The effects of inherent distribution of discontinuities and stress-induced anisotropy on pore water pressure distribution of rock slope.

Presenter: Chia-Yi Liu
Advisor: Jia-Jyun Dong
Date: 2022/09/30

Influence of geological conditions on stability of rock cuts

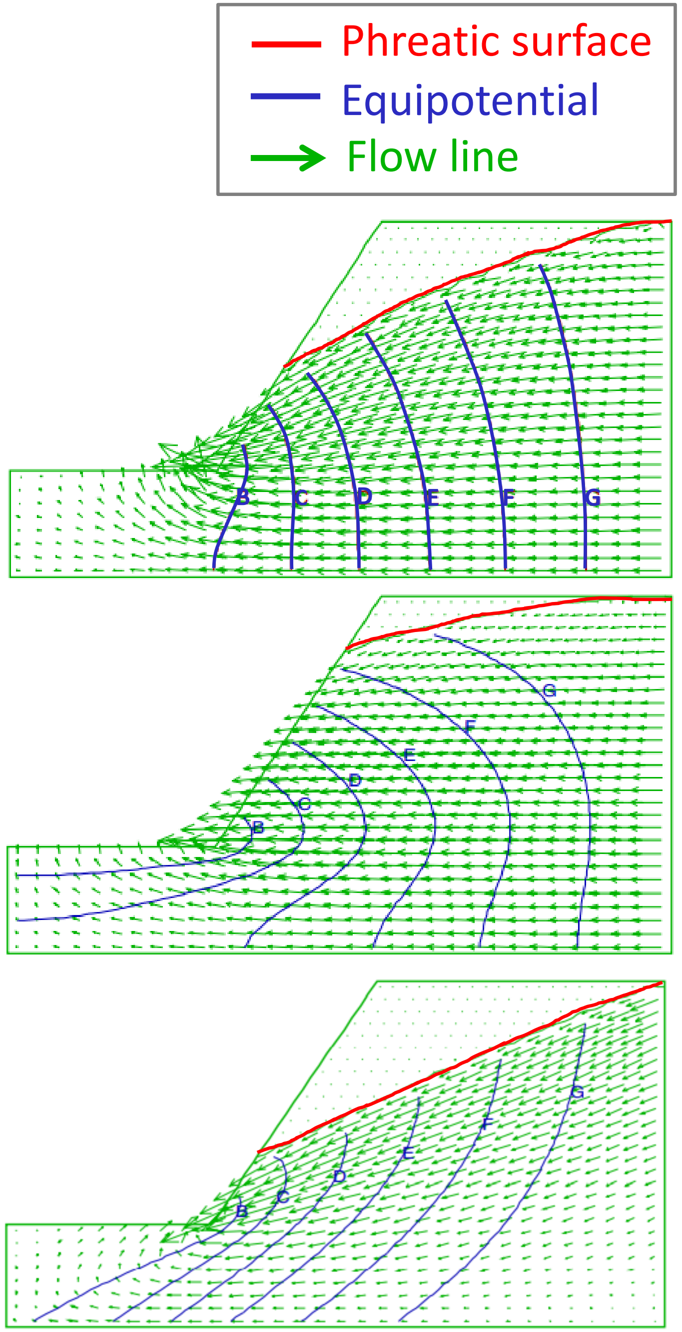
The distribution of discontinuities

(Duncan & Christopher, 2004)

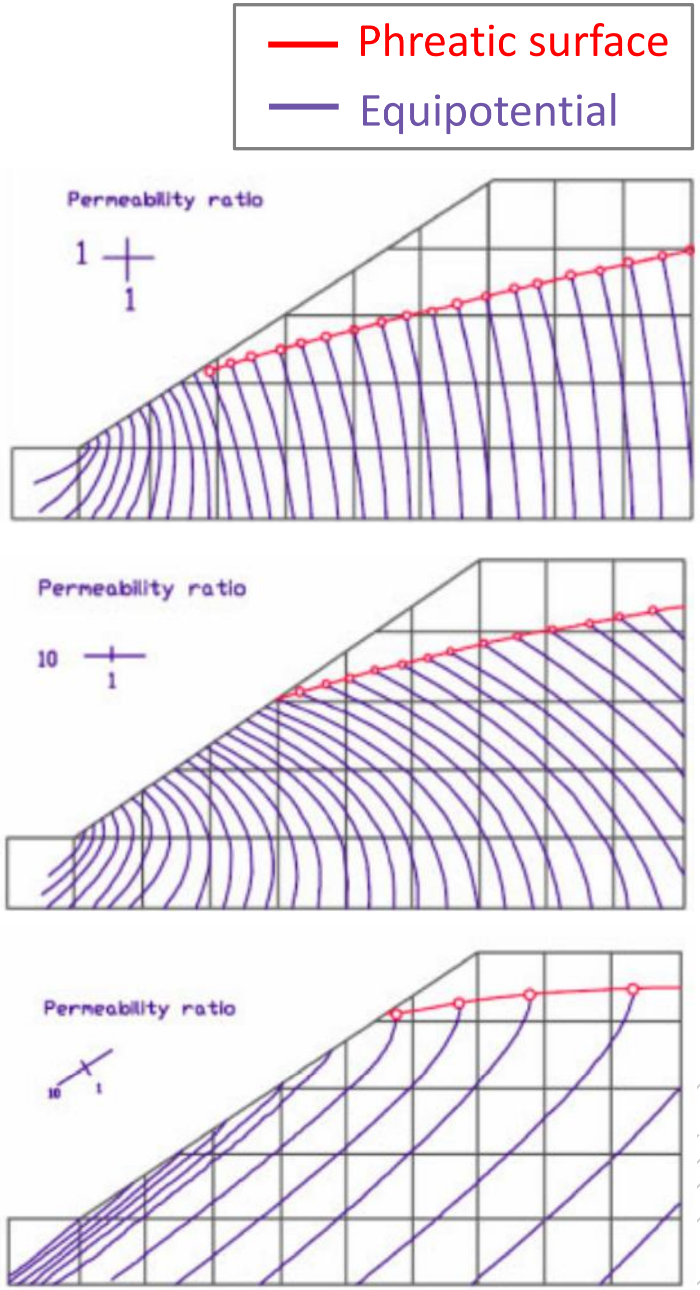
The distribution of discontinuities

The anisotropy of permeability

Slope stability

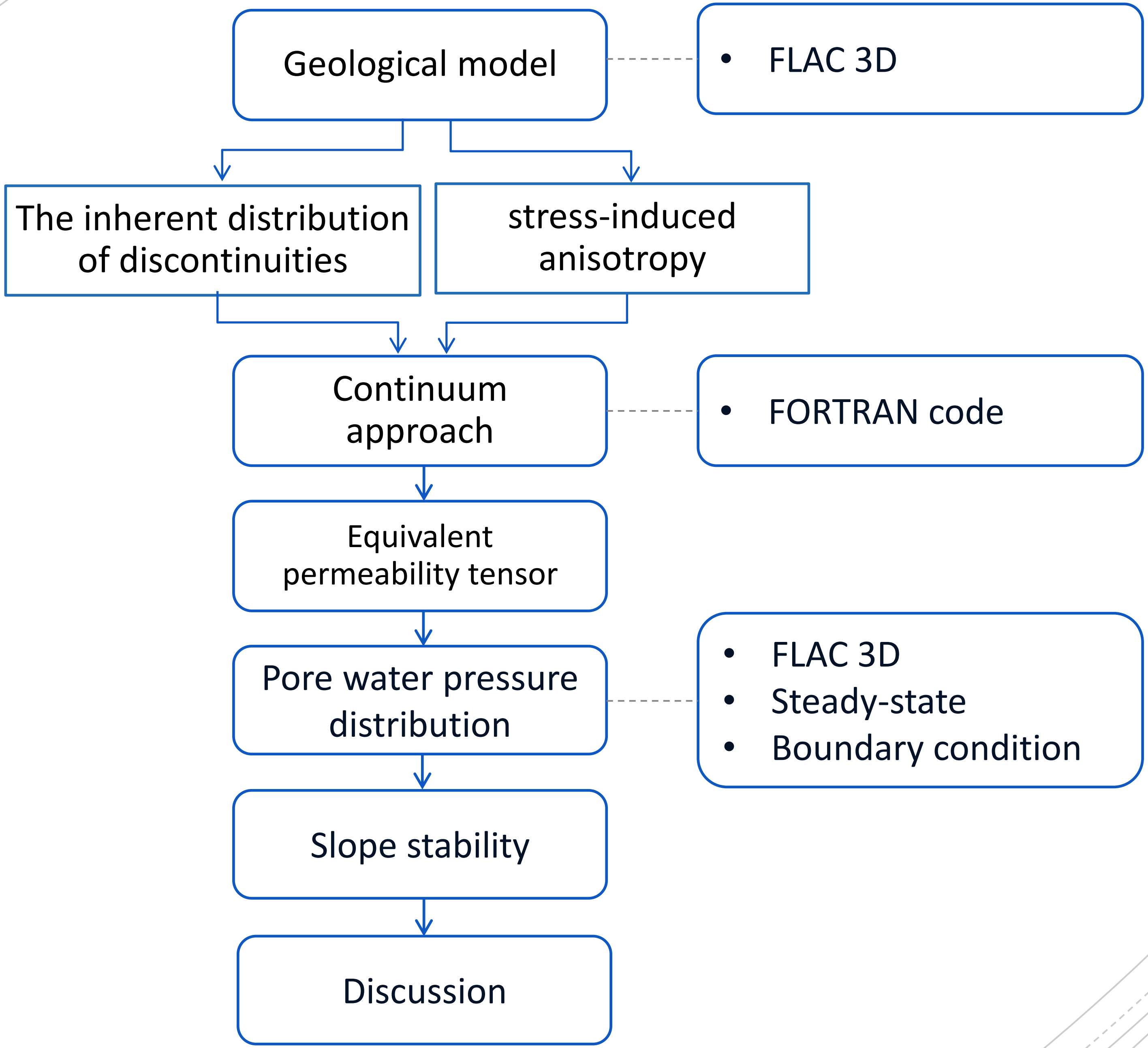


(modified from Sharp et al.,1972)



(modified from Hoek et al.,1981)

Flow chart



Geological model

- FLAC 3D

The inherent distribution of discontinuities

stress-induced anisotropy

Continuum approach

- FORTRAN code

Equivalent permeability tensor

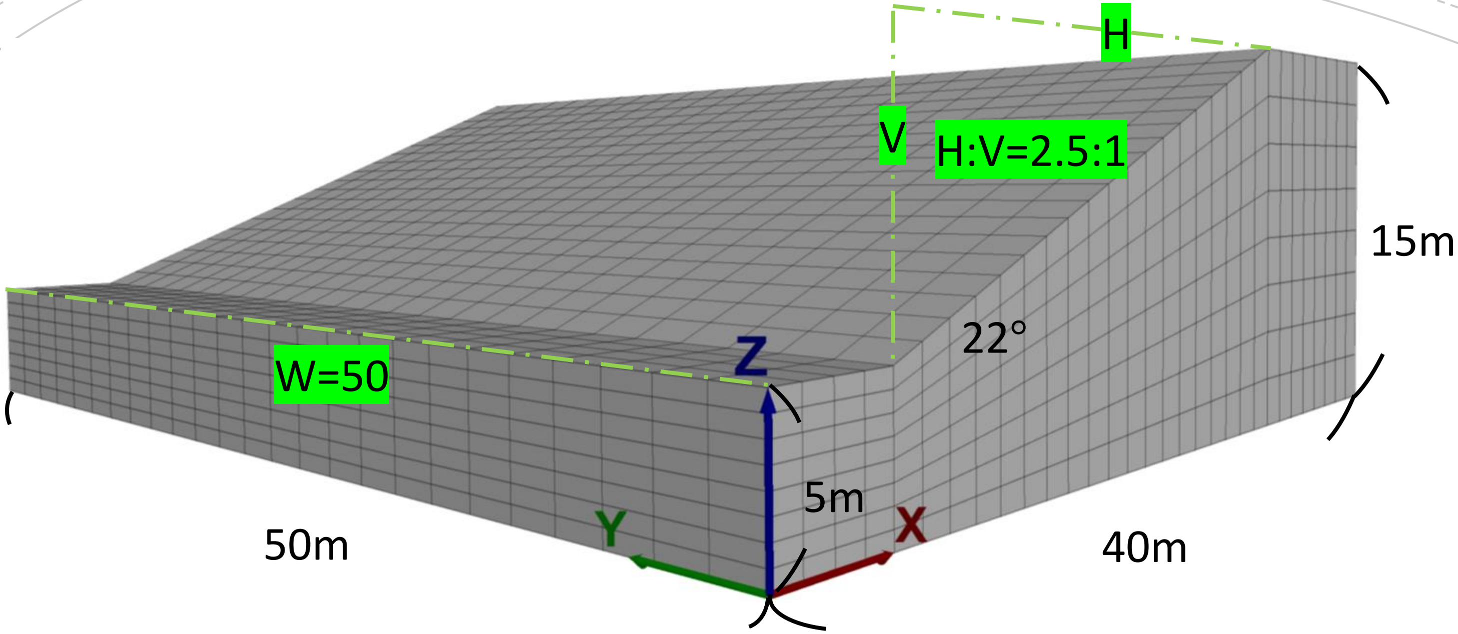
Pore water pressure distribution

- FLAC 3D
- Steady-state
- Boundary condition

Slope stability

Discussion

Model setting



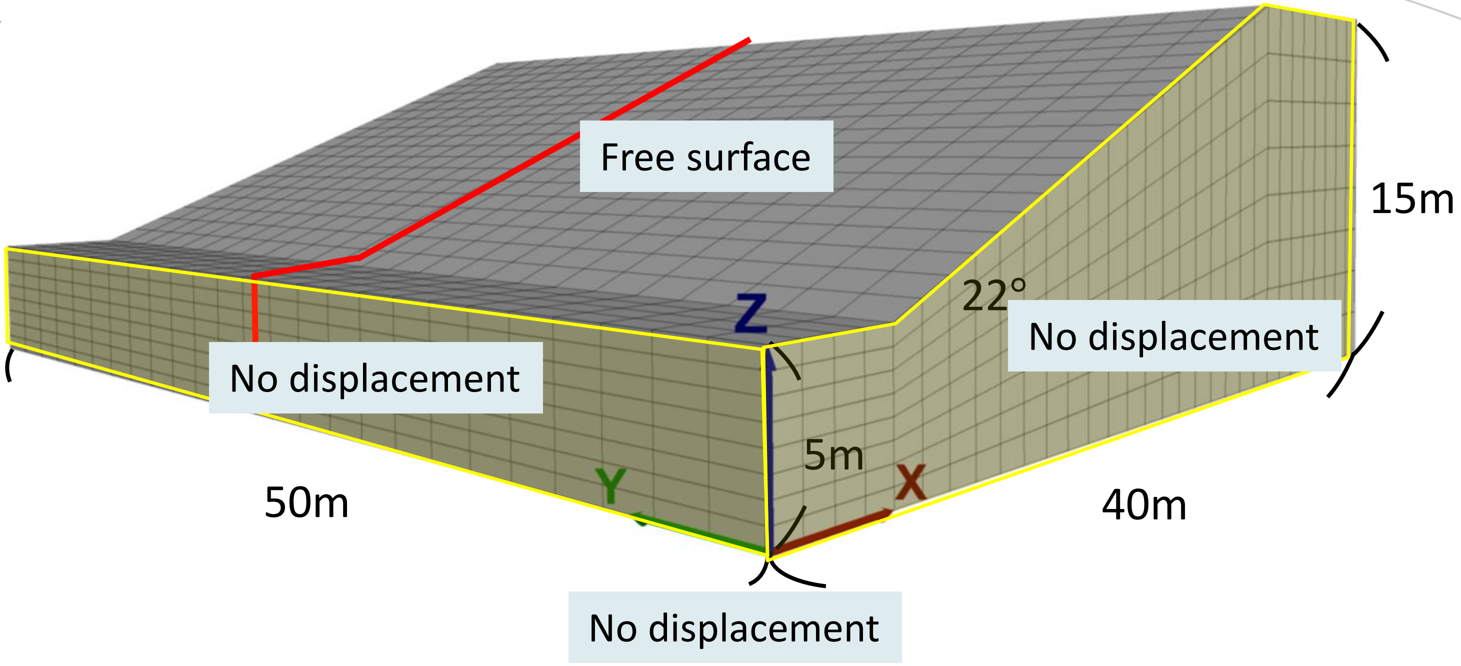
Mohr-Coulomb model

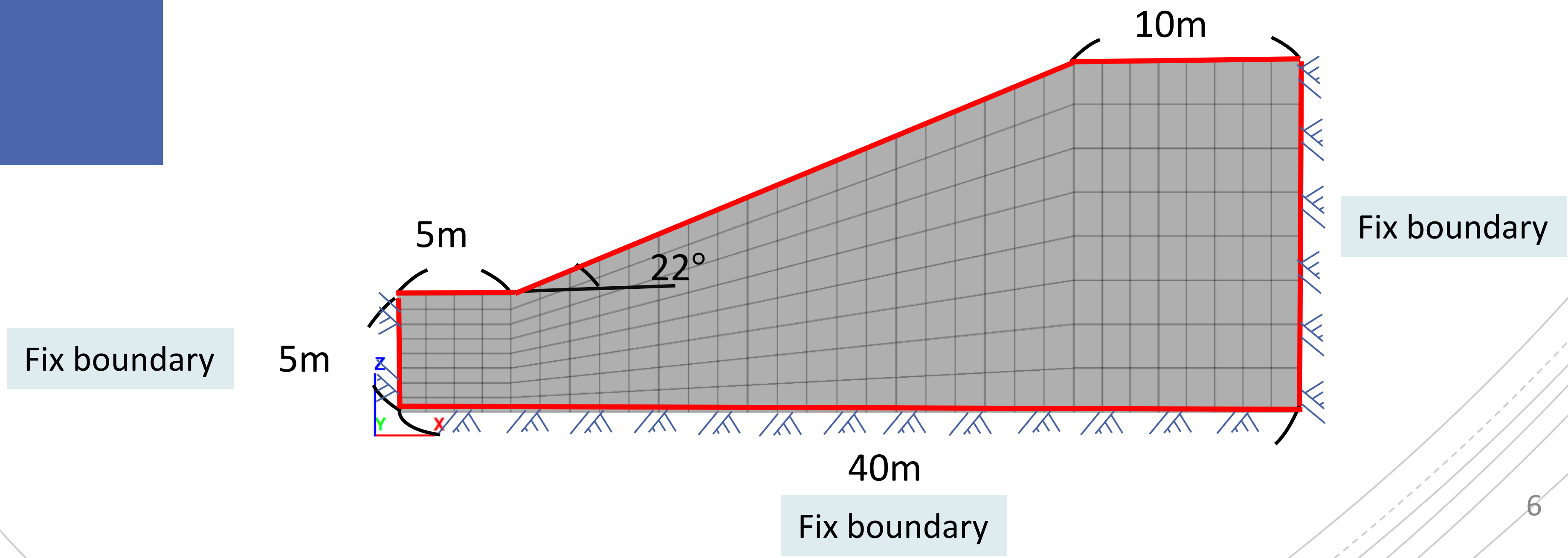
Material properties	**Value**
Dry density	18000 $(\frac{kg}{m^3})$
Young's modulus, E	200 (MPa)
Poison ratio, ν	0.33
Friction angle, ϕ	32°
cohesion, c	31.4 (kPa)
Stress ratio,$\overline{K}$	1.1

(CECI, 2021)(Chen C. C.& Yu C. W., 1994)

