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National Central University

NATIONAL CENTRAL UNIVERSITY - COLLEGE OF EARTH SCIENCES
GRADUATE INSTITUTE OF APPLIED GEOLOGY

Effect of stratigraphic model uncertainty at a given site on its liquefaction potential index: Comparing two random field approaches

Chien, W. Y., Lu, Y. C., Juang, C. H., Dong, J. J., & Hung, W. Y. (2022). Effect of stratigraphic model uncertainty at a given site on its liquefaction potential index: Comparing two random field approaches. *Engineering Geology*, ***309***, 106838.

Presenter: Hoai-Han Le

Advisor: Prof. Jia-Jyun Dong

Co-Advisor: Dr. Yu-Chen Lu

Date: 2023/01/13

CONTENT

1. INTRODUCTION

Liquefaction potential index (LPI)

- Assessment soil liquefaction hazards
- Integrated effect of soil liquefaction: 0-20m
- Data from: seismic parameters and in-situ testing data
- Expressed as an equation: FS = CRR / CSR

LPI	Damage potential
0	Very low
$0 < LPI \leq 5$	Low
$5 < LPI \leq 15$	High
$LPI > 15$	Very high

FS : factor of safety

CRR : cyclic resistance ratio

CSR : cyclic stress ratio

Examples of liquefaction consequences (NASEM, 2016)

What will occur if the project site is lack of in-situ data?

1. INTRODUCTION

What will occur if the project site is lack of in-situ data?

Interpolation & extrapolation data: geological strata and LPI distribution

High uncertainty
Geological information between boreholes may not change much

Difficulty
for

Evaluate LPI
Fully explain for spatial variability

1. INTRODUCTION

Solve this problems?

