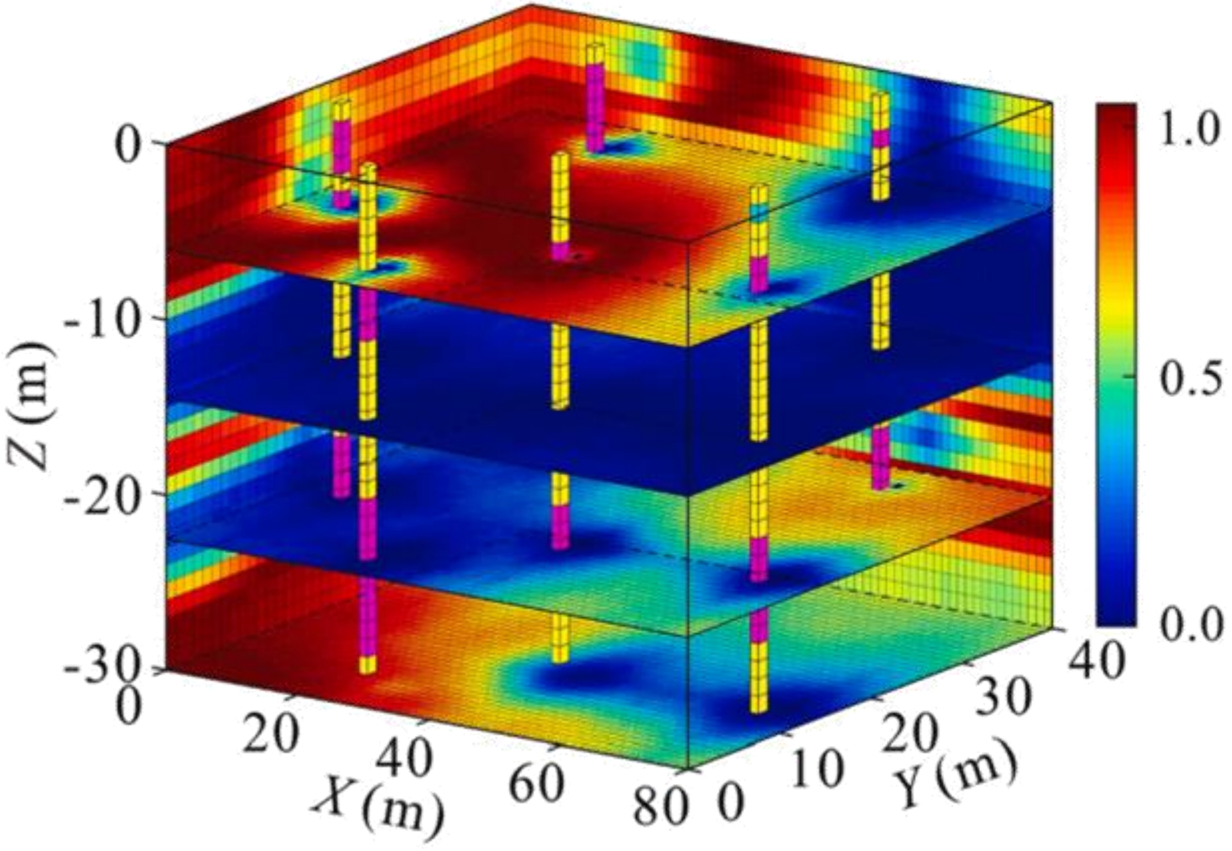


b) Most probable stratigraphic configuration

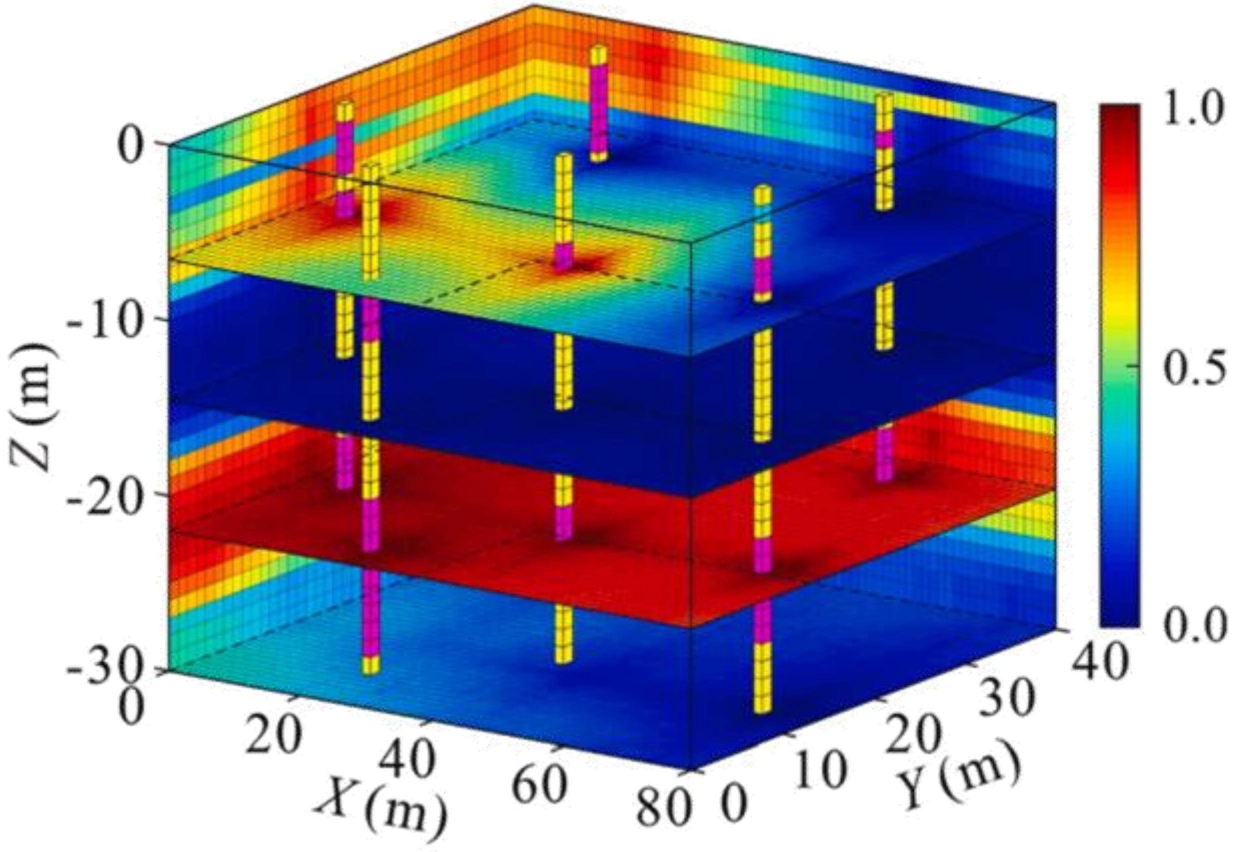
Stratigraphic uncertainty modeling results of the 3-D example site in Western Australia with the new method

3. RESULTS AND DISCUSSION

3.2. The proposed method for 3-D stratigraphic configuration characterazation



c) Spatial distribution of the modeled information entropy



d) Spatial distribution of the probability of the existence of clay

Stratigraphic uncertainty modeling results of the 3-D example site in Western Australia with the new method

CONTENT

4. CONCLUSIONS

- According to the results 2-D and 3-D of the paper, the author given a modified approach to overcome the limitations of the existing random field-based approach for characterizing the subsurface stratigraphic configuration and its uncertainty.

- The paper proposed 3 features to improve the existing random field-based approach.

- The proposed method is superior to the CMC, MRF, and random field-based approaches: the results is more consistent with the stratigraphic dips and the strata boundaries are smoother.

CONTENT

5. FUTURE WORKS

"Calibrating the parameters of MRF in the Taipei basin via the maximum likelihood concept"

- Choose the site in Taipei basin with maybe over 100 boreholes.
- Determine the soil layer of the boreholes.
- Using the random field to generate the model.

THANK
YOU

APPENDIX OF THIS REPORT

The limitations of random field-based approach in the past

1. Extensive manual interventions

2. Approaches developed in petroleum and mining engineering cannot be directly applied to the geotechnical engineering *(geological model)*