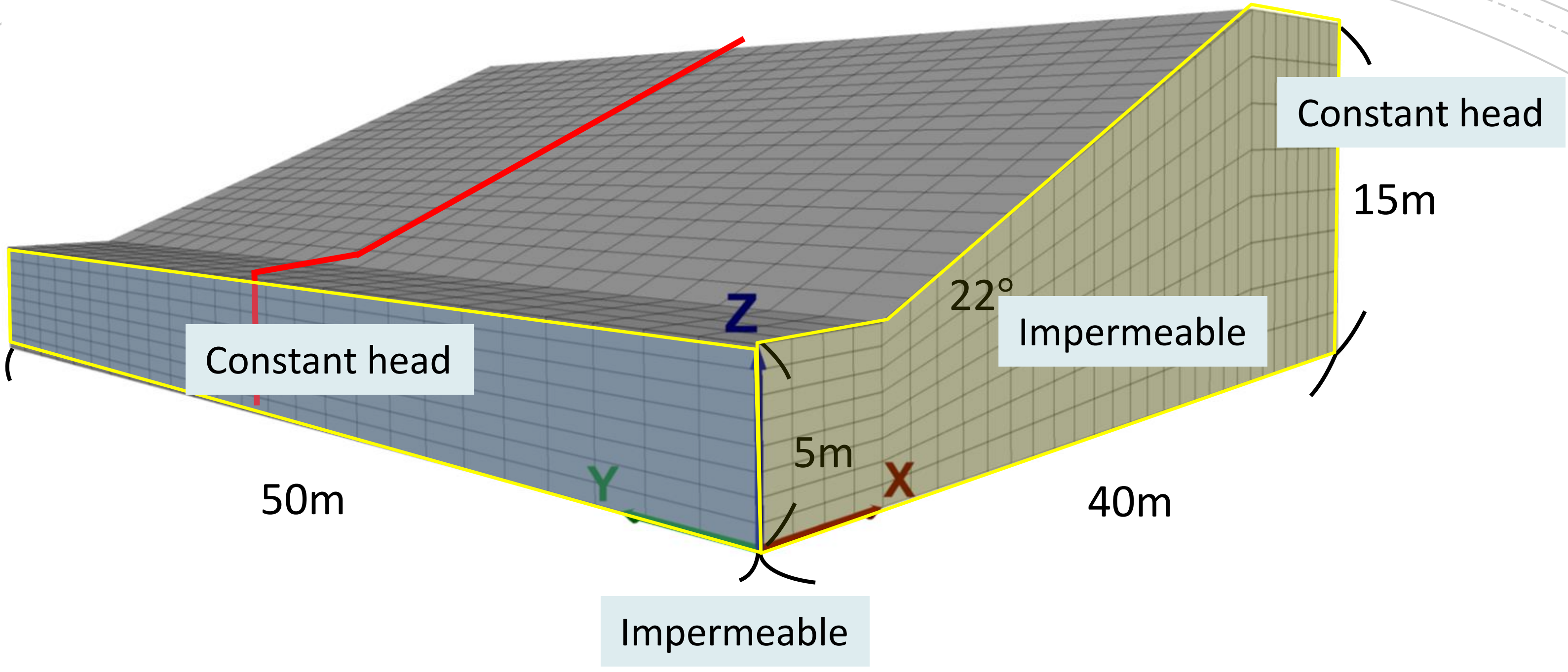
Material properties	**Value**
Fluid modulus,K_f	10 (kPa)
porosity, n	0.33
permeability, k (Mobility permeability)	$\frac{m^2}{Pa * sec}$
saturation,S	1

Mechanical boundary





Fluid boundary

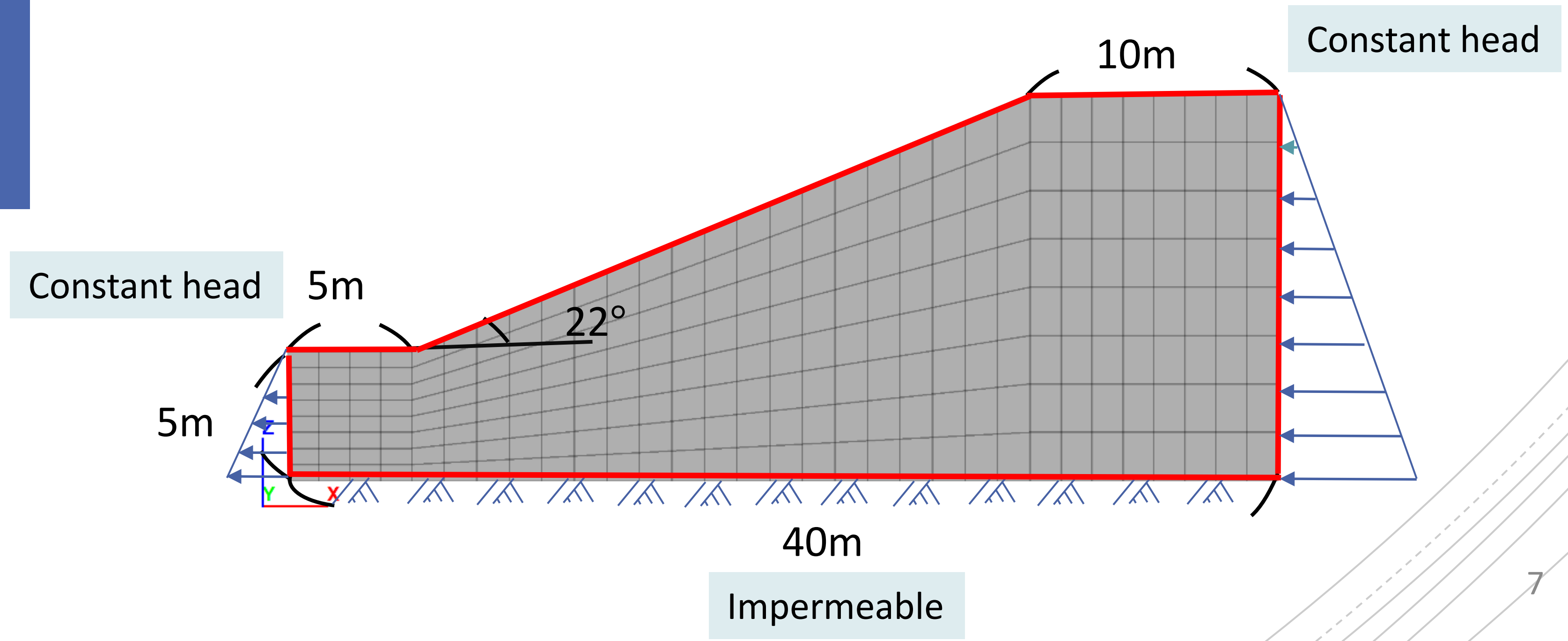


Constant head

Constant head

Impermeable

Impermeable



Constant head

Constant head

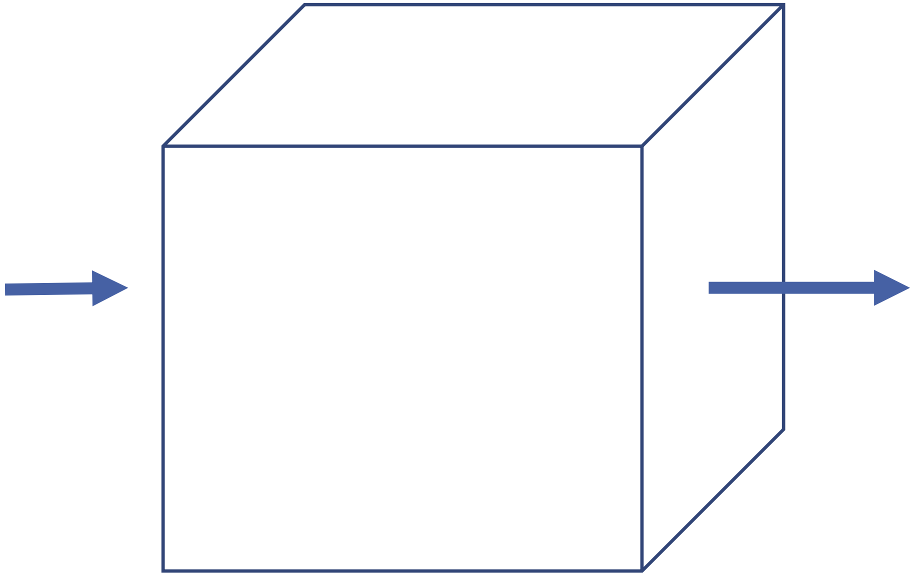
Impermeable

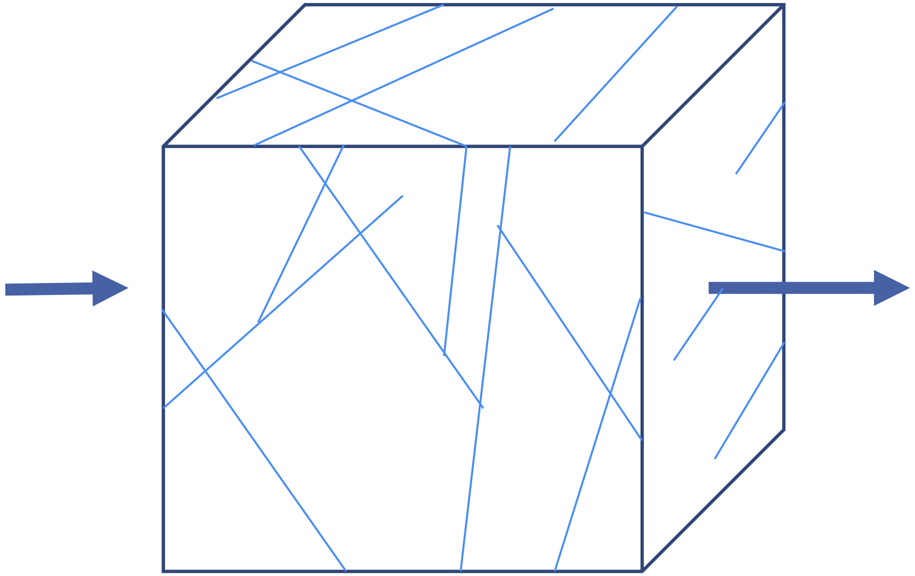
Methodology

Continuum approach

Continuum approach *(Oda,1985)*

Darcy's law





$$\bar{v_i} = k_{ij}J_j$$

the gradient of
total hydraulic head

permeability

$$\bar{v_i} = \frac{1}{V}\int_{V^c} v_i^c \, dV^c$$

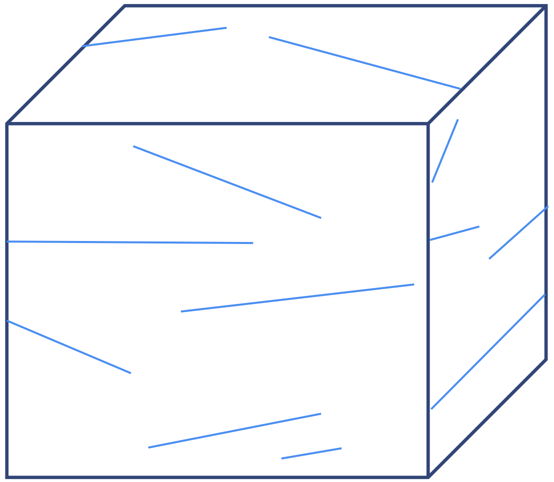
(Oda, 1985)

volume

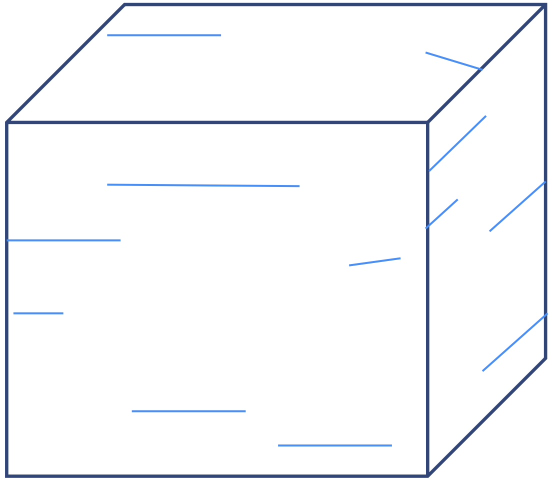
crack volume

local velocity
(the fluid flow through the crack)

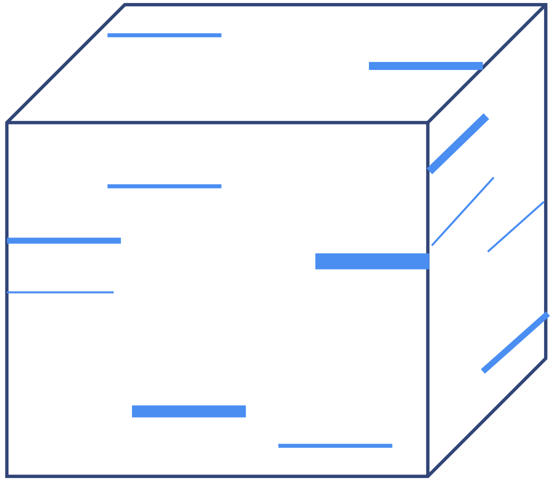
Fracture orientation$(\hat{n})$



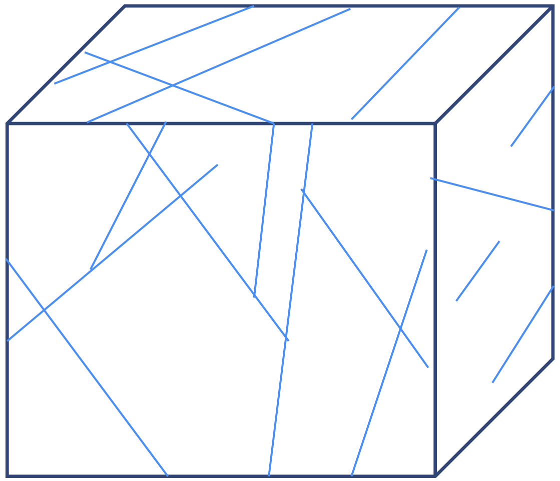
Fracture length(r)



Fracture aperture(t)



Volume density(ρ)



Crack tensor
P_{ij}

$$P_{ij} = \frac{\pi\rho}{4}\int_0^{t_m}\int_0^{r_m}\int_\Omega D^2 t^3 \hat{n}_i\hat{n}_j E(\hat{n},r,t)\,d\Omega\,dr\,dt$$

Darcy’s law

$$k_{ij} = \sum \frac{1}{12}(P_{kk}\delta_{ij} - P_{ij})$$

Equivalent permeability tensor
k_{ij}

Parameter sensitivity analysis *(Cheng,2006)*

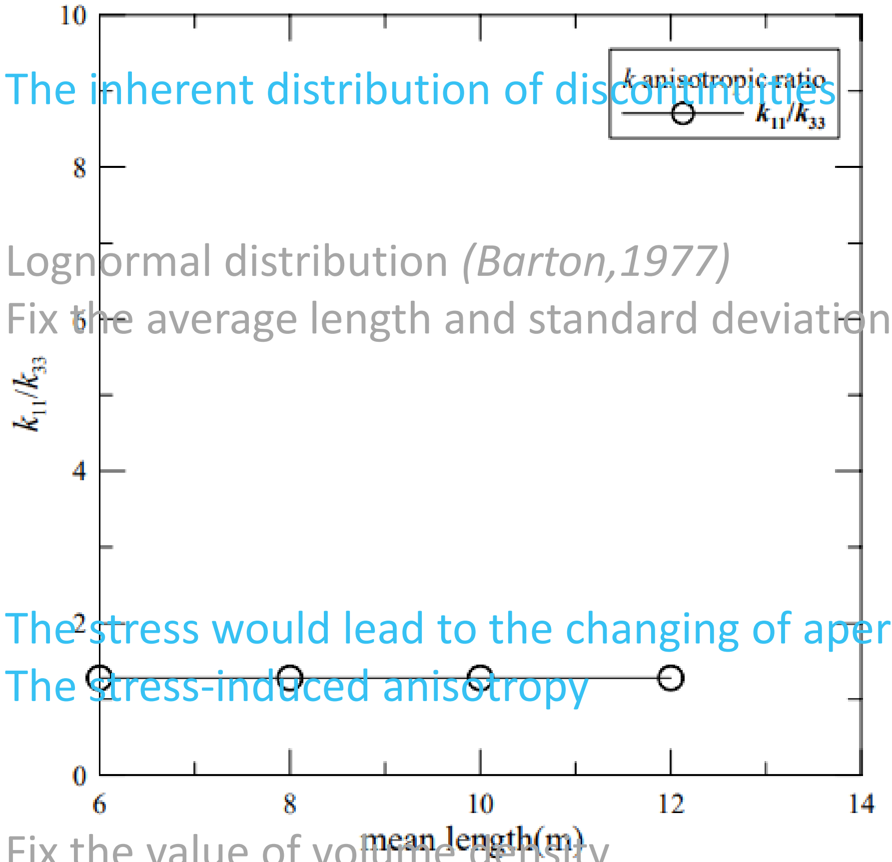
Equivalent permeability tensor

Orientation
$E(\hat{n})$

Length
$f(r)$

Aperture
$g(t)$

volume
density
ρ



- The inherent distribution of discontinuities

- Lognormal distribution *(Barton,1977)*
- Fix the average length and standard deviation

- The stress would lead to the changing of aperture
- The stress-induced anisotropy

- Fix the value of volume density
    - Lognormal distribution *(Barton,1977)*

The inherent distribution of discontinuities

- **The orientation of discontinuities, $\mathbb{E}(\hat{n})$**

The stress-induced anisotropy

- **The aperture of discontinuities, $g(t)$**

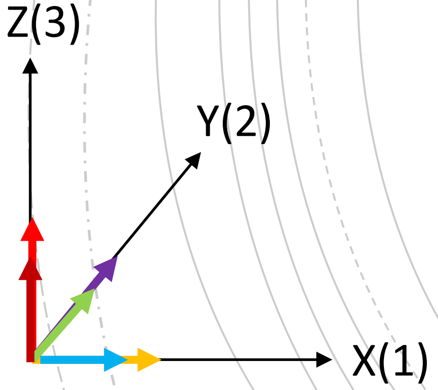
Directional density function,$\mathrm{E}(\hat{\mathrm{n}})$ *(Kanatani,1984)*

$$\int \mathrm{E}(\hat{\mathrm{n}})d\Omega = 1$$

- The vector $\hat{\mathrm{n}}$ accounts for the total proportion of all vectors

$$\mathrm{E}(\hat{\mathrm{n}}) = \frac{1}{4\pi}(1 + \mathrm{D_{ij}}\mathrm{n_i}\mathrm{n_j})$$

- Use the fabric tensor $(\mathrm{D_{ij}})$ as the coefficient to approximate the vector distribution
- The value of $\mathrm{D_{ij}}$ can represent the anisotropic degree



- ↑ The normal vector of discontinuity
- $\mathrm{D_{11}} = \mathrm{D_{22}} = \mathrm{D_{33}} = 0$, it has same number of discontinuities in all directions.

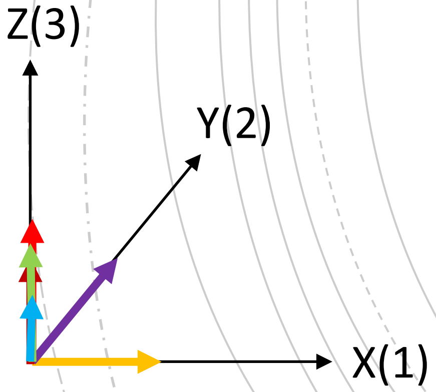
Directional density function,$\mathrm{E}(\hat{\mathrm{n}})$ *(Kanatani,1984)*

$$\int \mathrm{E}(\hat{\mathrm{n}})d\Omega = 1$$

- The vector $\hat{\mathrm{n}}$ accounts for the total proportion of all vectors

$$\mathrm{E}(\hat{\mathrm{n}}) = \frac{1}{4\pi}(1 + \mathrm{D_{ij}n_in_j})$$

- Use the fabric tensor $(\mathrm{D_{ij}})$ as the coefficient to approximate the vector distribution
- The value of $\mathrm{D_{ij}}$ can represent the anisotropic degree



- ↑ The normal vector of discontinuity
- $\mathrm{D_{11}} = \mathrm{D_{22}} = \mathrm{D_{33}} = 0$, it has same number of discontinuities in all directions.
- $\mathrm{D_{33}} > \mathrm{D_{11}} = \mathrm{D_{22}}$, the number of normal vector in the vertical direction is more than the number of normal vector in the horizontal direction

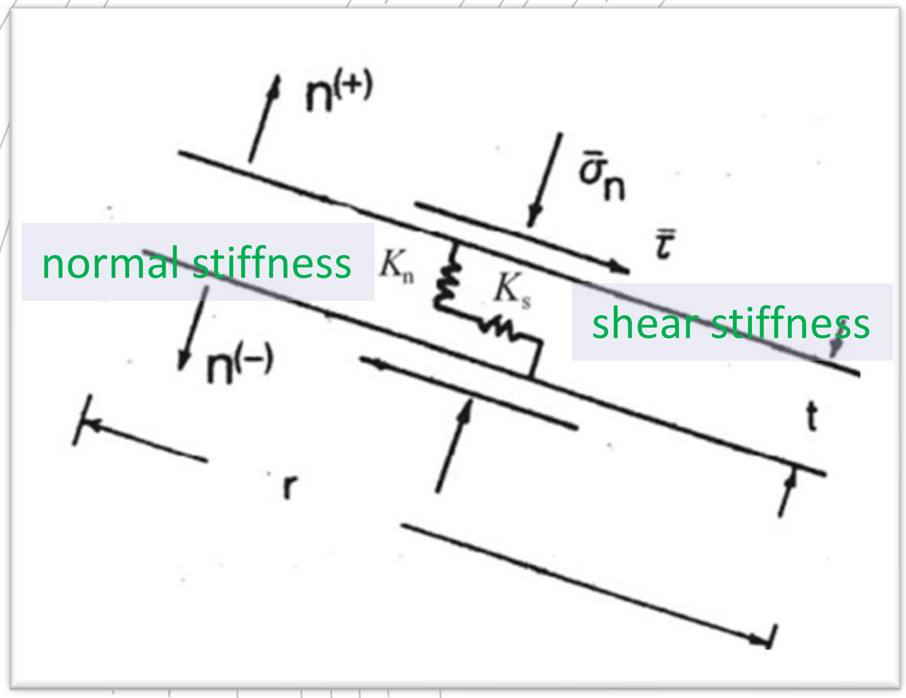
The inherent distribution of discontinuities