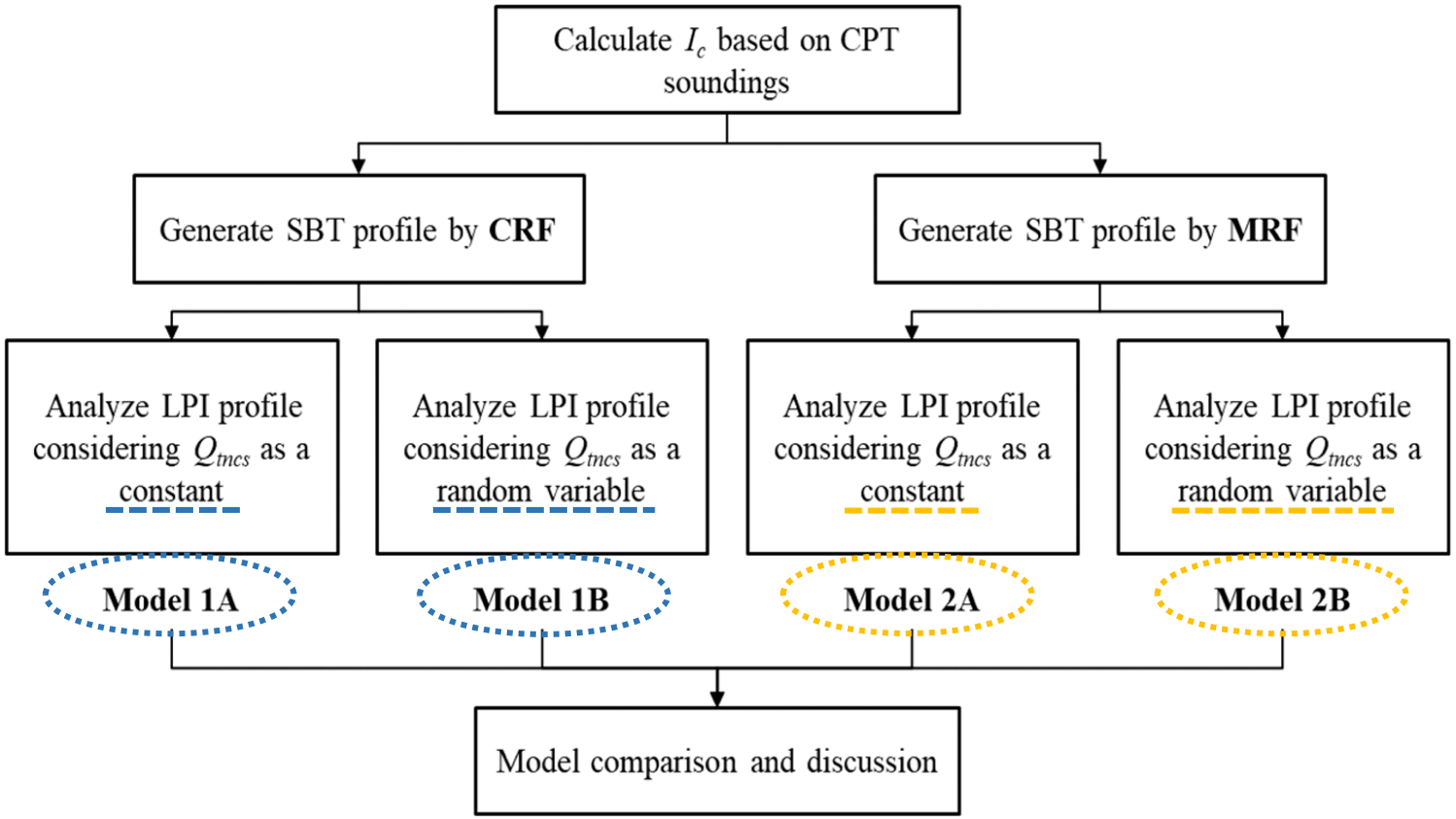
Random field theory

Continuous random field (conditional random field, CRF): using continuous functions
Discontinuous random field (Markov random field, MRF): using discrete functions

- Stratigraphic models
- Uncertainty of stratigraphic models
- Liquefaction potential index LPI
- Influence of relationship between uncertainties in the stratigraphic models & LPI

