Explicit analytical solution for the advection-dispersion transport equation in a radial two-zone confined aquifer

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Outline

- Introduction
- Methodology
- Results and Discussion
- Future work

Introduction

Background

Freshwater is essential for human survival and well-being

Global distribution of the world's water

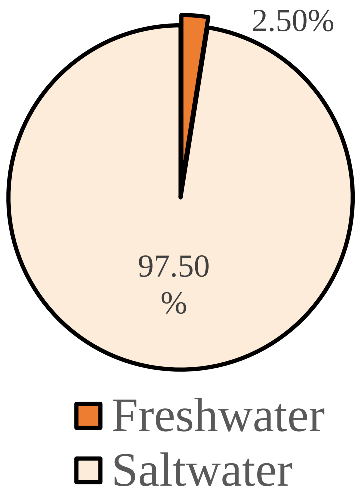


Fig. 1

Proportion of freshwater

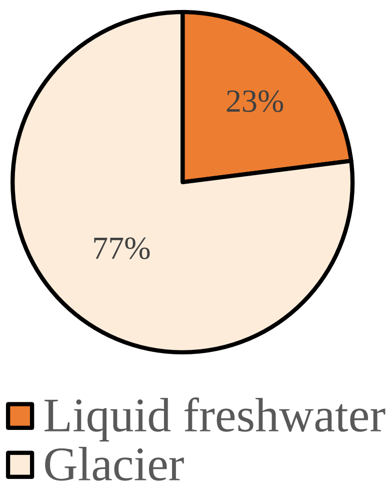


Fig. 2

Proportion of liquid freshwater

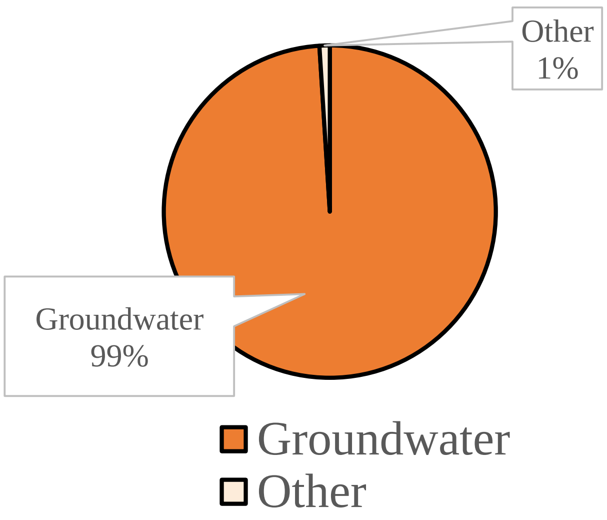


Fig. 3

Introduction

Background

Problems with groundwater contamination have grown increasingly severe worldwide over the past few decades.

Impact: Human health

- Contaminated groundwater will pose a hazard to human health.

Impact: Study on groundwater system

- It has motivated many theoretical and experimental studies on the transport of contaminants dissolved in the groundwater system.

Introduction

Background

Analytical models in which the advection dispersion equations (ADE) is solved is continually being developed .

$$D\frac{\partial^2 c}{\partial x^2} - v\frac{\partial c}{\partial x} = R\frac{\partial c}{\partial t}$$

One-dimensional for ADE

- Analytical models can be used for a modeling tool for contaminant transport in groundwater system.

Introduction

Literature

Uniform flow field: Analytical solutions

- Analytical solutions for advective dispersive transport problems in a uniform flow field have been reported in the literature. (Van Genuchten,1982; Batu,1993; Chen and Liu,2011)