- The orientation of discontinuities, $\mathbb{E}(\hat{n})$

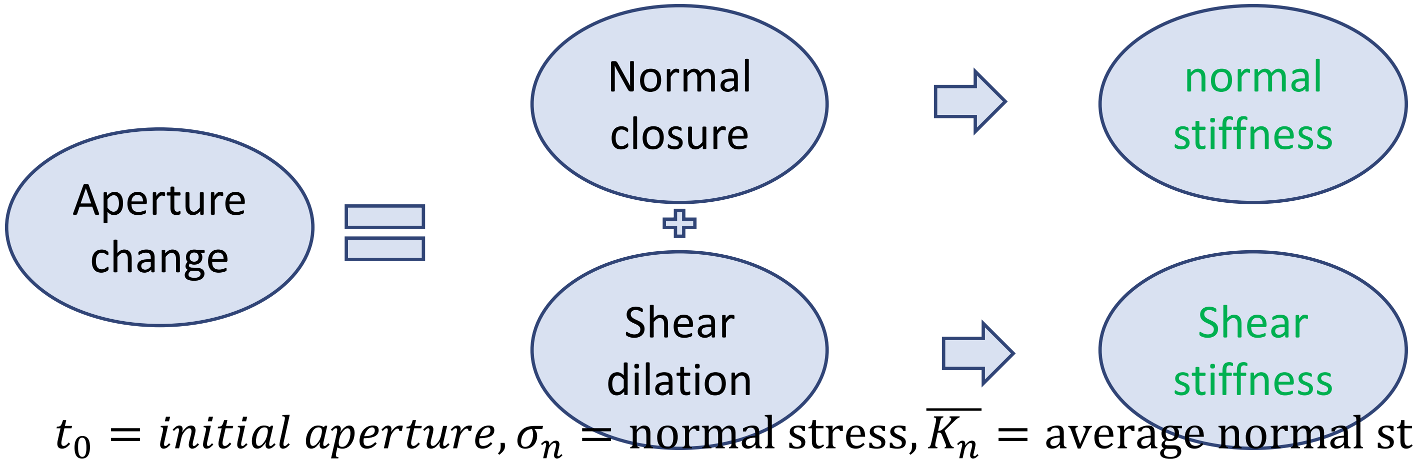
The stress-induced anisotropy

- The aperture of discontinuities, $g(t)$

The function of aperture,$g(t)$ *(Oda,1986)*



- Assume each fracture are two plates connected by springs



Aperture change = Normal closure + Shear dilation

Normal closure ⇨ normal stiffness

Shear dilation ⇨ Shear stiffness

$t_0 = initial\ aperture, \sigma_n$ = normal stress, $\overline{K_n}$ = average normal stiffnes

- $t = t_0 - \left(\frac{\overline{\sigma_n}}{\overline{K_n}}\right)$ Normal closure, $\overline{K_n} \uparrow \Delta t \downarrow$
- $\overline{K_n}$ ()is depend on strength of material

$$g(t) = t(\hat{n}) = t_0 - \left(\frac{\overline{\sigma_n}}{\overline{K_n}}\right)$$

Result

The inherent distribution of discontinuities

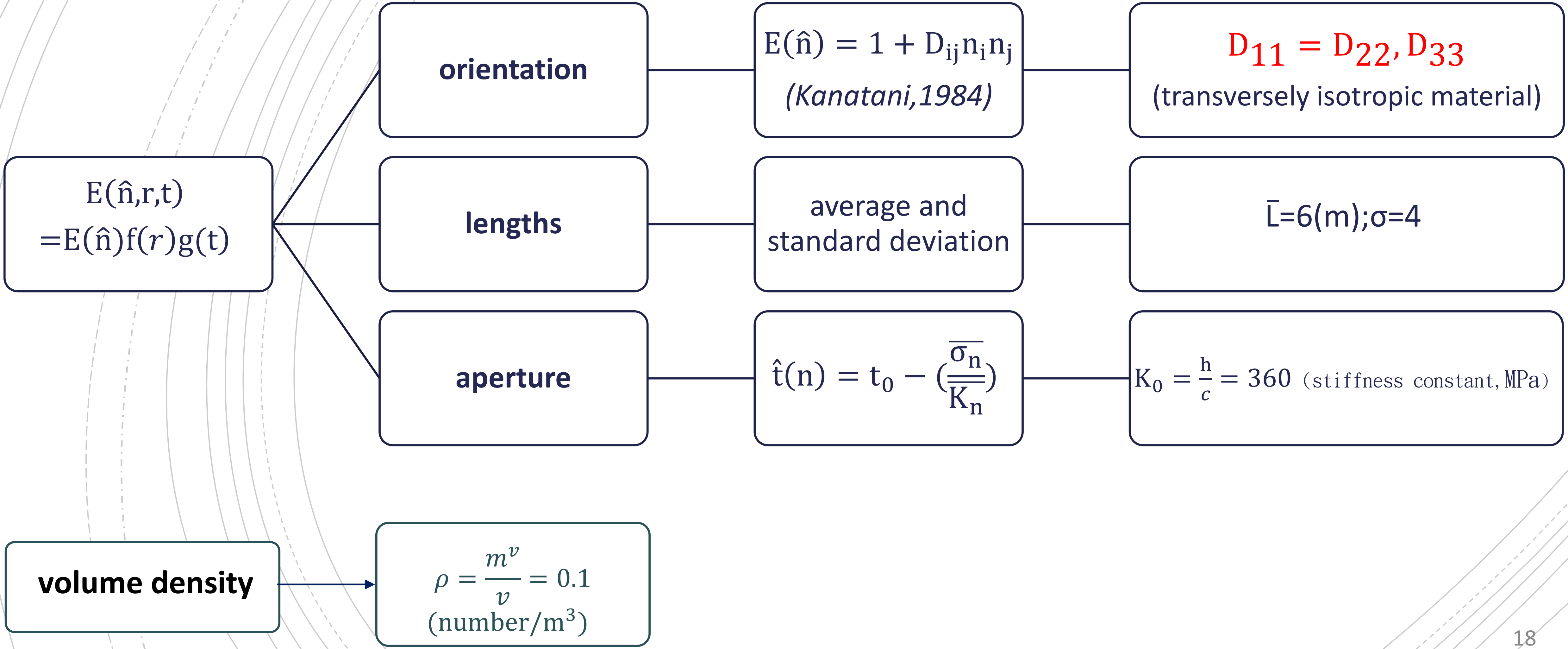
- Continuum approach (crack tensor)
- Equivalent permeability tensor
- Pore water pressure distribution

Continuum approach (crack tensor)

Equivalent permeability tensor

Pore water pressure distribution

(Assume the rock mass under uniform stress $\sigma_n = 0.25MPa$)



$E(\hat{n},r,t)$
$=E(\hat{n})f(r)g(t)$

orientation

$E(\hat{n}) = 1 + D_{ij}n_in_j$
(Kanatani,1984)

$D_{11} = D_{22}, D_{33}$
(transversely isotropic material)

lengths

average and standard deviation

$\bar{L}$=6(m);σ=4

aperture

$\hat{t}(n) = t_0 - (\frac{\overline{\sigma_n}}{\overline{K_n}})$

$K_0 = \frac{h}{c} = 360$ (stiffness constant, MPa)

volume density

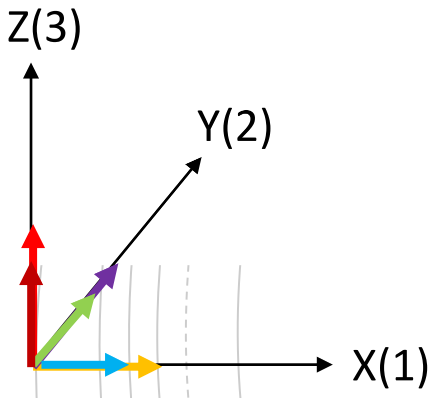
$\rho = \frac{m^v}{v} = 0.1$
(number/m^3)

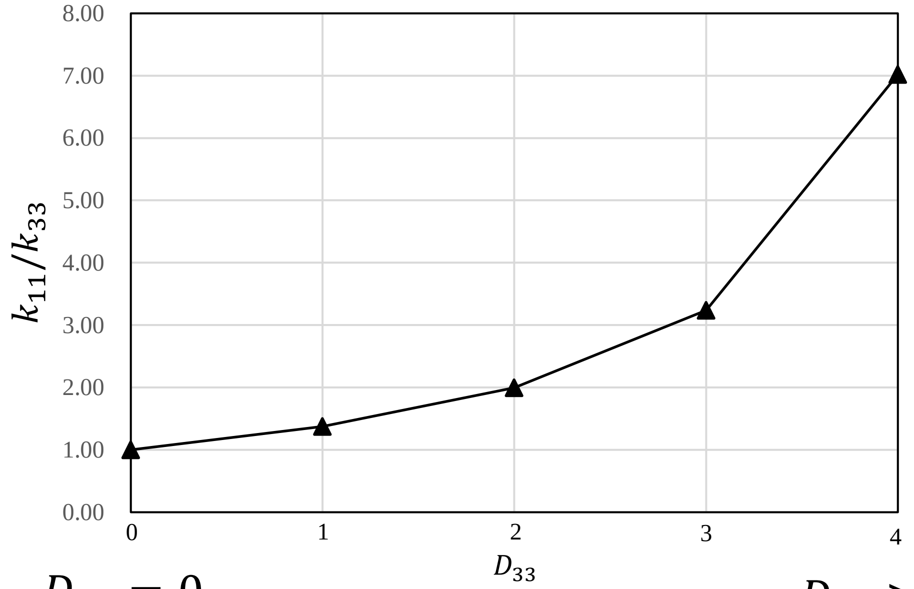
Continuum approach (crack tensor)

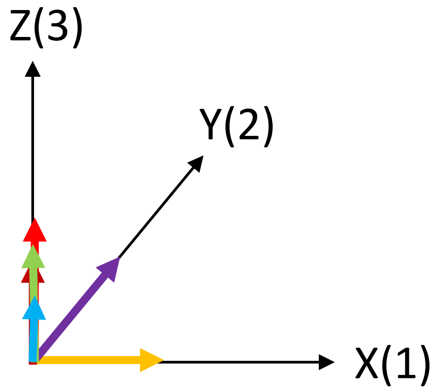
Equivalent permeability tensor

Pore water pressure distribution

(Assume the rock mass under uniform stress $\sigma_n = 0.25MPa$)







$D_{33} = 0$ isotropy → anisotropy $D_{33} > 0$

The discontinuities distributed same number at direction 1,2,3
= isotropic permeability

The normal vector of discontinuities account for a large proportion in direction 3
= anisotropic permeability

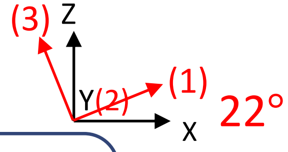
$(unit\ of\ k = \frac{m^2}{Pa * sec})$

Continuum approach (crack tensor)

Equivalent permeability tensor

Pore water pressure distribution

(Assume the direction of maximum principal permeability parallel the slope surface)



Isotropic permeability

$k_{11} = k_{22} = k_{33} = 1.23 * 10^{-10}$

anisotropic permeability($D_{33} = 4$)

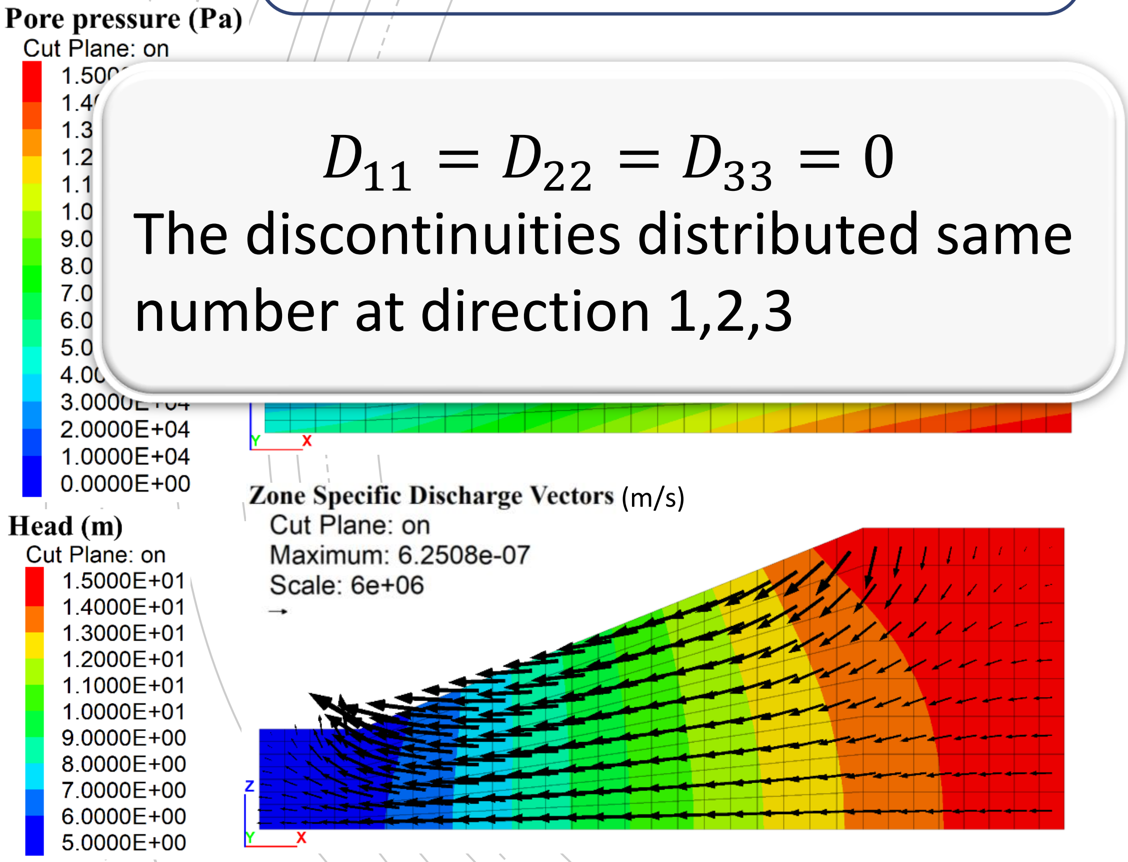
$k_{11} = k_{22} = 1.72 * 10^{-10}; k_{33} = 2.45 * 10^{-11}$

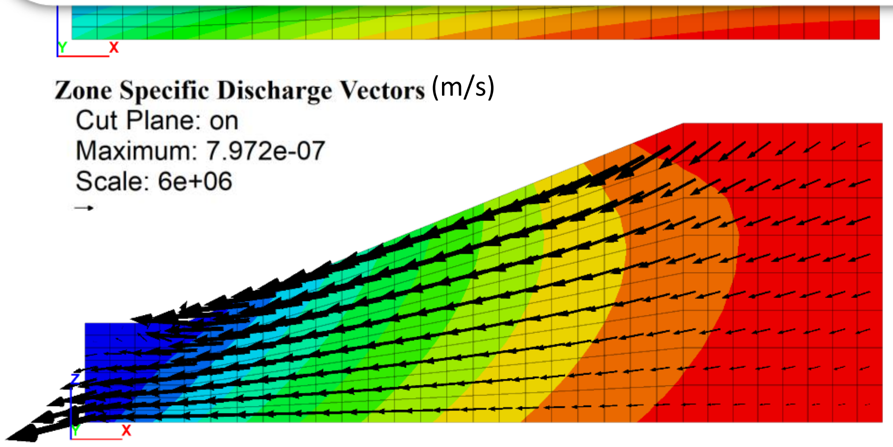
$$D_{11} = D_{22} = D_{33} = 0$$

The discontinuities distributed same number at direction 1,2,3

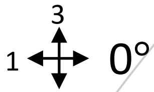
$$D_{11} = D_{22} = -2 < D_{33} = 4$$

The normal vector of discontinuities account for a large proportion in direction 3





$(unit\ of\ k = \frac{m^2}{Pa * sec})$



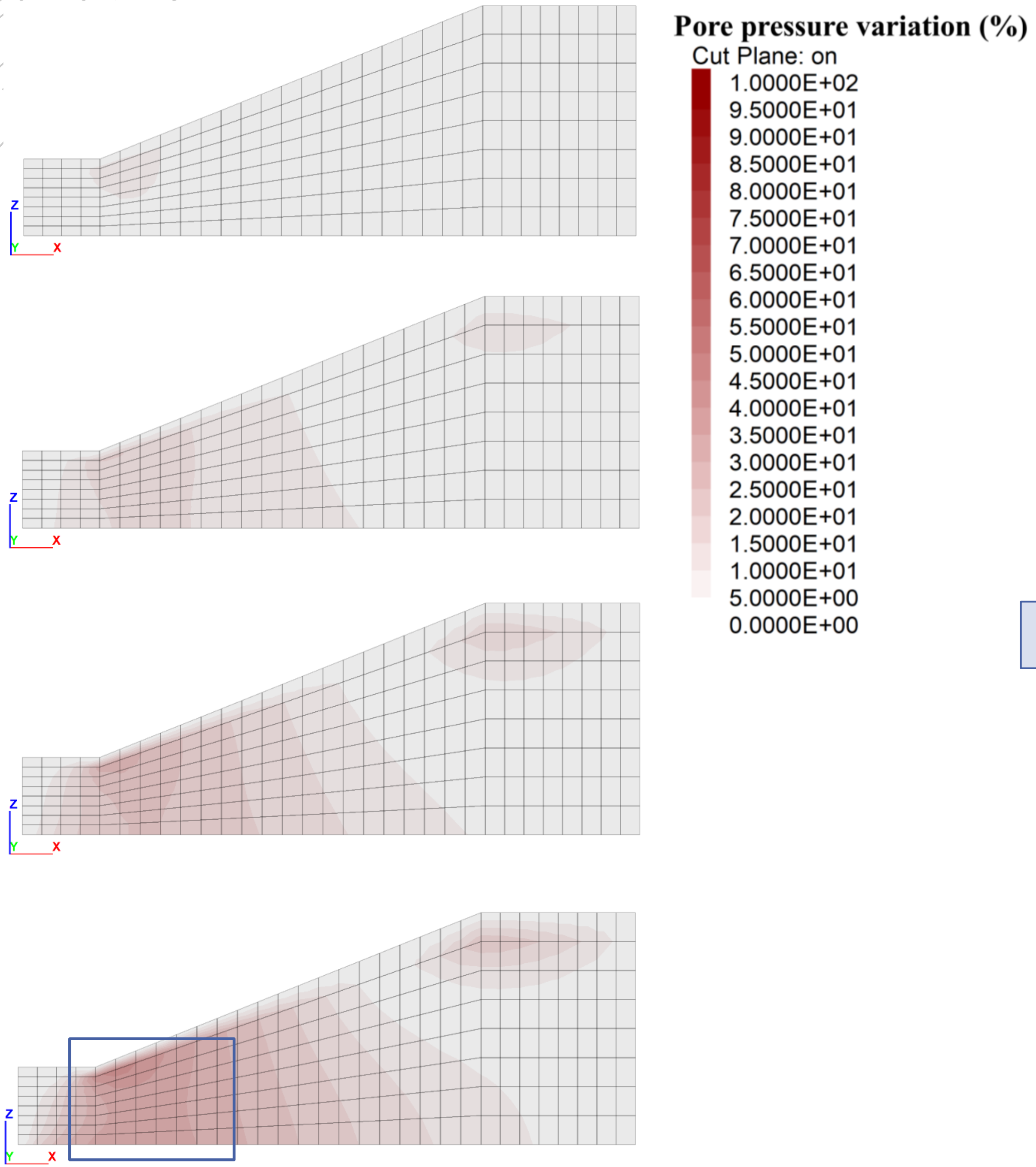
Relative variation

$$\frac{|P(D_{33}=1)-P(D_{33}=0)|}{P(D_{33}=0)}$$

$$\frac{|P(D_{33}=2)-P(D_{33}=0)|}{P(D_{33}=0)}$$

$$\frac{|P(D_{33}=3)-P(D_{33}=0)|}{P(D_{33}=0)}$$

$$\frac{|P(D_{33}=4)-P(D_{33}=0)|}{P(D_{33}=0)}$$



D_{33} ↗, variation ↗

Maximum variation:44%

Result

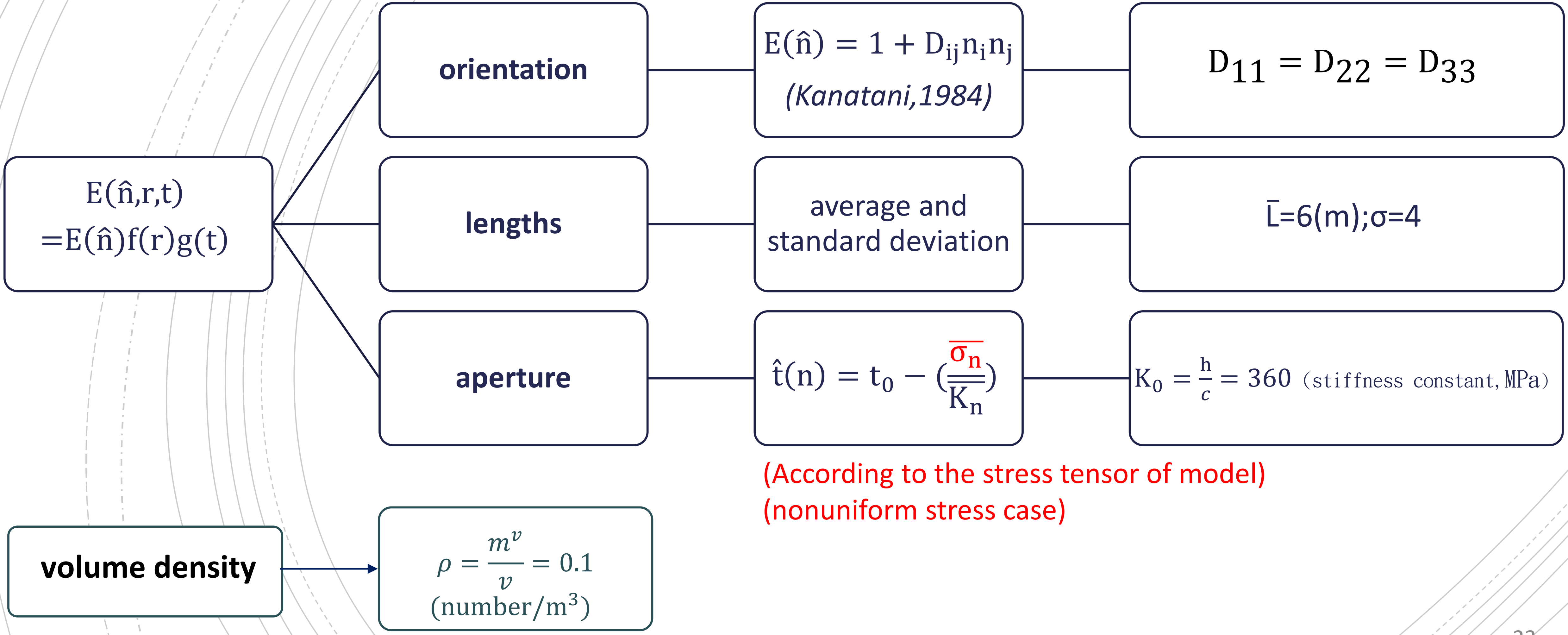
Stress-induced anisotropy

- Continuum approach (crack tensor)
- Equivalent permeability tensor
- Pore water pressure distribution