CONTENT

2. METHODOLOGY



I_c : Soil behavior type index

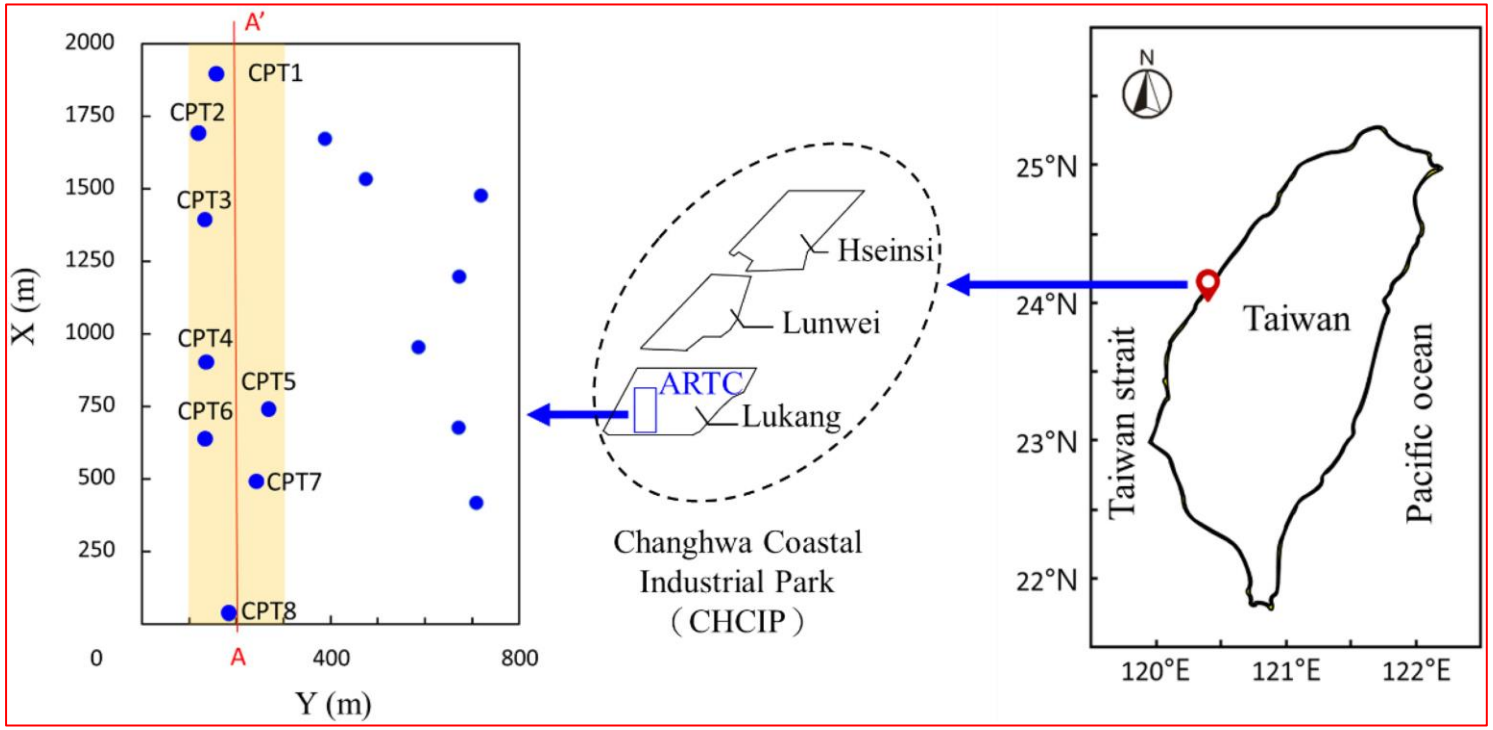
SBT : Soil Behavior Type

Q_{tncs} : corrected cone tip resistance (q_c)

Zone: Soil behavior types (SBT) (Robertson, 1990)

1. Sensitive, fine grained
2. Organic soils-peats
3. Clays-clay to silt clay
4. Silt mixtures clayey silt to silt clay
5. Sand mixtures; silt sand to sandy silt
6. Sands; clean sands to silt sands
7. Gravelly sand to sand
8. Very stiff sand to clayey sand
9. Very stiff fine grained

2. METHODOLOGY



Soil behavior type(SBT)	Soil behavior type index(I_c)	Description
3	$2.95 < I_c < 3.60$	Clay
4	$2.60 < I_c < 2.95$	Silt mixture: clayey silt to silty clay
5	$2.05 < I_c < 2.60$	Sand mixture: silty sand to sandy silt
6	$1.31 < I_c < 2.05$	Sands: clean sand to silty sand

Fig. Liquefiable site at Lukang township, Changhua county, Taiwan (ARTC site)