3. Geological heterogeneity

2020b). The early attempt in the characterization of the stratigraphic uncertainty was reported by Evans (1982), Tang et al. (1989), Halim (1991), Hansen et al. (2007), and De Marsily et al. (2005). However, broader applications of these geostatistical methods were hindered by the difficulty in determining the spatial correlation structures of geological heterogeneity (Carle, 2000; Caers and Zhang, 2004). To overcome this obstacle, the probabilistic modeling approaches, based on the coupled Markov chain (CMC) (Carle, 2000; Hu and Huang, 2007; Deng et al., 2017; Li et al., 2019) and stochastic Markov random field (MRF) (Norberg et al., 2002; Li et al., 2016c; Wang et al., 2018; Wang, 2020), have recently been proposed. Furthermore, Crisp et al. (2019)

data (e.g., geological setting, borehole data, seismic data, and gravity data) are integrated (Caumon et al., 2009; Wu et al., 2015; Schweizer et al., 2017). The probabilistic approaches that are based on the Bayesian inference (Arnold et al., 2013&2019) and support vector machine (Jung et al., 2018) have been recently developed to characterize the stratigraphic uncertainty in petroleum and mining engineering. A potential limitation of these probabilistic approaches is that extensive manual interventions are required (Wu et al., 2015; Arnold et al., 2019). Furthermore, the approaches developed in petroleum and mining engineering cannot be directly applied to the geotechnical engineering. For example, the data involved in the geotechnical practice are often limited.