Continuum approach (crack tensor)

Equivalent permeability tensor

Pore water pressure distribution



$E(\hat{n},r,t)$
$=E(\hat{n})f(r)g(t)$

orientation

$E(\hat{n}) = 1 + D_{ij}n_in_j$
(Kanatani,1984)

$D_{11} = D_{22} = D_{33}$

lengths

average and standard deviation

$\bar{L}$=6(m);σ=4

aperture

$\hat{t}(n) = t_0 - (\frac{\overline{\sigma_n}}{K_n})$

$K_0 = \frac{h}{c} = 360$ (stiffness constant, MPa)

(According to the stress tensor of model)
(nonuniform stress case)

volume density

$\rho = \frac{m^v}{v} = 0.1$
$(\text{number}/\text{m}^3)$

Equivalent permeability tensor

{ Material properties **Mohr-Coulomb criterion** } + { Boundary condition *(Chugh et al.,2003)* }

gravitational equilibrium

⬇

{ The stress tensor of all zones }

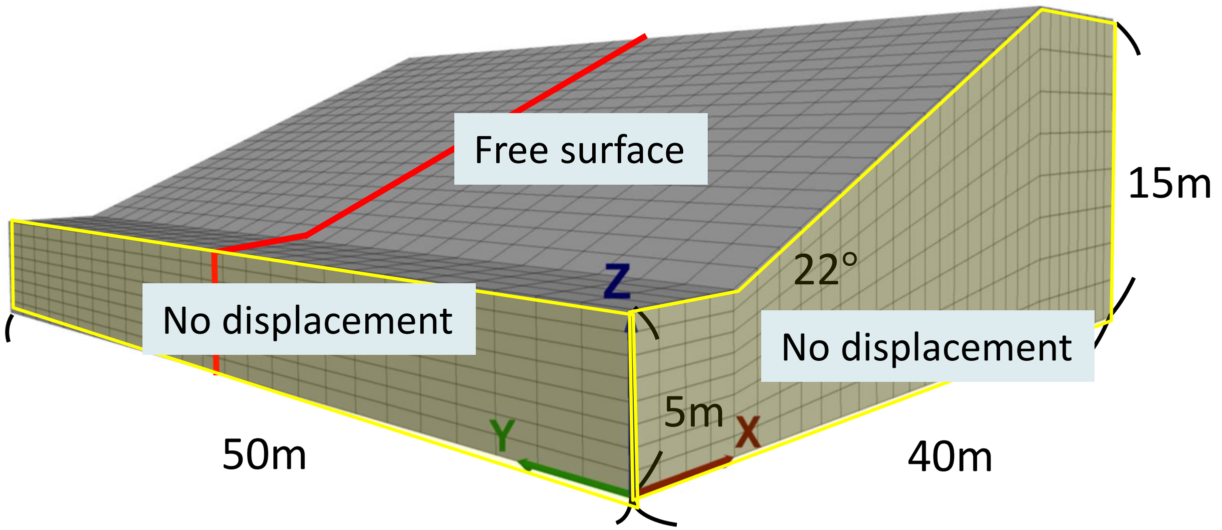
Zone 3416

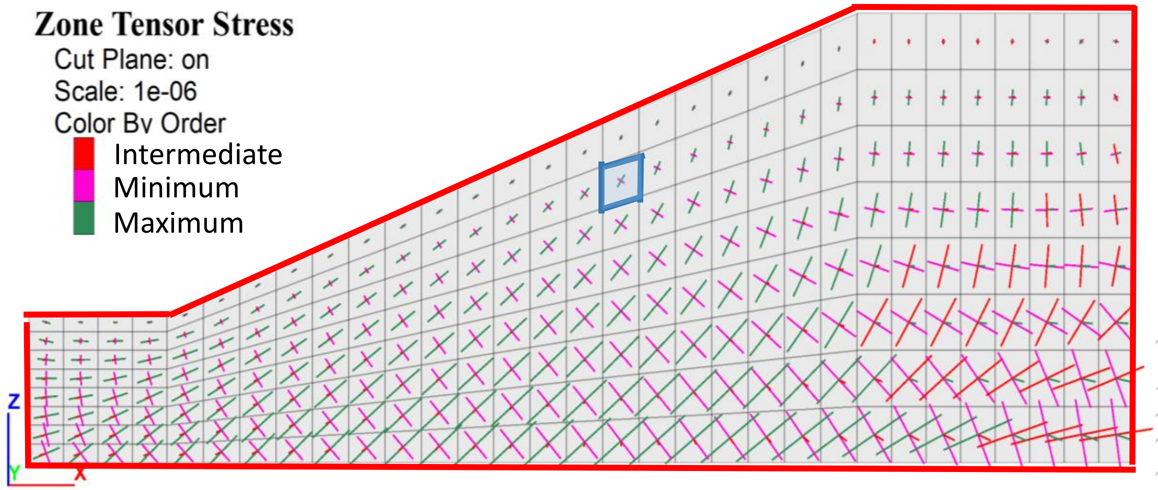
$$\overline{\sigma_{ij}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$g(t) = t(\hat{n}) == t_0 - \left(\frac{\overline{\sigma_n}\hat{n}_i\hat{n}_j}{K_n}\right)$$

Crack tensor, P_{ij}

Equivalent permeability tensor , k_{ij}





Continuum approach (crack tensor)

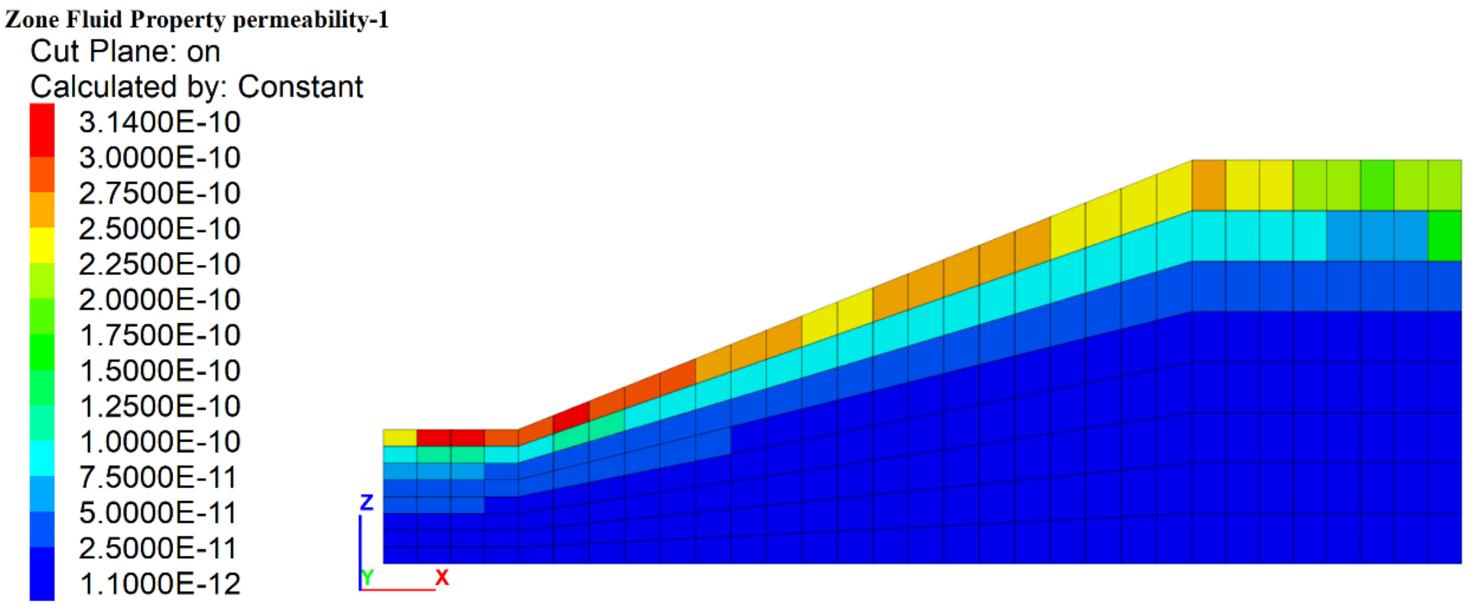
Equivalent permeability tensor

Pore water pressure distribution

aperture closure

⇨ Depth ↑, the value of permeability ↓

⇨ average value of k_{11}/k_{33}=1.22



(The maximum principal permeability of each zone)

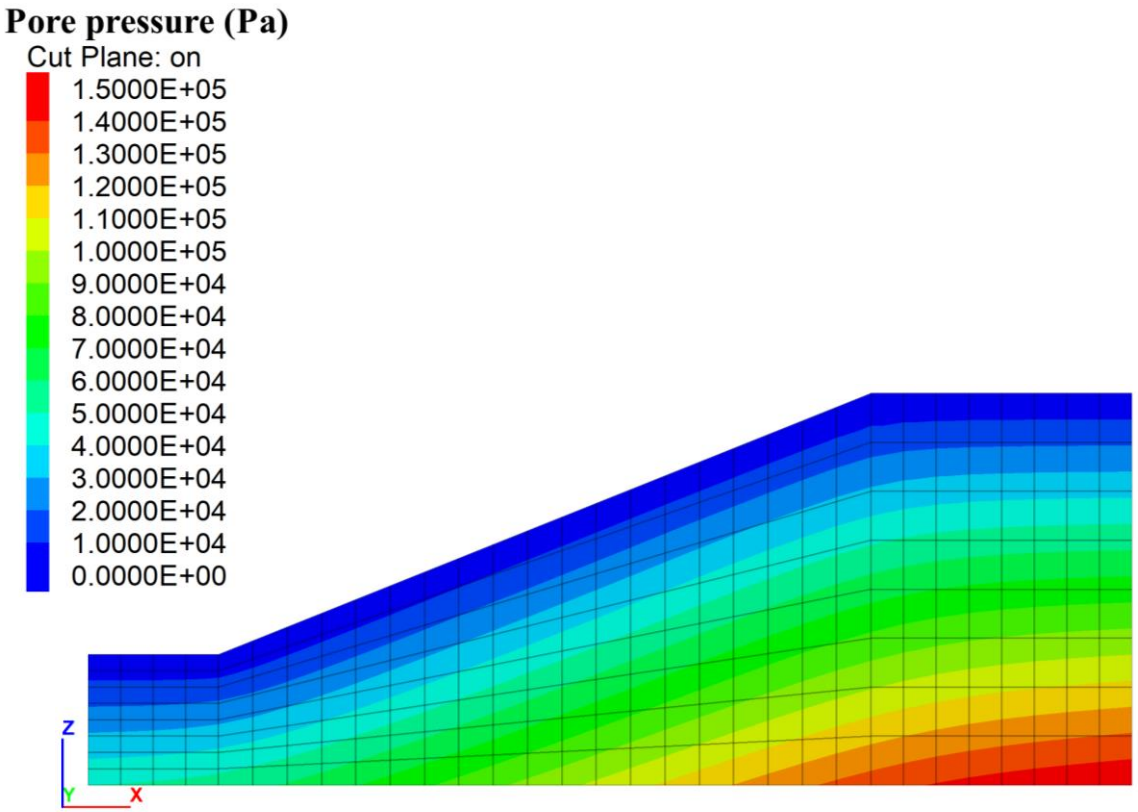
$(unit\ of\ k = \frac{m^2}{Pa * sec})$

Continuum approach (crack tensor)

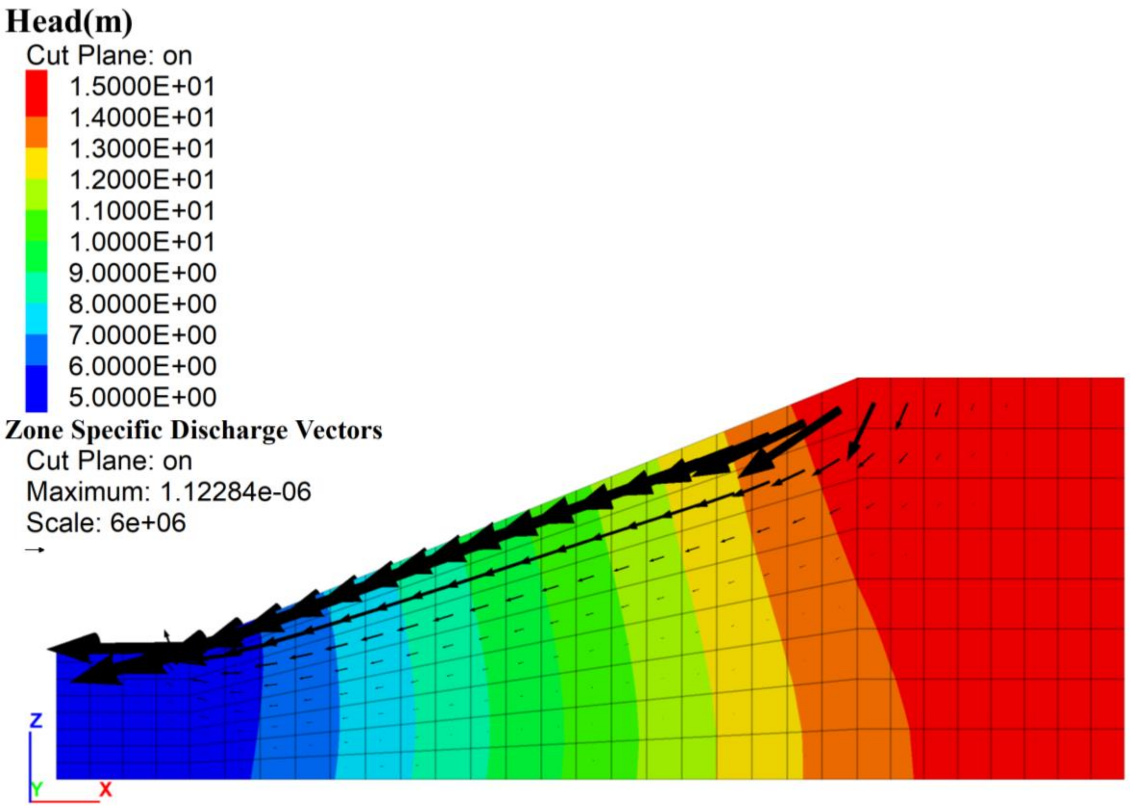
Equivalent permeability tensor

Pore water pressure distribution

The distribution of pore water pressure



The distribution of water head



Conclusion

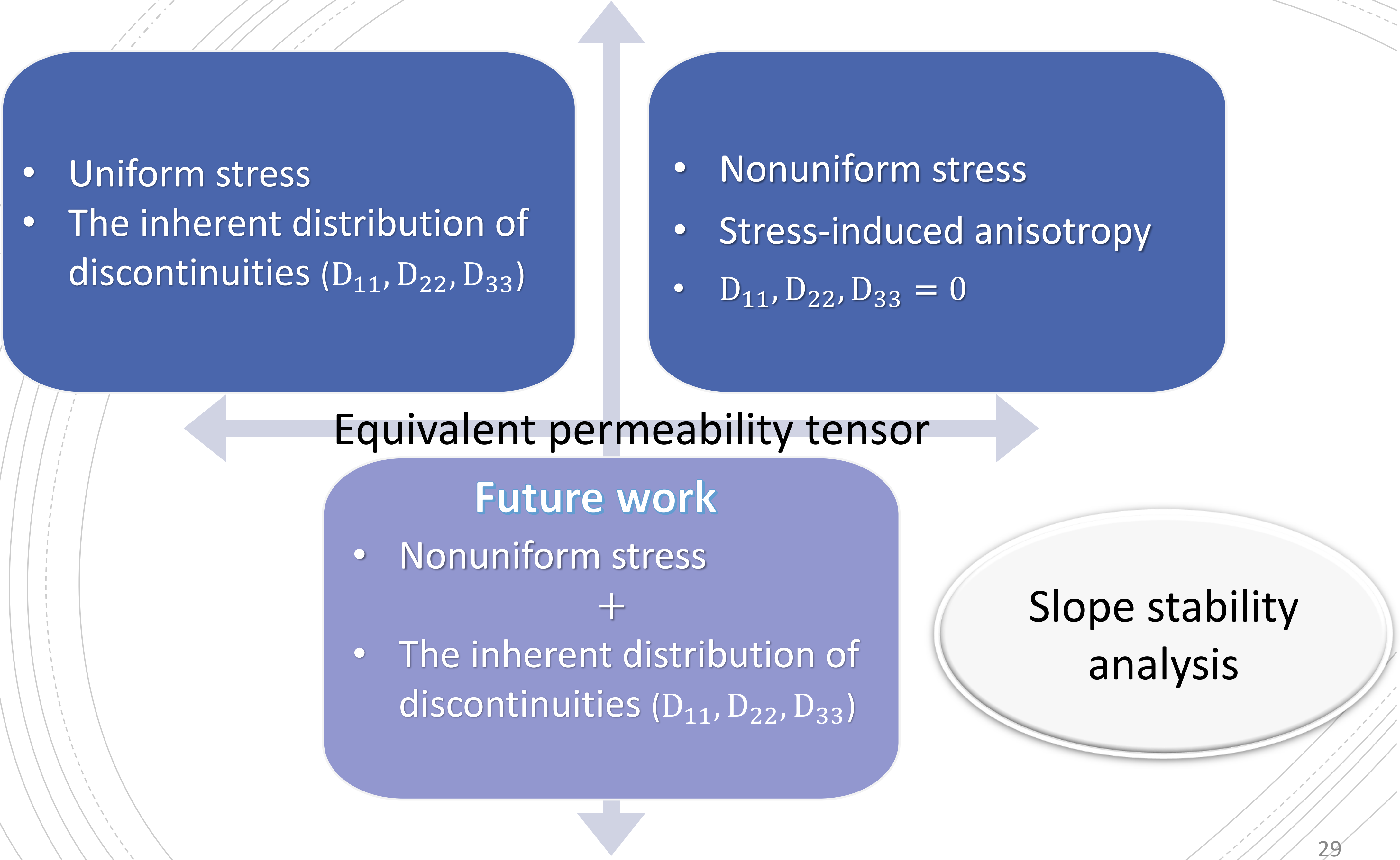
Equivalent permeability tensor

- Uniform stress
- The inherent distribution of discontinuities $(\mathrm{D}_{11}, \mathrm{D}_{22}, \mathrm{D}_{33})$