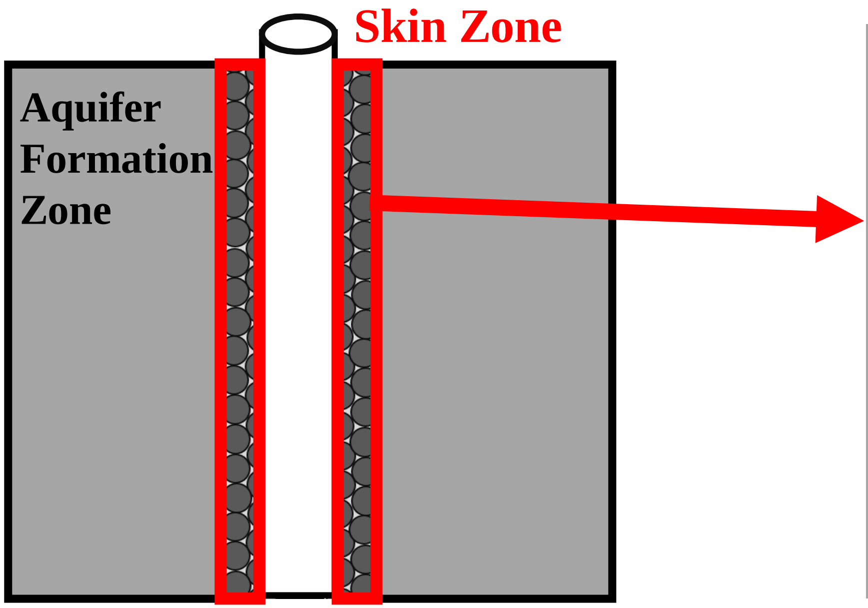
Non-uniform flow field: Complicated and difficult

- Deriving analytical solutions for solute transport in a non-uniform flow field is complicated and difficult.
- Because of the dependence of the flow field on the spatial locations. (Chen et al., 2002)
- Solute transport in a radial flow field created by an injection well can be viewed as a special case of solute transport in a non-uniform flow field.

Literature

Skin zone: Small area around the well screen

- It is known that there is a small area around the well screen showing anomalous hydrogeological properties called the skin zone. (Barker and Herbert, 1982;Houben, 2015)



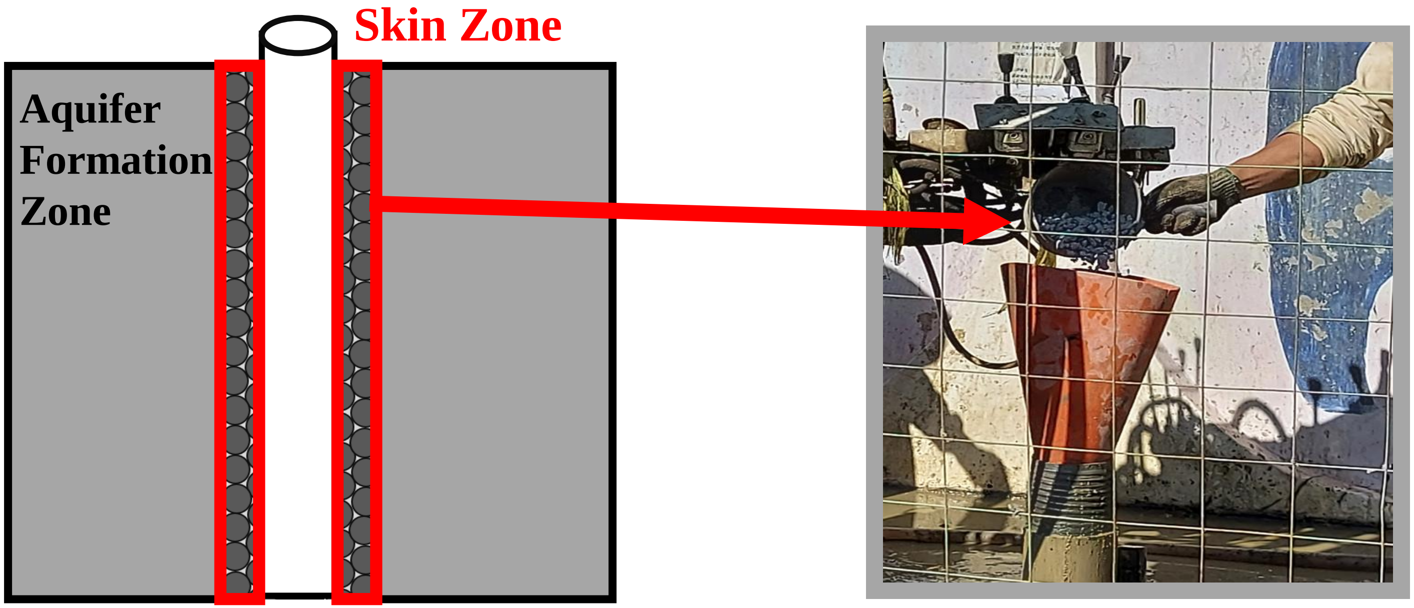
The radial thickness of the skin zone generally extends from a few millimeters to several meters.