- Nonuniform stress
- Stress-induced anisotropy
- $\mathrm{D}_{11}, \mathrm{D}_{22}, \mathrm{D}_{33} = 0$

- When D_{33} ↑, pore pressure variation ↑

- When depth ↑, the principal permeability ↓



- Uniform stress
- The inherent distribution of discontinuities (D_{11}, D_{22}, D_{33})

- Nonuniform stress
- Stress-induced anisotropy
- $D_{11}, D_{22}, D_{33} = 0$

Equivalent permeability tensor

Future work

- Nonuniform stress

$+$

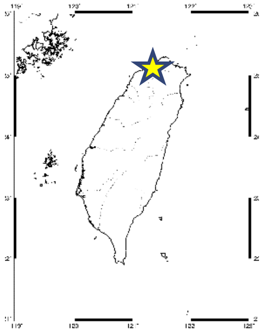
- The inherent distribution of discontinuities (D_{11}, D_{22}, D_{33})

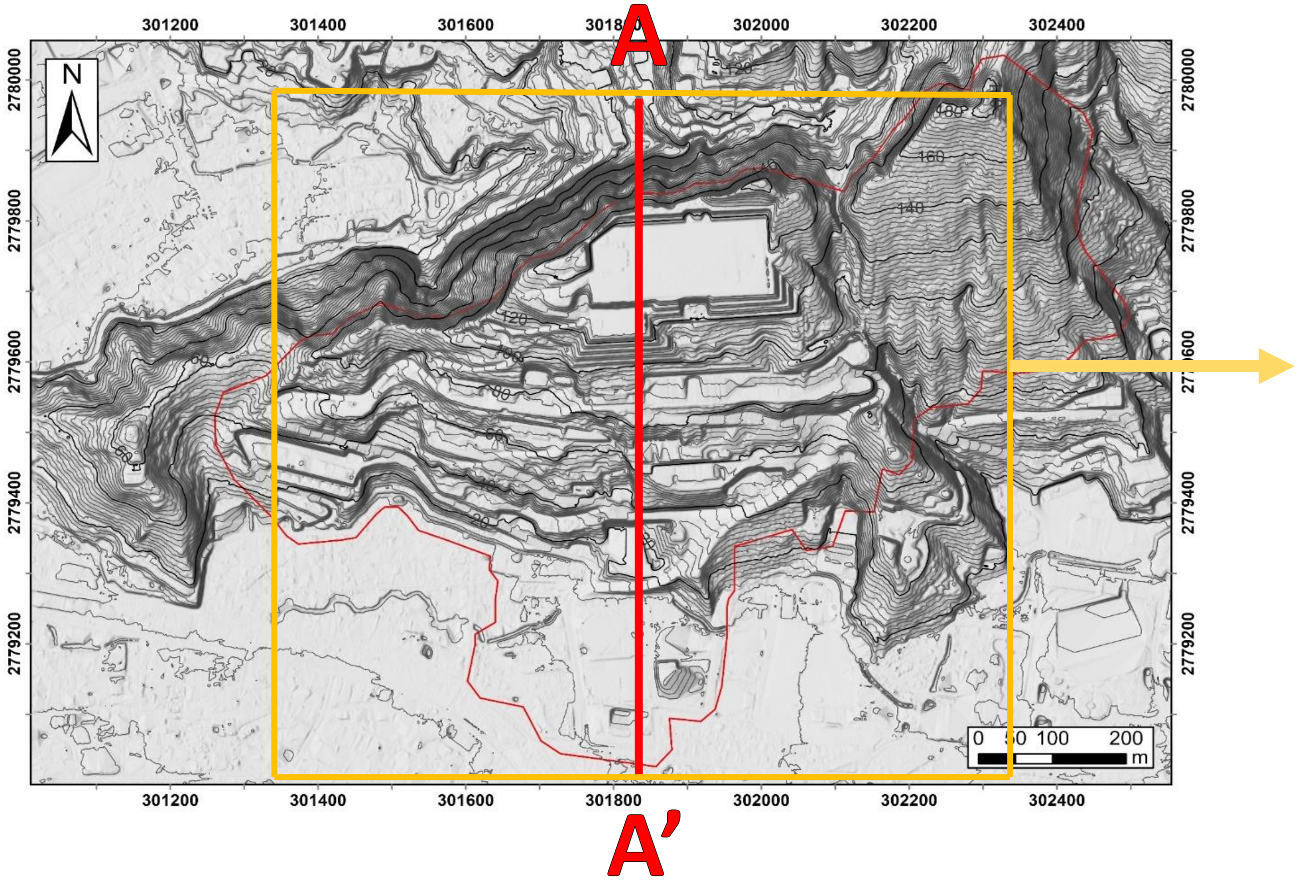
Pore pressure distribution

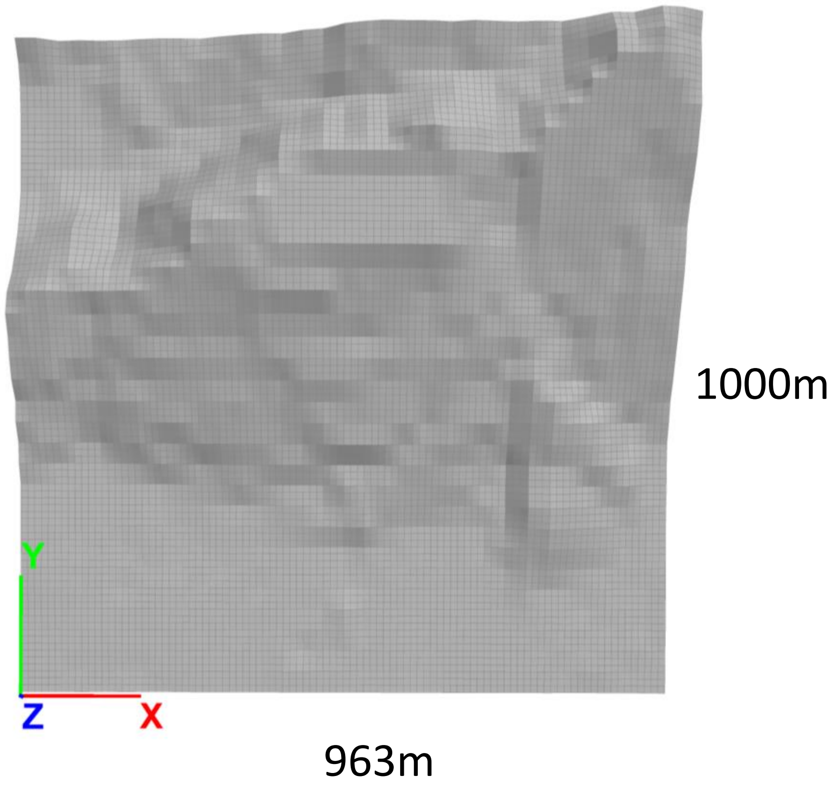
Slope stability analysis

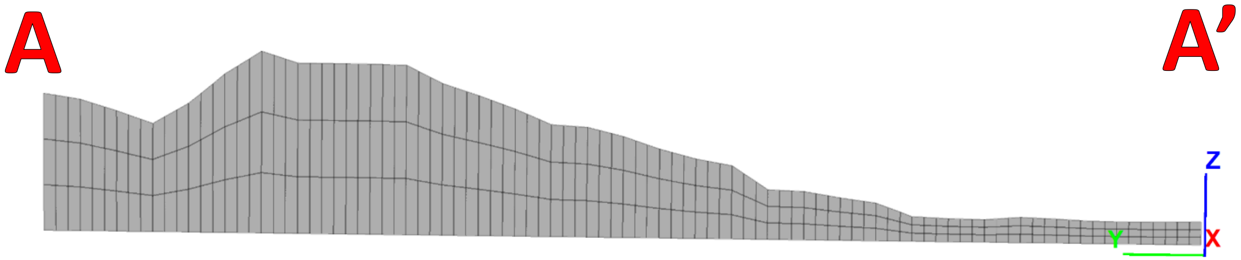
Thank you for your attention.

Case study

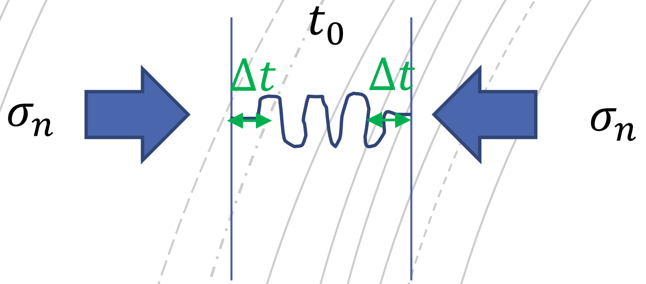



A
301200
301400
301600
302000
302200
302400
2780000
2779800
2779600
2779400
2779200
N
120
160
140
100
80
0 50 100 200
m
A'


1000m
Y
Z
X
963m


A
A'
Z
Y
X

The function of aperture,g(t) *(Oda,1986)*



$t_0 = initial\ aperture, \sigma_n =$ normal stress, $\overline{K_n}$ = average normal stiffness

- $t = t_0 - \left(\frac{\overline{\sigma_n}}{\overline{K_n}}\right)$ Normal closure, $\overline{K_n} \uparrow \Delta t \downarrow$
- $\overline{K_n}$ ()is depend on strength of material

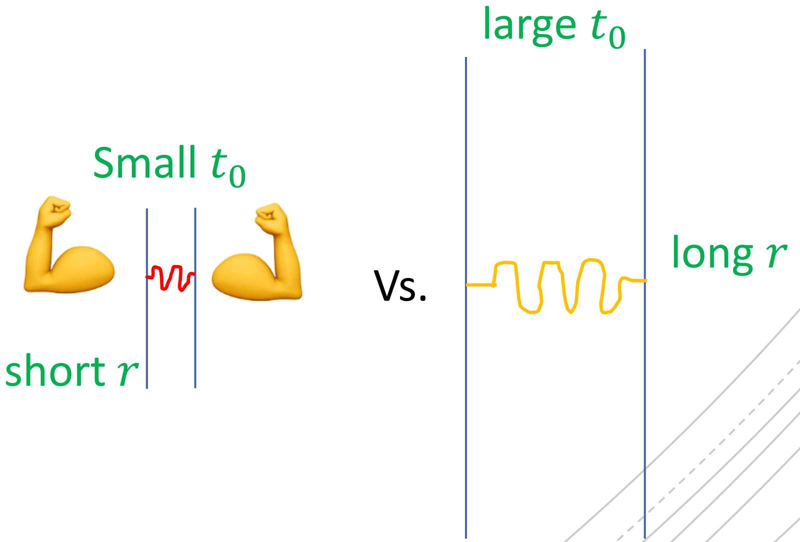
- $g(t) = t(\hat{n}) = t_0 - \left(\frac{\overline{\sigma_n}}{\overline{K_n}}\right) = r(\frac{1}{c} - \frac{\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\overline{h}})$

normal stiffness coefficient, h

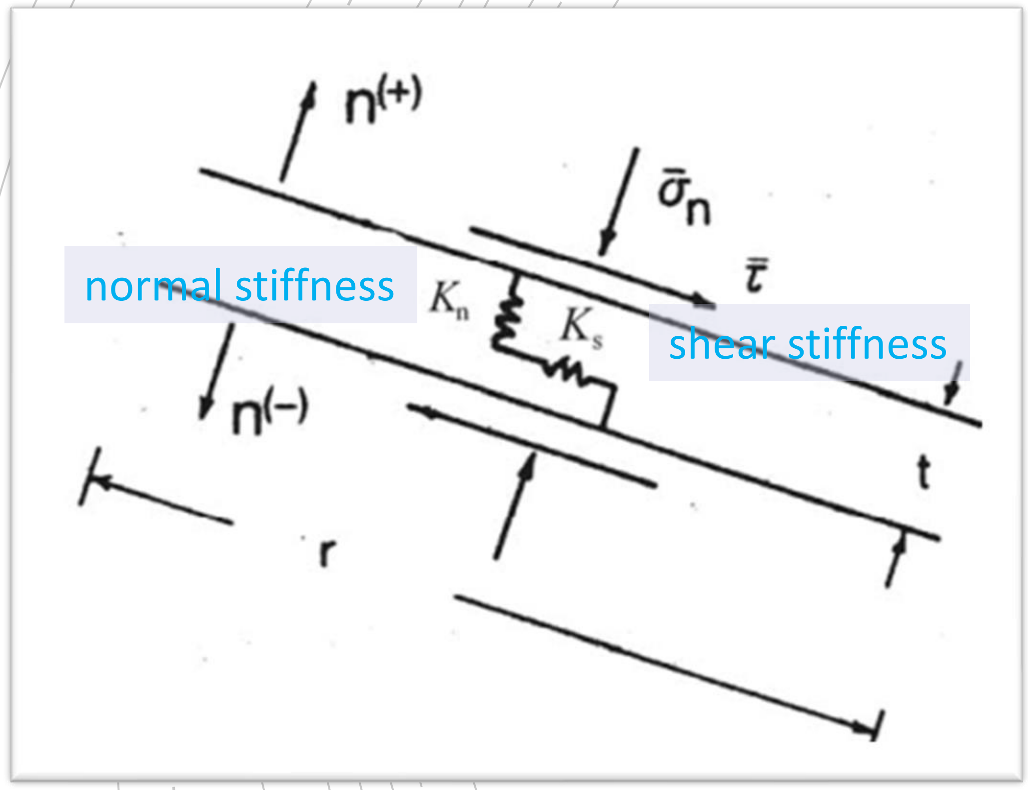
- $h = K_0 * r$, where $\mathrm{K}_0 =$ initial normal stiffness, $r =$ fracture length, $r \uparrow K_0 \downarrow$

aspect ratio, c

- $c = \frac{r}{t_0}$, where $r =$ fracture length, $t_0 =$ initial aperture, $r \uparrow t_0 \uparrow$



The density function of aperture,g(t) (Oda,1986)



σ_n = normal stress, $\overline{K_n}$ = average stiffness

- $t = t_0 - \left(\frac{\overline{\sigma_n}'}{\overline{K_n}}\right)$ Normal closure δ
- $K_n = \frac{\overline{\sigma_n}}{\delta} = \frac{1+b\overline{\sigma_n}}{a} = \frac{1+b\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{a} = \frac{1+\left(\frac{1}{t_0K_0}\right)\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\frac{1}{\mathrm{K}_0}} = \frac{\mathrm{h}+\mathrm{c}\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\mathrm{r}}$

experimentally determined

- $\overline{\sigma_n} = 0, a = \frac{1}{K_0}$, K_0 initial stiffness
- $\overline{\sigma_n} \to \infty, b = \frac{1}{t_0K_0}$, t_0 initial aperture

aspect ratio, c

- $c = \frac{r}{t_0}$, where r = crack length, t_0 = initial aperture, $r \uparrow t_0 \uparrow$

normal stiffness coefficient, h

- $K_0 = \frac{h}{r}$, where $r \uparrow K_0 \downarrow$
- $\overline{K_n} = \int_{\Omega} K_n E(\hat{n}) d\Omega = \frac{\mathrm{h}+\mathrm{c}\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\mathrm{r}} \equiv \frac{\bar{h}}{r}$
- $g(t) = t(\hat{n}) = t_0 - \left(\frac{\overline{\sigma_n}'}{\overline{K_n}}\right) = \frac{\mathrm{r}}{\mathrm{c}} - \left(\mathrm{r}\frac{\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\bar{h}}\right) = r(\frac{1}{\mathrm{c}} - \frac{\overline{\sigma_{ij}}\widehat{n_i}\widehat{n_j}}{\bar{h}})$

The density function of aperture,g(t) (Oda,1986)

$g(t)$

aspect ratio, c

$$c = \frac{r}{t_0}, r \uparrow t_0 \uparrow$$

normal stiffness coefficient, h

$$K_0 = \frac{h}{r}, r \uparrow K_0 \downarrow$$

- Cheng & Toksoz(1979) was used wave velocity through rock mass to determine the aspect ratio.
- The rock has smaller value of porosity, it would has smaller wave velocity, then the aspect ratio would has smaller value
- The aspect ratio of Navajo sandstone roughly equal to 1000

- Using the experimental results of uniaxial test, it can access the value of initial normal stiffness by the uniaxial strength of different material.
- The initial normal stiffness of fresh sandstone to medium-weathering sandstone is between the range 3.6~25.6MPa/mm, so the range of h equal to 360~2560MPa.

(where r = crack length, t_0 = initial aperture, K_0 = initial normal stiffness)

The density function of aperture,g(t) *(Oda,1986)*

g(t)

aspect ratio, c

$$c = \frac{r}{t_0}, r \uparrow t_0 \uparrow$$

- Fortin(2005) defined aspect ratio in the range $10^2 \sim 10^4$ approximately.

normal stiffness coefficient, h

$$K_0 = \frac{h}{r}, r \uparrow K_0 \downarrow$$

- The initial normal stiffness of fresh sandstone to medium-weathering sandstone is between the range 3.6~25.6MPa/mm, so the range of h equal to 360~2560MPa. (Chen,2005)

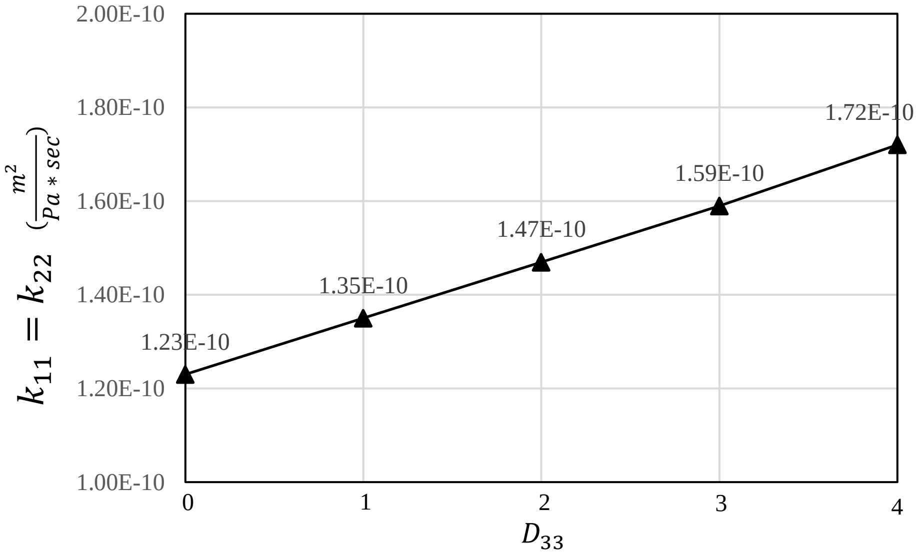
(where r = crack length, t_0 = initial aperture, K_0 = initial normal stiffness)

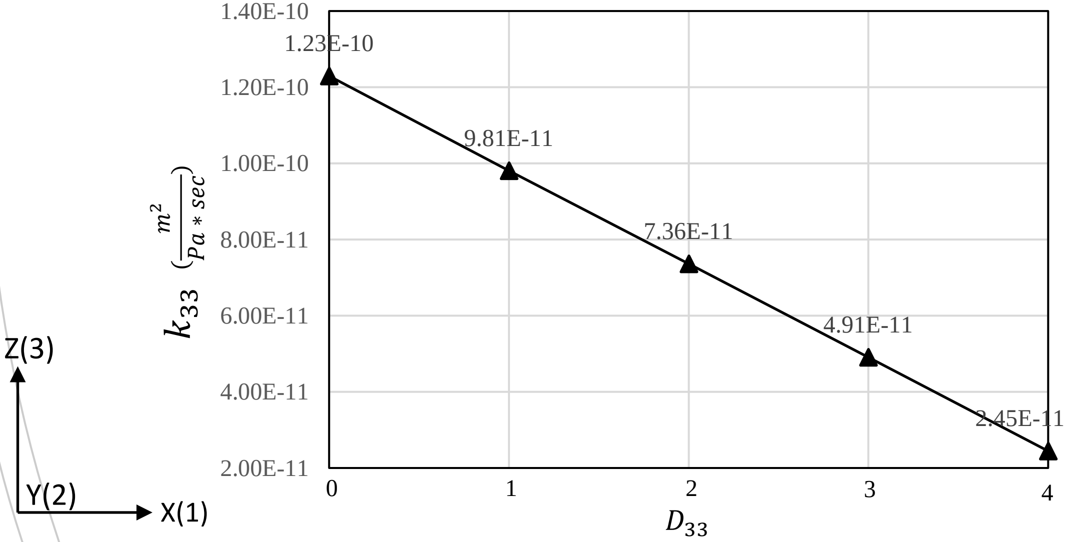
Continuum approach (crack tensor)

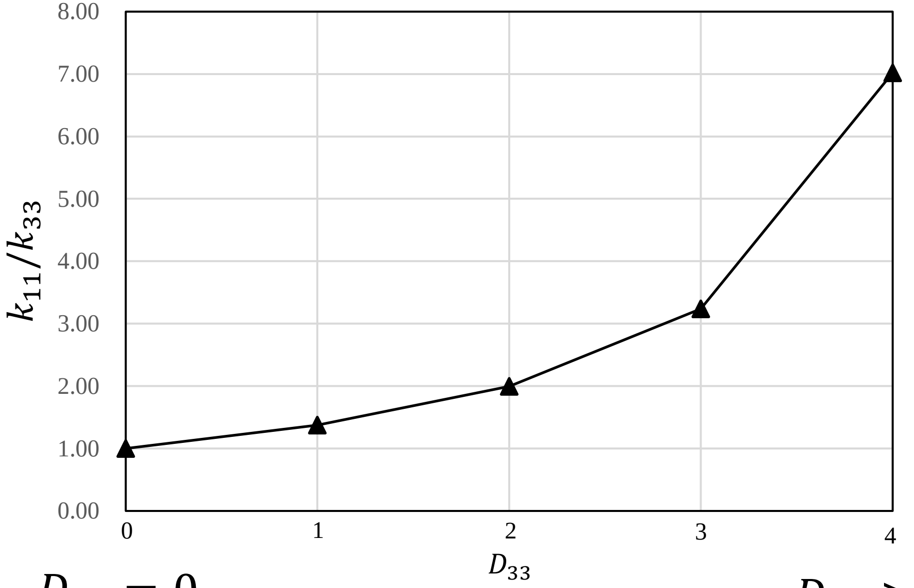
Equivalent permeability tensor

Pore water pressure distribution

(Assume the rock mass under uniform stress $\sigma_n = 0.25MPa$)







$D_{33} = 0$ isotropy → anisotropy $D_{33} > 0$

The discontinuities distributed same amount at direction 1,2,3
= isotropic permeability

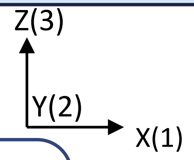
The normal vector of discontinuities account for a large proportion in direction 3
= anisotropic permeability

Continuum approach (crack tensor)

Equivalent permeability tensor

Pore water pressure distribution

(Assume the direction of maximum principal permeability parallel horizontal)



0°

Isotropic permeability

$$k_{11} = k_{22} = k_{33} = 1.23 * 10^{-10}$$

anisotropic permeability($D_{33} = 4$)

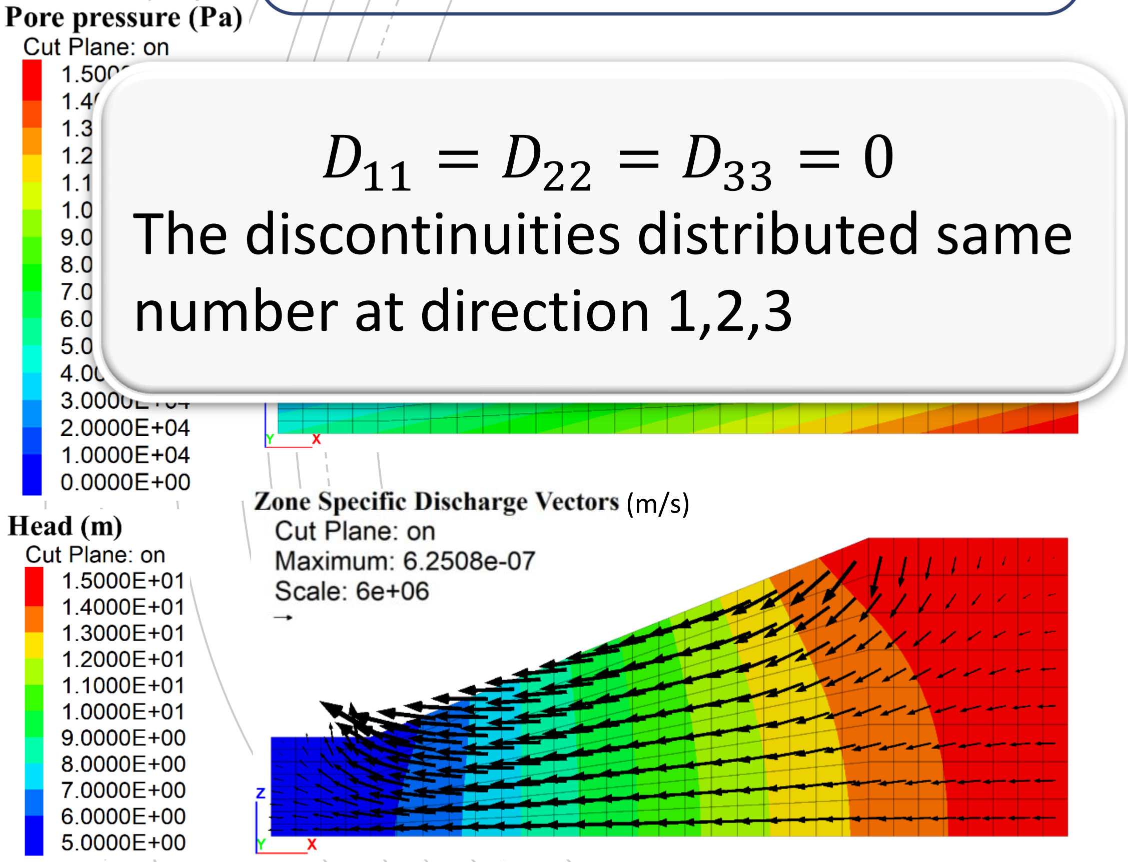
$$k_{11} = k_{22} = 1.72 * 10^{-10}; k_{33} = 2.45 * 10^{-11}$$

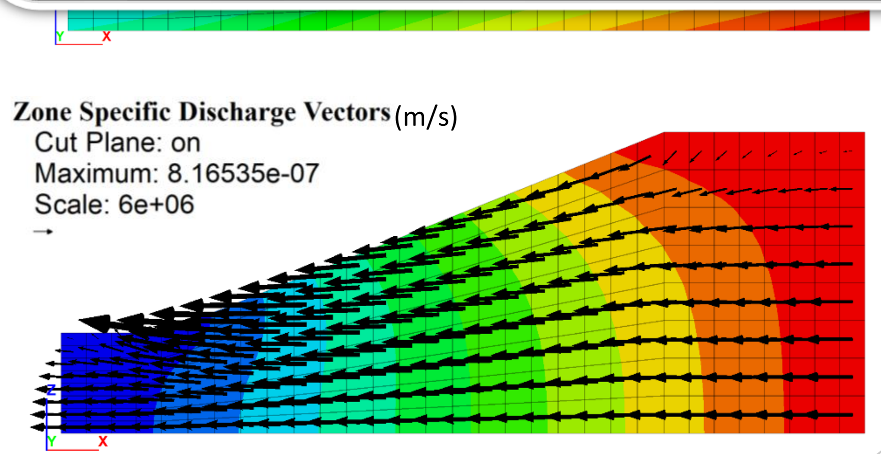
$$D_{11} = D_{22} = D_{33} = 0$$

The discontinuities distributed same number at direction 1,2,3

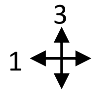
$$D_{11} = D_{22} = -2 < D_{33} = 4$$

The normal vector of discontinuities account for a large proportion in direction 3





$(unit\ of\ k = \frac{m^2}{Pa * sec})$



0°

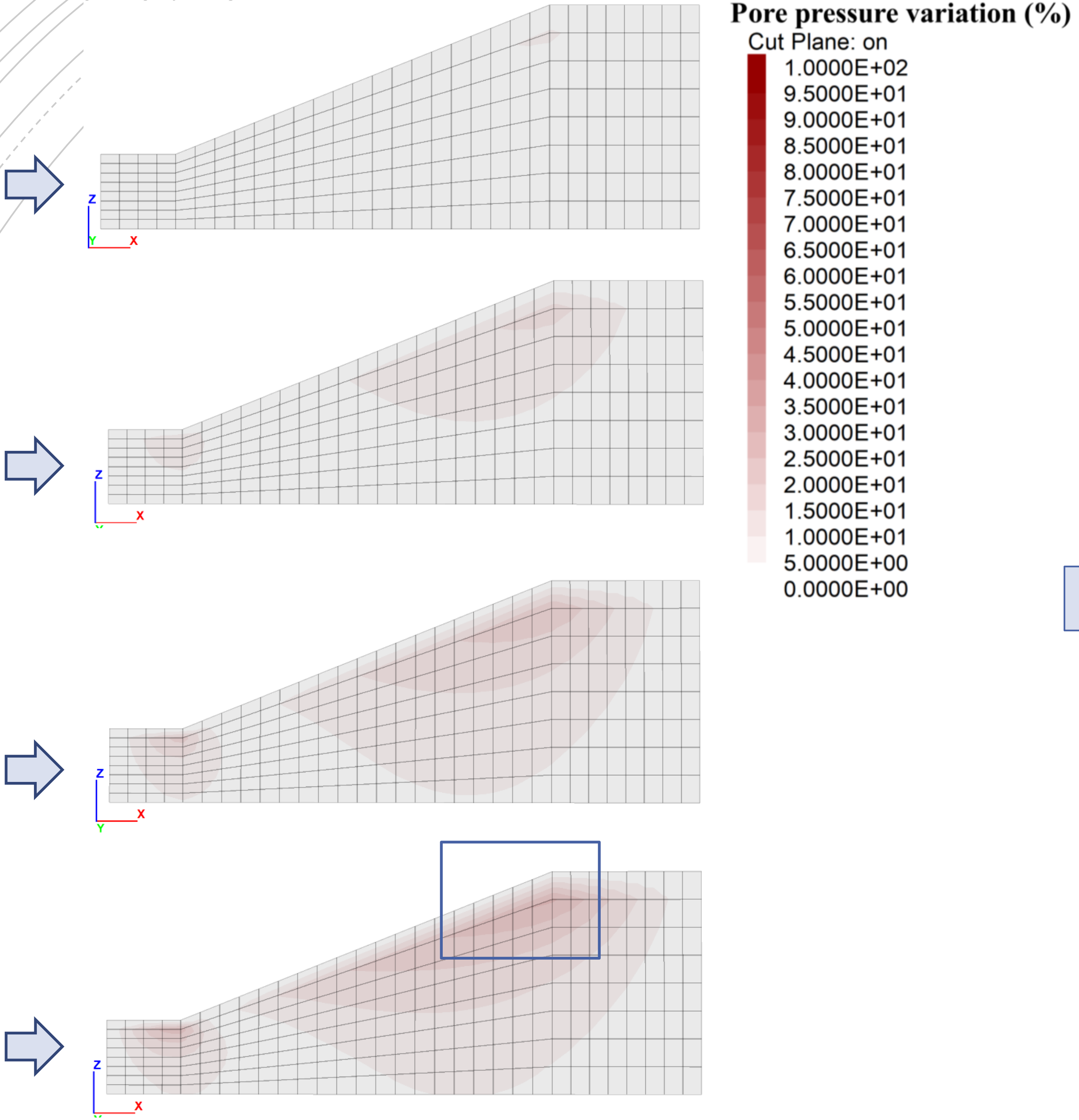
Relative variation

$$\frac{|P(D_{33}=1)-P(D_{33}=0)|}{P(D_{33}=0)}$$

$$\frac{|P(D_{33}=2)-P(D_{33}=0)|}{P(D_{33}=0)}$$

$$\frac{|P(D_{33}=3)-P(D_{33}=0)|}{P(D_{33}=0)}$$

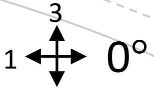
$$\frac{|P(D_{33}=4)-P(D_{33}=0)|}{P(D_{33}=0)}$$

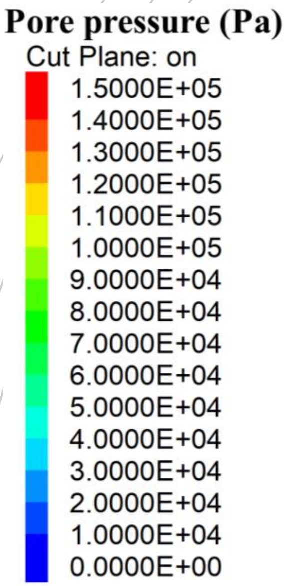


D_{33} ↗, variation ↗

Maximum variation:26%

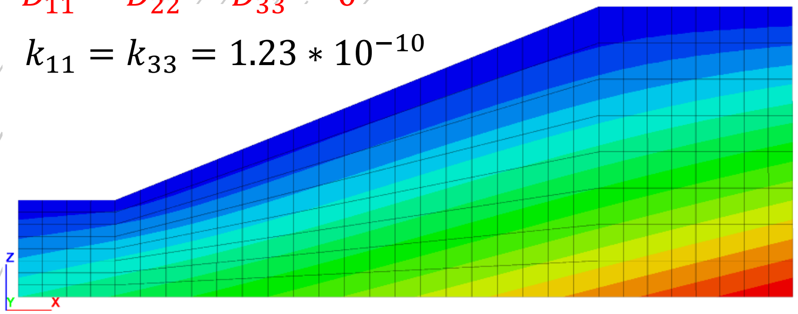
Result Ⅰ: The direction of maximum principle permeability at horizontal



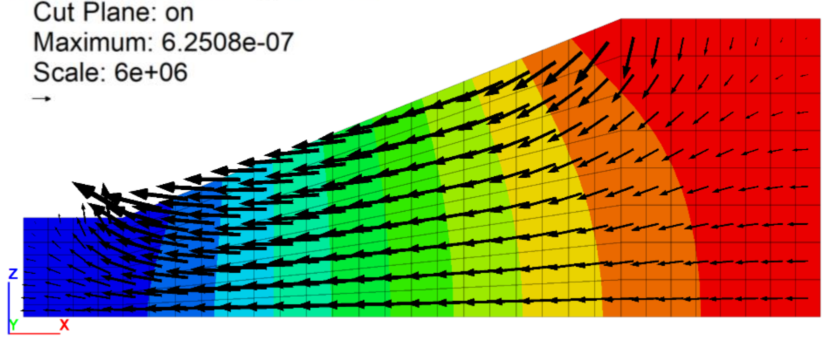


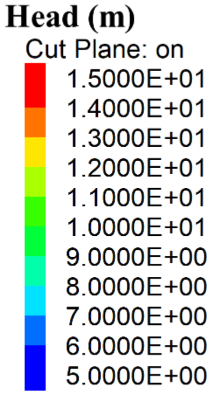
$D_{11} = D_{22} = D_{33} = 0$

$k_{11} = k_{33} = 1.23 * 10^{-10}$



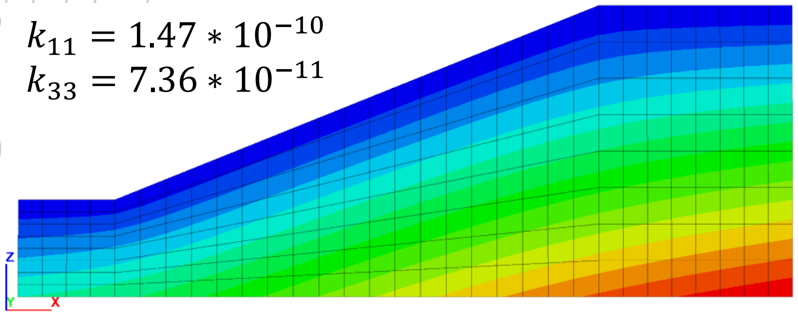
Zone Specific Discharge Vectors (m/s)
Cut Plane: on
Maximum: 6.2508e-07
Scale: 6e+06



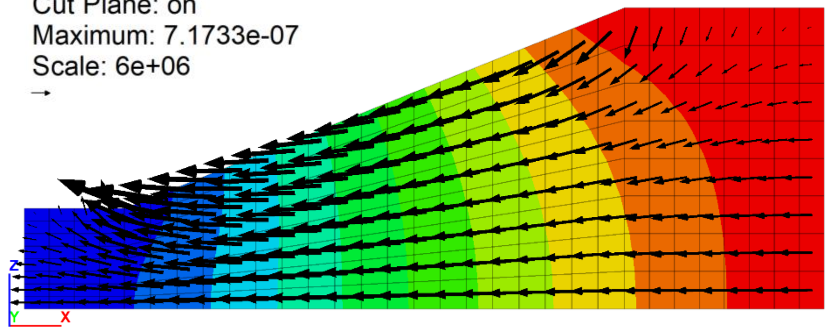


$D_{11} = D_{22} = -1; D_{33} = 2$

$k_{11} = 1.47 * 10^{-10}$
$k_{33} = 7.36 * 10^{-11}$

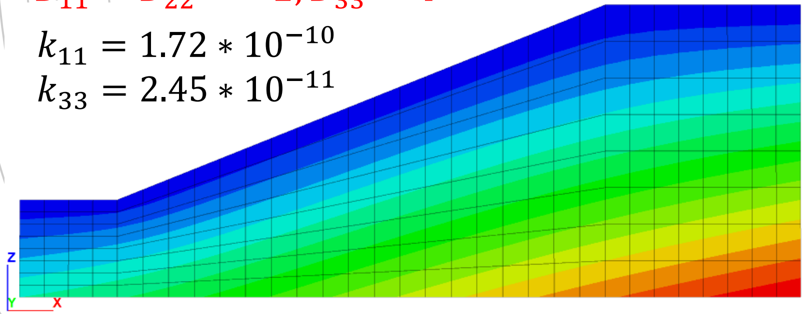


Zone Specific Discharge Vectors (m/s)
Cut Plane: on
Maximum: 7.1733e-07
Scale: 6e+06

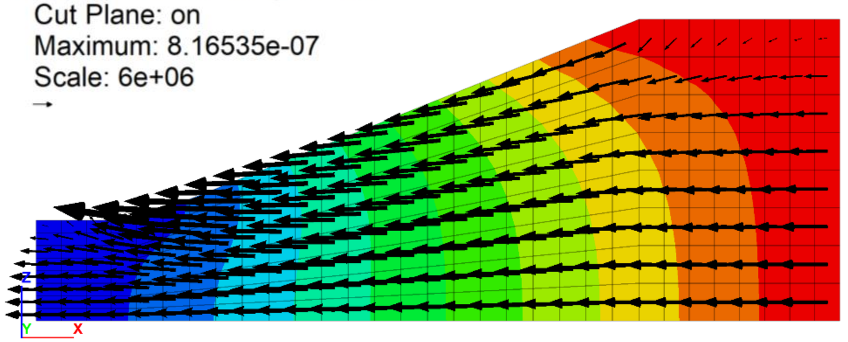


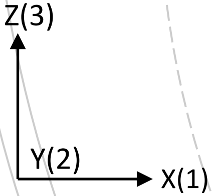
$D_{11} = D_{22} = -2; D_{33} = 4$

$k_{11} = 1.72 * 10^{-10}$
$k_{33} = 2.45 * 10^{-11}$

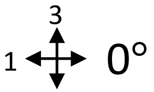


Zone Specific Discharge Vectors (m/s)
Cut Plane: on
Maximum: 8.16535e-07
Scale: 6e+06





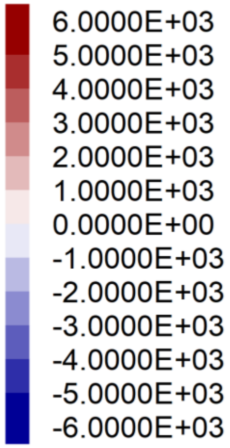
$\sigma_{33} = 0.25MPa$; Unit of $k = m^2/(Pa * sec)$



Absolute variation

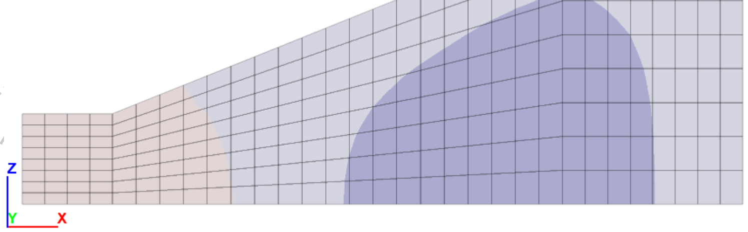
Pore pressure variation(Pa)