Introduction

Literature

Skin zone: Effect

- There may be changes in porosity, permeability and dispersivity in the skin zone due to the intrusion of drilling mud or extensive well development. (Novakowski,1989;Yeh et al,1982;Chen et al,2012)



Introduction

Literature

Analytical solutions for solute transport in a radial flow field have a variety of practical applications related to the study of Well Tracer Test and aquifer remediation by pumping.

Well Tracer Test	Detection of tracer concentration in produced water over time
Aquifer remediation by pumping	Remove contaminants from aquifers

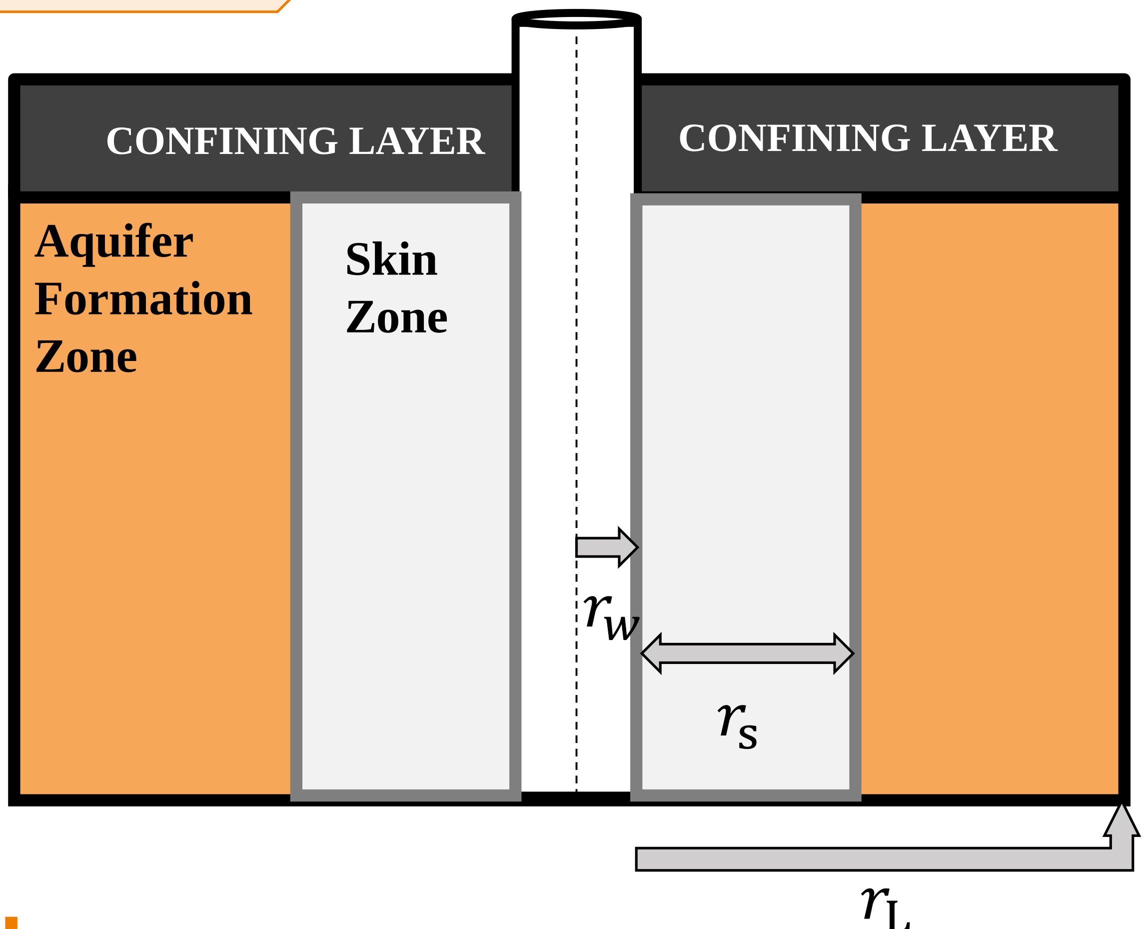
Introduction

Objection

Develop explicit analytical solutions for reactive transport in a radial two-zone confined aquifer will be derived for equilibrium-controlled sorption processes.