Cut Plane: on

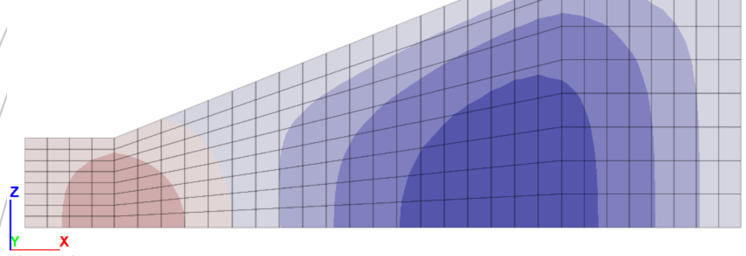


$P(D_{33} = 1) - P(D_{33} = 0)$

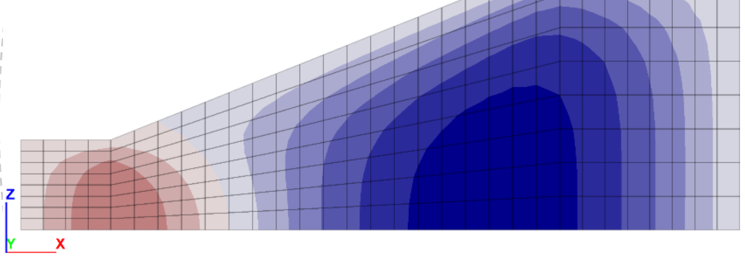
$P(D_{33} = 1) - P(D_{33} = 0)$



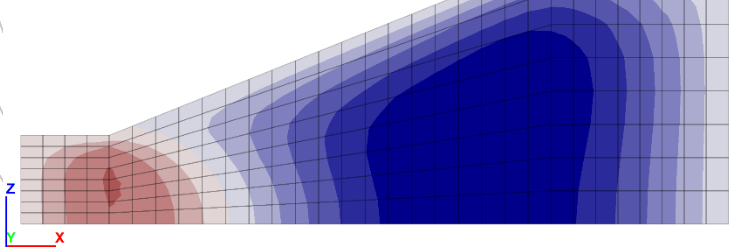
$P(D_{33} = 2) - P(D_{33} = 0)$

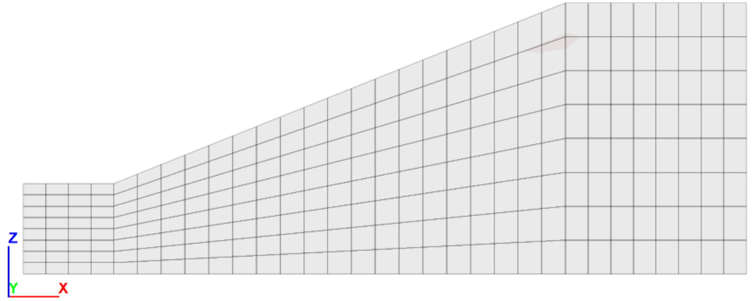


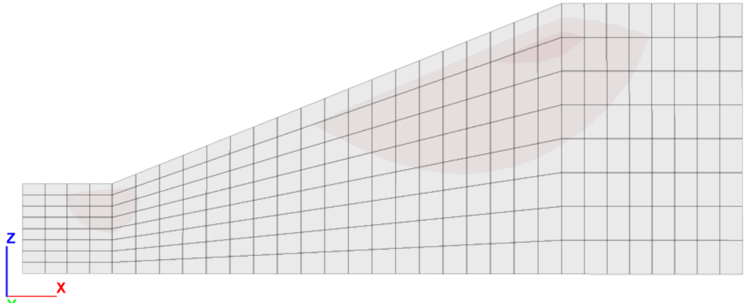
$P(D_{33} = 3) - P(D_{33} = 0)$

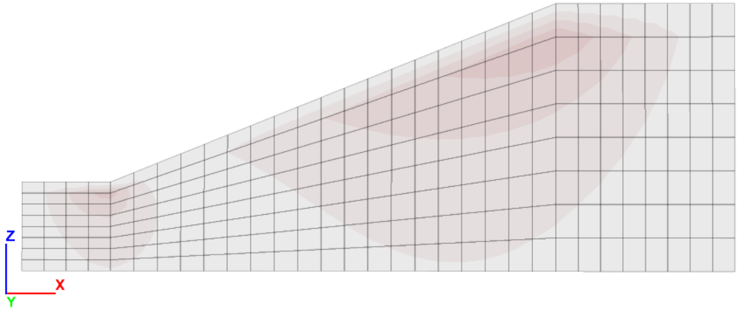


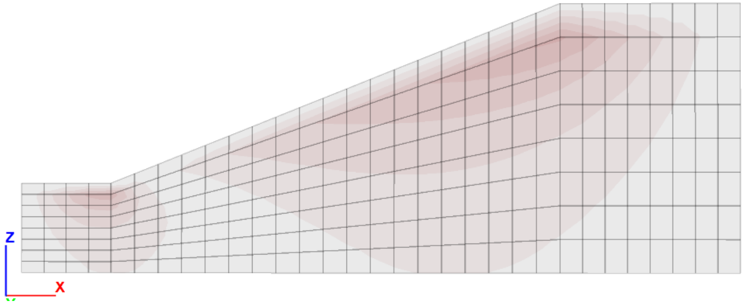
$P(D_{33} = 4) - P(D_{33} = 0)$







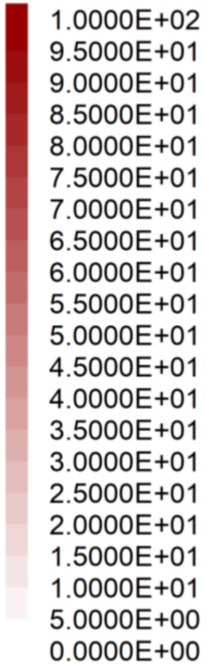




Relative variation

Pore pressure variation (%)

Cut Plane: on

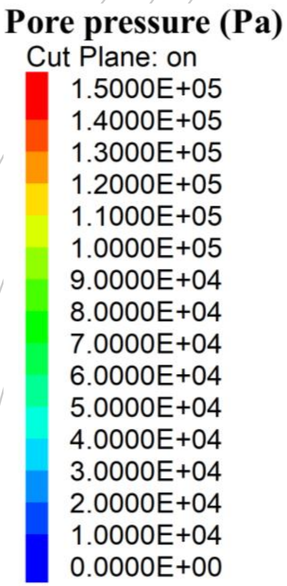


$$\frac{|P(D_{33} = 1) - P(D_{33} = 0)|}{P(D_{33} = 0)}$$

D_{33} ↗, variation ↗

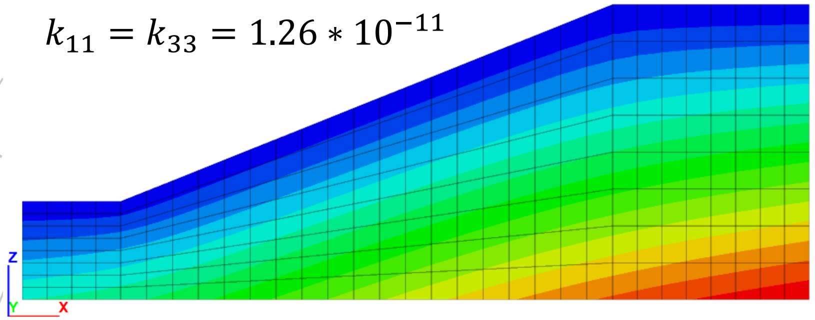
Maximum variation:26%

Result Ⅱ: The direction of maximum principle permeability parallel slope surface



$D_{11} = D_{22} = D_{33} = 0$

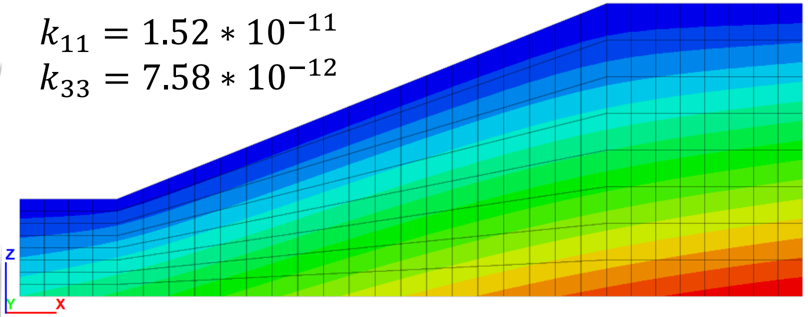
$k_{11} = k_{33} = 1.26 * 10^{-11}$



$D_{11} = D_{22} = -1; D_{33} = 2$

$k_{11} = 1.52 * 10^{-11}$

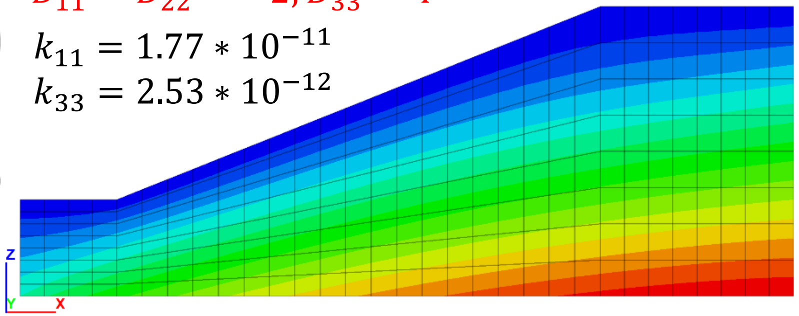
$k_{33} = 7.58 * 10^{-12}$

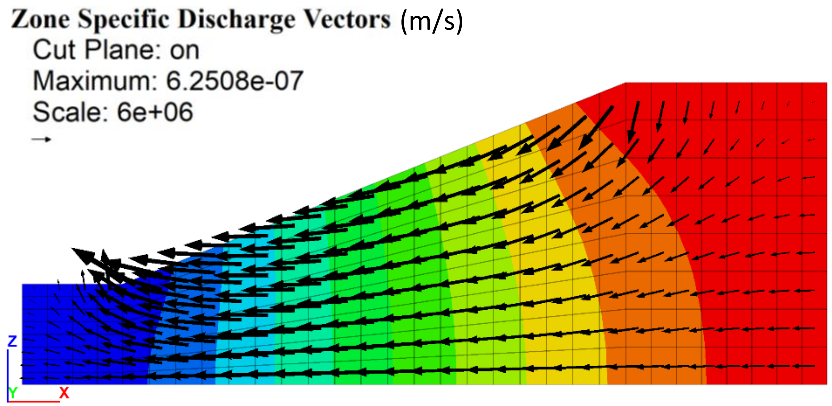


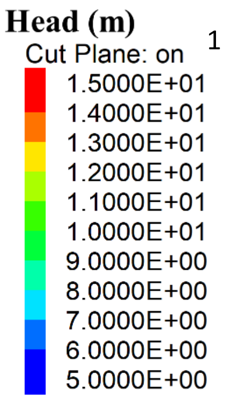
$D_{11} = D_{22} = -2; D_{33} = 4$

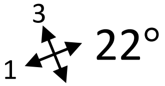
$k_{11} = 1.77 * 10^{-11}$

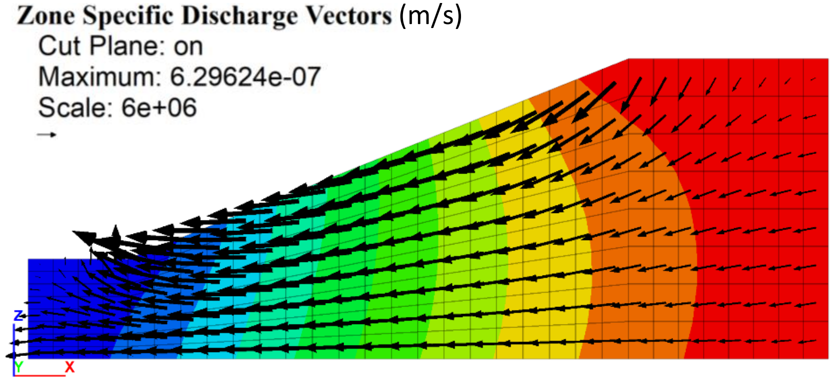
$k_{33} = 2.53 * 10^{-12}$

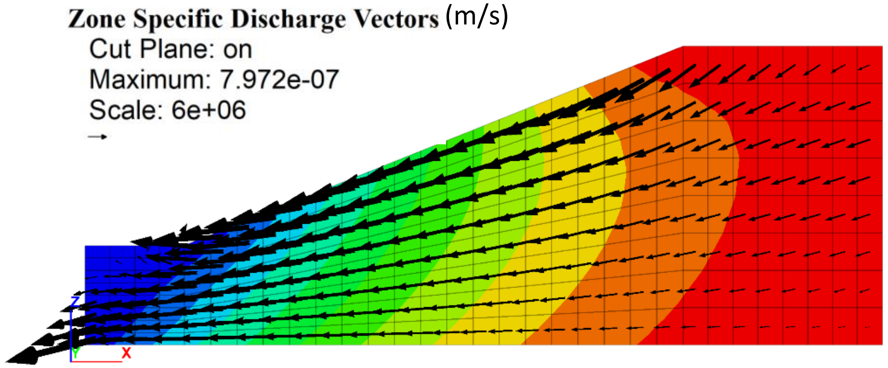


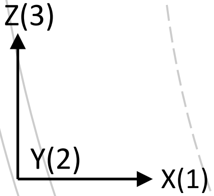




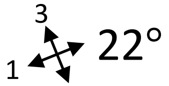








$\sigma_{33} = 0.25MPa$; Unit of $k = m^2/(Pa * sec)$



22°

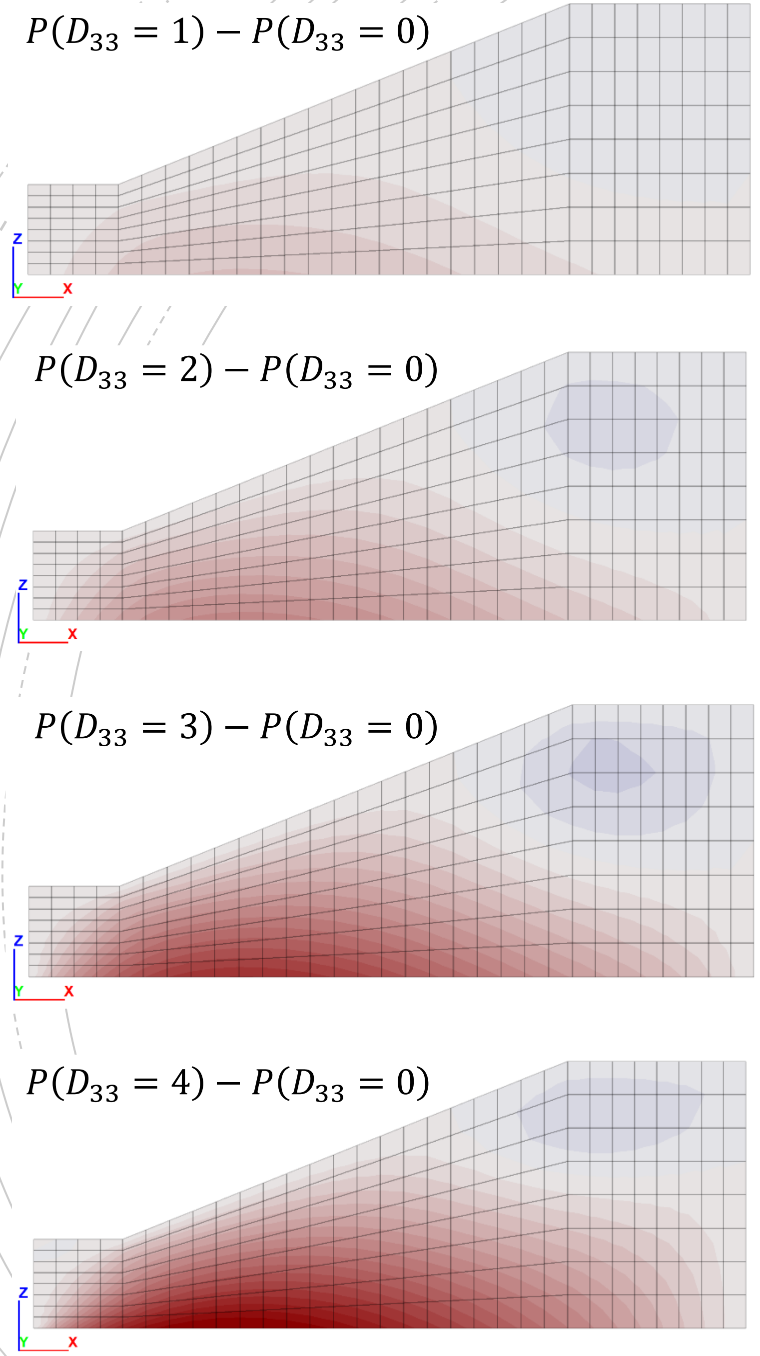
Absolute variation

Pore pressure variation(Pa)

Cut Plane: on

1.8000E+04
1.6000E+04
1.4000E+04
1.2000E+04
1.0000E+04
8.0000E+03
6.0000E+03
4.0000E+03
2.0000E+03
0.0000E+00
-2.0000E+03
-4.0000E+03
-6.0000E+03
-8.0000E+03
-1.0000E+04
-1.2000E+04
-1.4000E+04
-1.6000E+04
-1.8000E+04

$P(D_{33}=1)-P(D_{33}=0)$

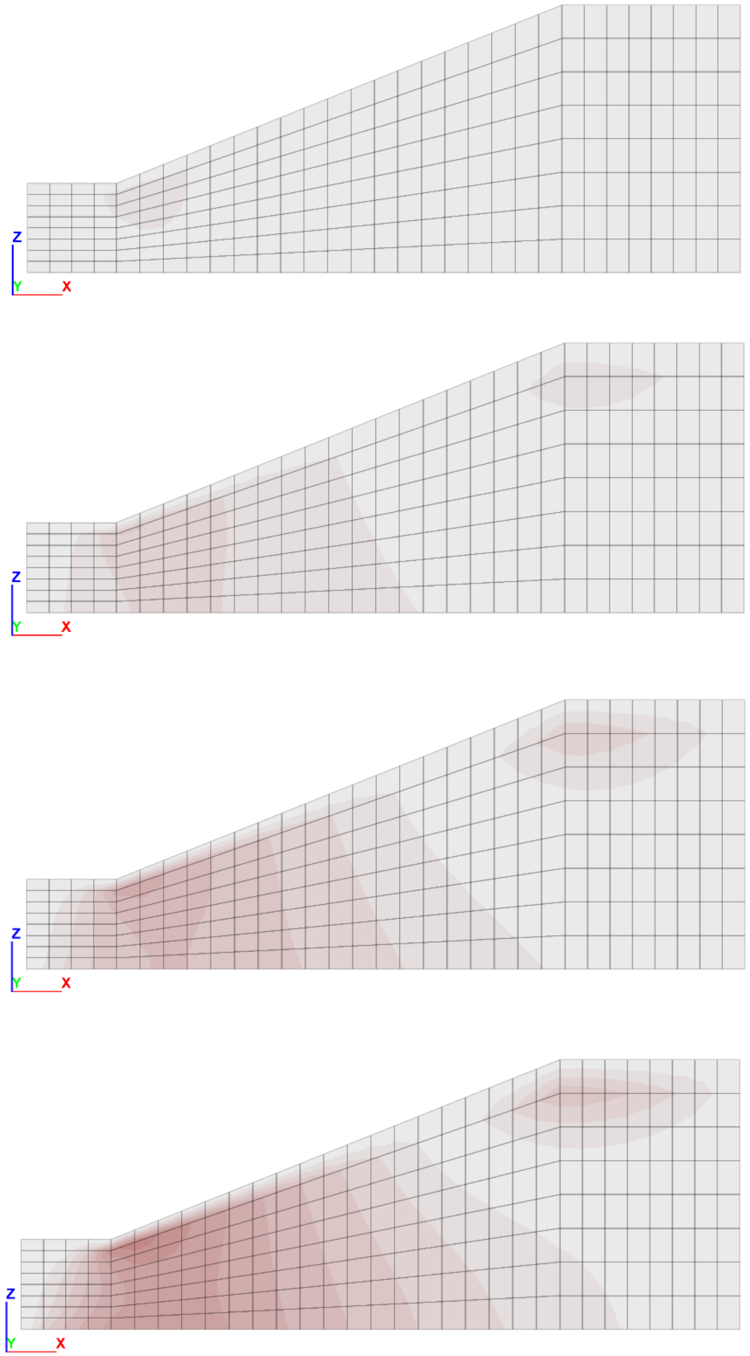


$P(D_{33}=1)-P(D_{33}=0)$

$P(D_{33}=2)-P(D_{33}=0)$

$P(D_{33}=3)-P(D_{33}=0)$

$P(D_{33}=4)-P(D_{33}=0)$



Relative variation

Pore pressure variation (%)

Cut Plane: on

1.0000E+02
9.5000E+01
9.0000E+01
8.5000E+01
8.0000E+01
7.5000E+01
7.0000E+01
6.5000E+01
6.0000E+01
5.5000E+01
5.0000E+01
4.5000E+01
4.0000E+01
3.5000E+01
3.0000E+01
2.5000E+01
2.0000E+01
1.5000E+01
1.0000E+01
5.0000E+00
0.0000E+00

$$\frac{|P(D_{33}=1)-P(D_{33}=0)|}{P(D_{33}=0)}$$

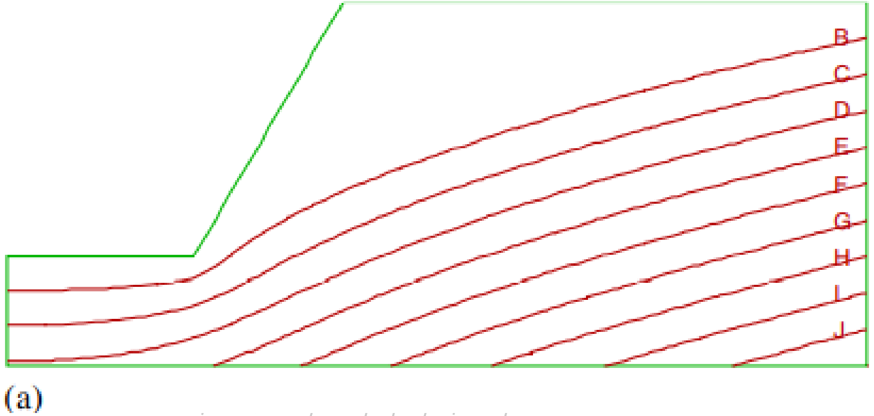
D_{33} ↗, variation ↗

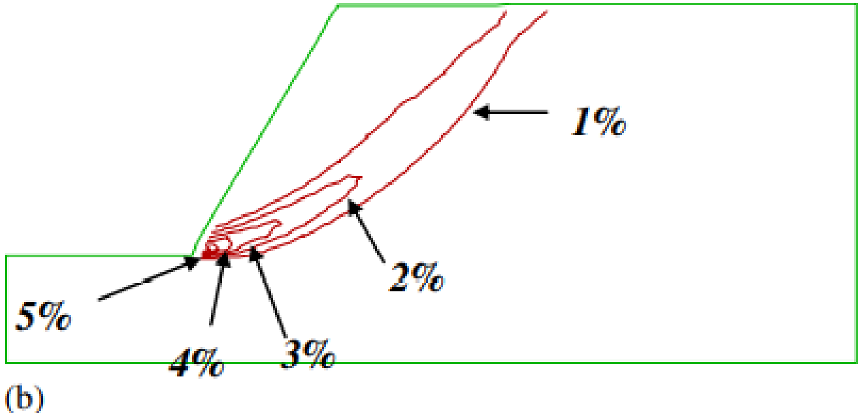
Maximum variation:44%

Effects of anisotropic of hydraulic conductivity on slope stability.