Methodology

Conceptual model



- r_w : well radial [L]
- r_s : skin zone thickness [L]
- r_L : total length [L]

Methodology

Skin zone function:

$$\frac{A_s \alpha_s}{r} \frac{\partial^2 C_1(r,t)}{\partial r^2} - \frac{A_s}{r} \frac{\partial C_s(r,t)}{\partial r} = R_s \frac{\partial C_s(r,t)}{\partial t}, r_w \leq r \leq r_s + r_w, t > 0$$

Where $A_s = \frac{Q}{2\pi \phi_s B}$

s : skin zone
C : concentration $[ML^{-3}]$
t : time since injection $[T]$
r : radial distance $[L]$
Q : constant injection rate $[L^3 T^{-1}]$
B : aquifer thickness $[L]$
ϕ : porosity [-]
α : radial dispersivity $[L]$

Methodology

Aquifer zone function:

$$\frac{A_f \alpha_f}{r}\frac{\partial^2 C_f(r,t)}{\partial r^2} - \frac{A_f}{r}\frac{\partial C_f(r,t)}{\partial r} = R_f \frac{\partial C_f(r,t)}{\partial t}, r_s + r_w \leq r \leq r_L, t > 0$$

Where $A_f = \frac{Q}{2\pi\phi_f B}$

f : aquifer formation zone
C : concentration $[ML^{-3}]$
t : time since injection $[T]$
r : radial distance $[L]$
Q : constant injection rate $[L^3T^{-1}]$
B : aquifer thickness $[L]$
ϕ : porosity [-]
α : radial dispersivity $[L]$

Methodology

Initial conditions:

- $C_1(r,t) = 0, r_w \leq r \leq r_\mathrm{s} + r_\mathrm{w}, t > 0$
- $C_2(r,t) = 0, r_\mathrm{s} + r_\mathrm{w} \leq r \leq r_L, t > 0$

Boundary conditions:

- $C_1(r = r_w, t) = C_0$
- $\frac{\partial C_2(r = r_L, t)}{\partial r} = 0$
- $C_1(r = r_1, t) = C_2(r = r_1, t)$
- $\alpha_1 \frac{\partial C_1(r = r_\mathrm{s} + r_\mathrm{w}, t)}{\partial r} = \alpha_2 \frac{\partial C_2(r = r_\mathrm{s} + r_\mathrm{w}, t)}{\partial r}$

Methodology

Analytical solution derivation

1. Use the Laplace transform
2. Use variable changes
3. Use Generalized Integral Transform Technique(GITT)
4. Use a series of Inverse transform
5. Resulting in the explicit analytical solution

→ Eigenvalue Discussion

↓

Convergence test

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Mathematical mode

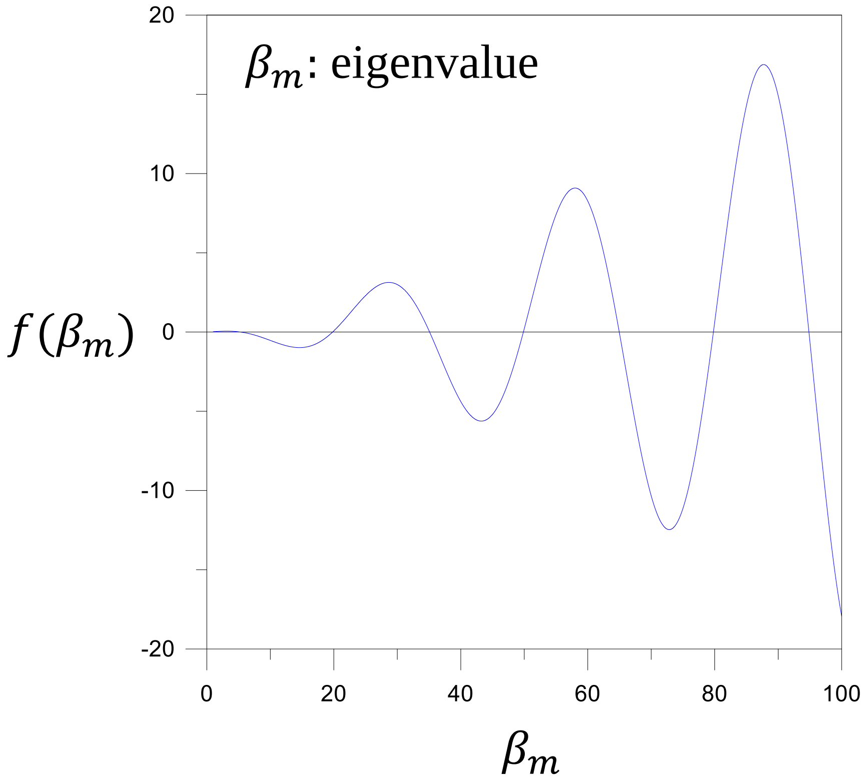
The solution strategy adopted herein is by Chen et al. (2016)