Isotropic hydraulic conductivity

FOS=2.0

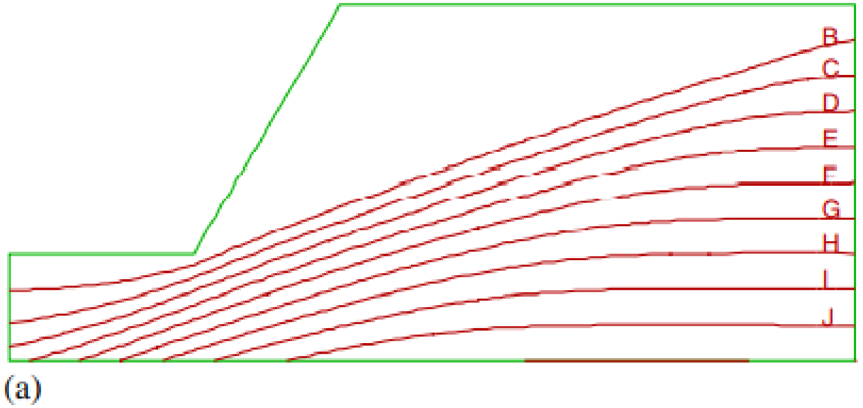


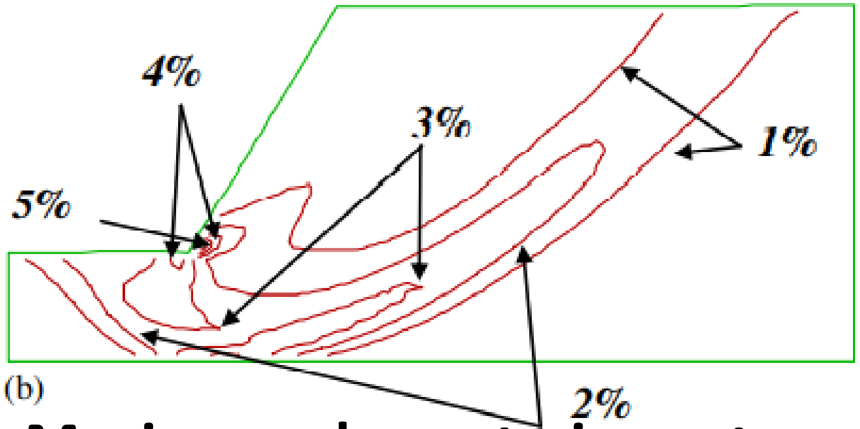


The currently selected domain for mechanical analysis may be insufficiently large to eliminate the boundary constraint on the development of the failure surface (bottom(b)).

Anisotropic ratio of hydraulic conductivity $k_x/k_y = 100$

FOS=2.0



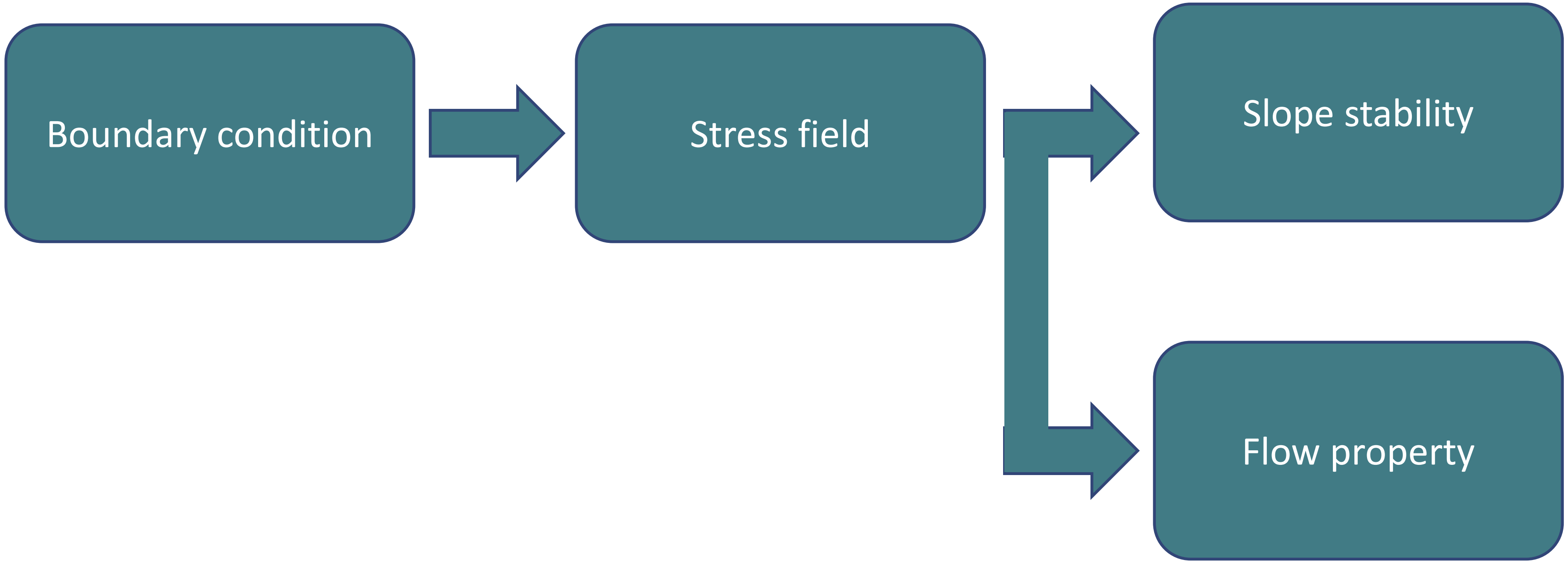


the simulated results still revealed that the effect of the hydraulic conductivity
anisotropy on the PWP has a significant influence on effective normal stress and, thus, shear strength along the failure surface.

PWP contours **Maximum shear strain contours**

(Dong et al., 2006)

Boundary effect

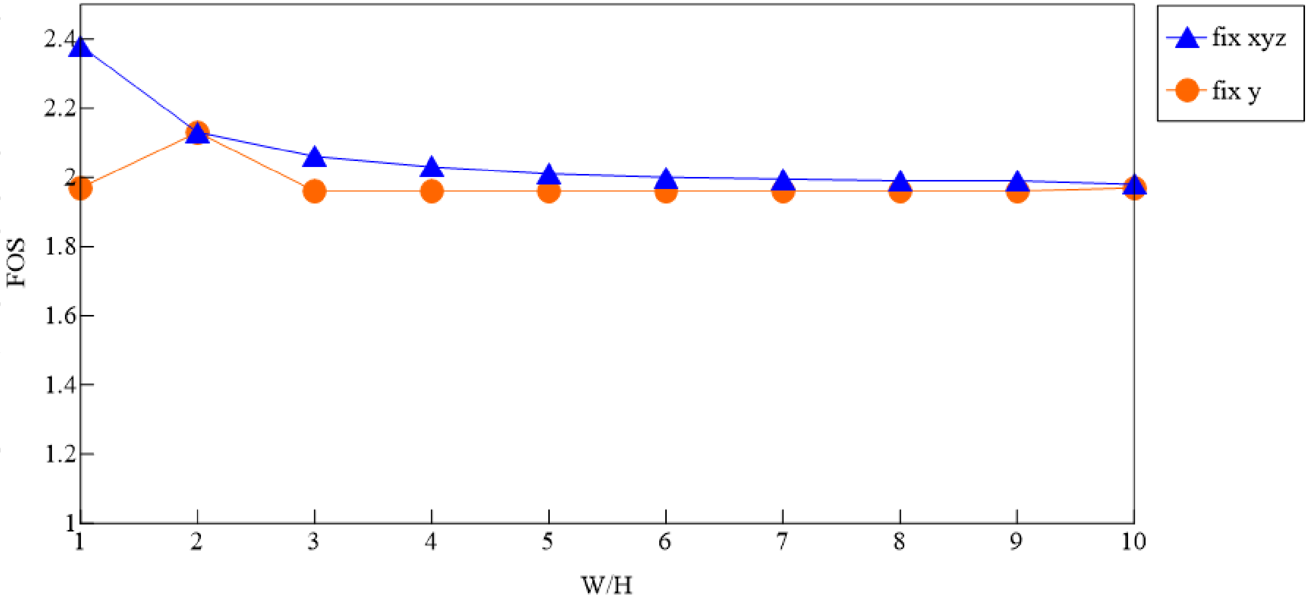


Boundary condition

Stress field

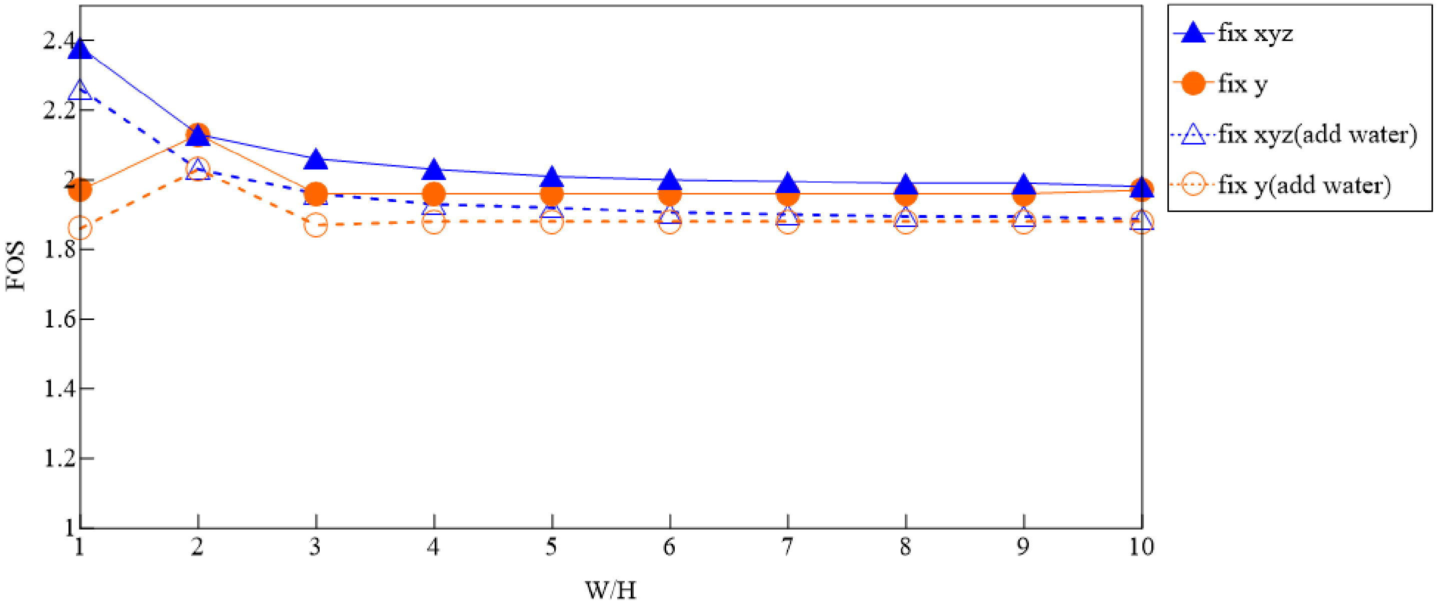
Slope stability

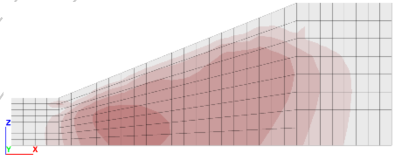
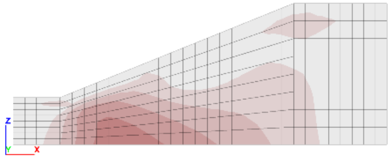
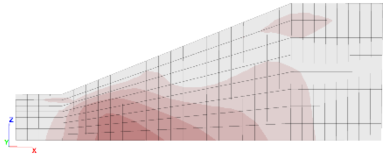
Flow property

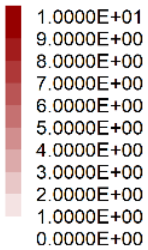


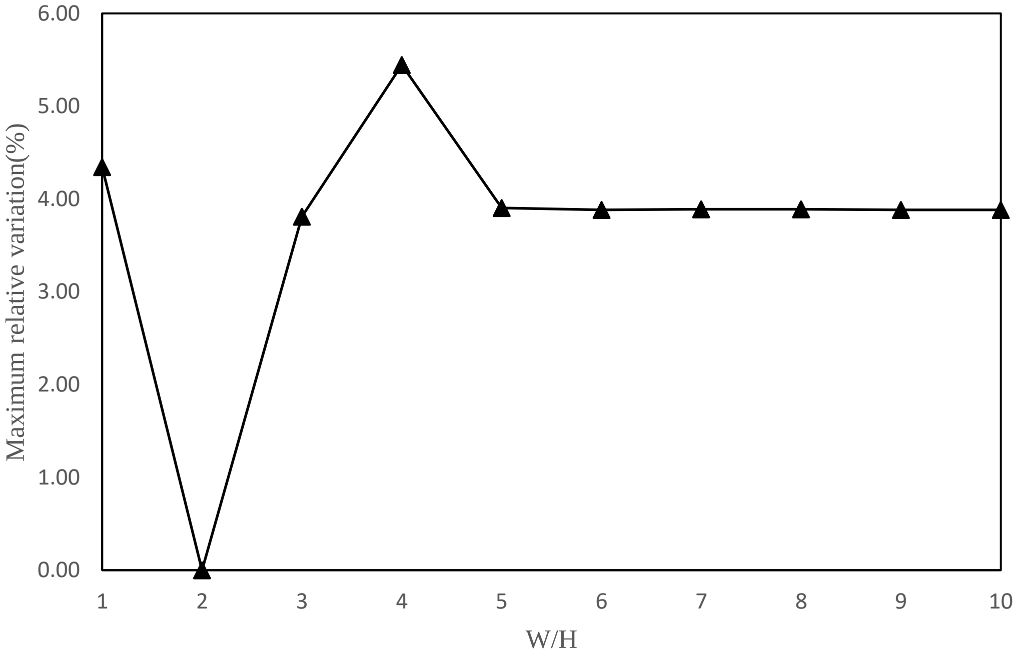
Chugh, A. K. (2003). On the boundary conditions in slope stability analysis. Int. J. Numer. Anal. Methods Geomech.

- For W =H ratio less than 5, the differences between 2-D and 3-D FoS values are significant.

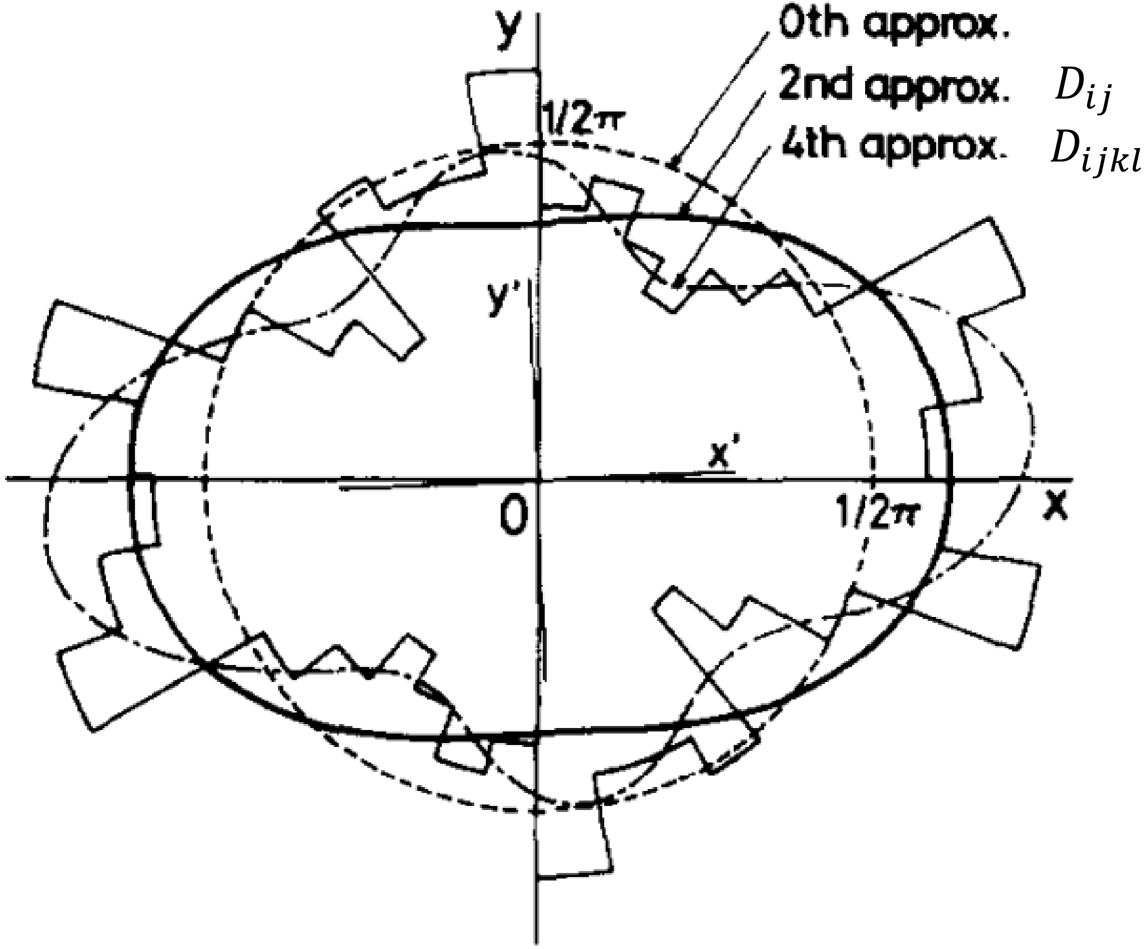


Comparison of different boundary condition (fix y vs. fix xyz)			
Differences in pore water pressure	W/H=1 (at y=5) 	W/H=5 (at y=1) 	W/H=10 (at y=1) 
Maximum relative variation(%)	4.34	4.7	4.7





Directional density function



Normal vector $\boldsymbol{N}_{i_1 i_2 \ldots i_n} = \langle n_{i_1} n_{i_2} \ldots n_{i_n} \rangle$

“distribution density” $f(\boldsymbol{n}) = \frac{1}{N} \sum_{\alpha=1}^{N} \delta(\boldsymbol{n} - \boldsymbol{n}^{(\alpha)}).$

$$\int f(\boldsymbol{n})\,\mathrm{d}\boldsymbol{n} = 1, \quad \int n_{i_1} \ldots n_{i_n} f(\boldsymbol{n})\,\mathrm{d}\boldsymbol{n} = \langle n_{i_1} \ldots n_{i_n} \rangle$$

$$f(\boldsymbol{n}) = \frac{1}{4\pi}[D + D_{ij} n_i n_j + D_{ijkl} n_i n_j n_k n_l + \cdots]$$

$$f(\boldsymbol{n}) = \frac{1}{4\pi}[D + D_{ij} n_i n_j]$$