Results and Discussion

Results and Discussion

Eigenvalue problem

Eigenfunction graph

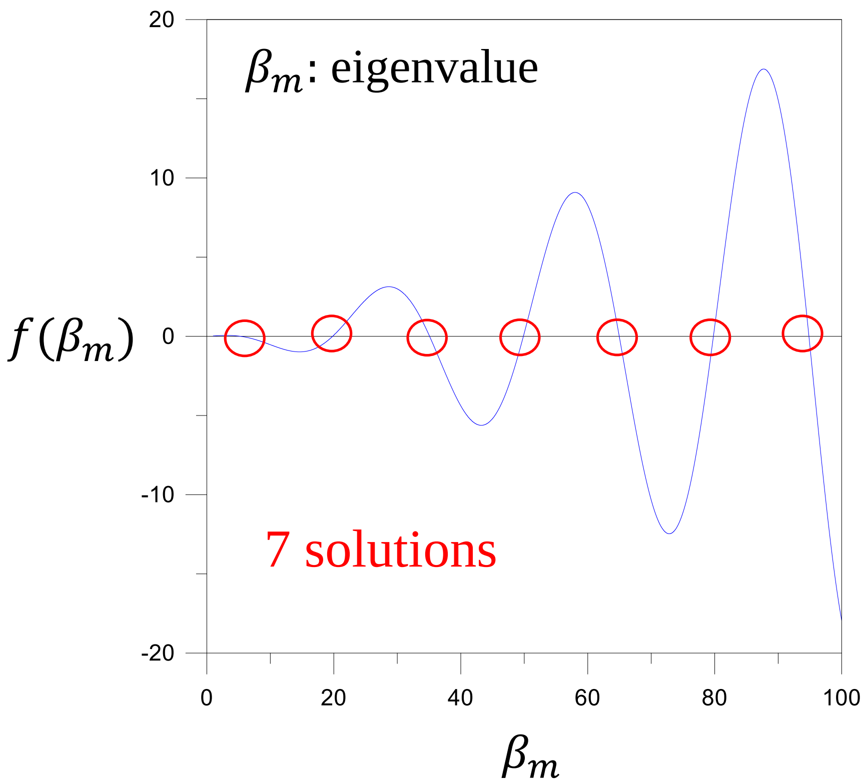


Parameters	Values
well radial , r_w [L]	25
skin zone thickness, r_s[L]	0.5
total length, r_L [L]	0.1
initial concentration, $C_0[ML^{-3}]$	40
radial dispersivity in skin zone, α_s $[L]$	2.5
radial dispersivity in aquifer zone, α_f $[L]$	2.5
porosity, ϕ_1 [-]	0.1
porosity, ϕ_2 [-]	0.1
constant injection rate, $Q[L^3T^{-1}]$	5

Need to choose the eigenvalue that to get $f(\beta_m)$ =0

Eigenvalue problem

Eigenfunction graph



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Need to choose the eigenvalue that to get $f(\beta_m)$ =0

Future work

- Use the convergence tests on eigenvalues.
- Use the fortran-based computer program code for the derived analytical solutions.
- Development of the numerical model to check the explicit analytical solutions.

Thanks