- Materials: silty sand, fine sand

- 15 CPT points: cone tip resistance, sleeve friction, pore water pressure

- AA': 8 CPT points with depth: 10m

2. METHODOLOGY

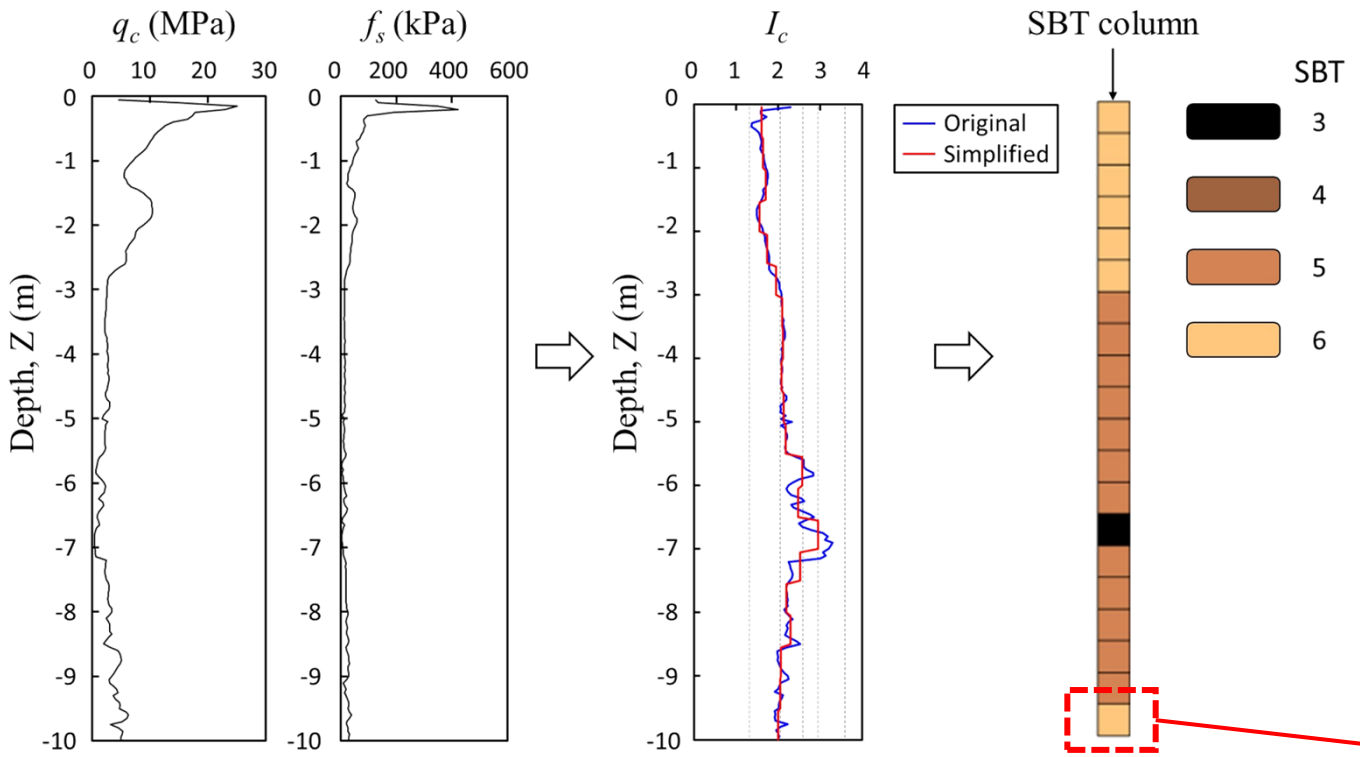


Fig. CPT data and the derived SBT and I_c (Using CPT8 as an example)

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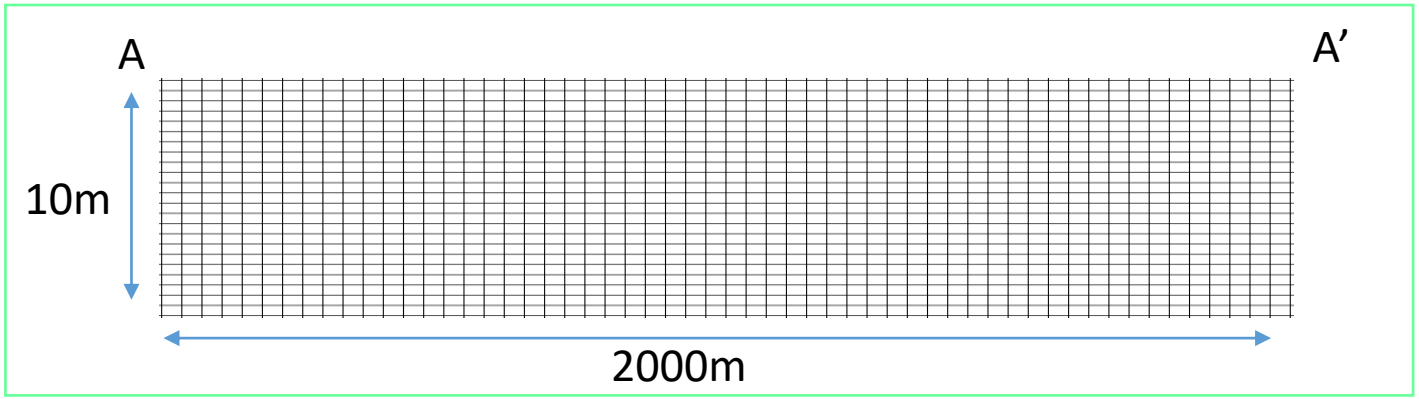

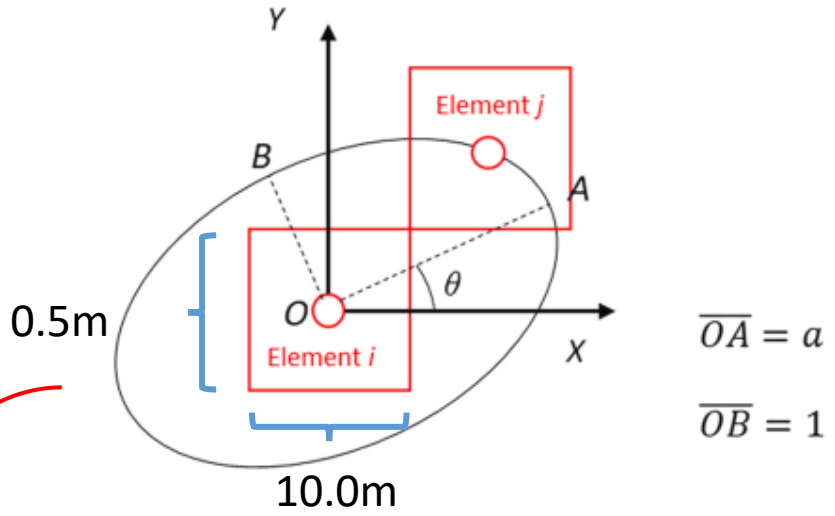

Spatial Correlation Model
(after Li et al., 2016)

2. METHODOLOGY

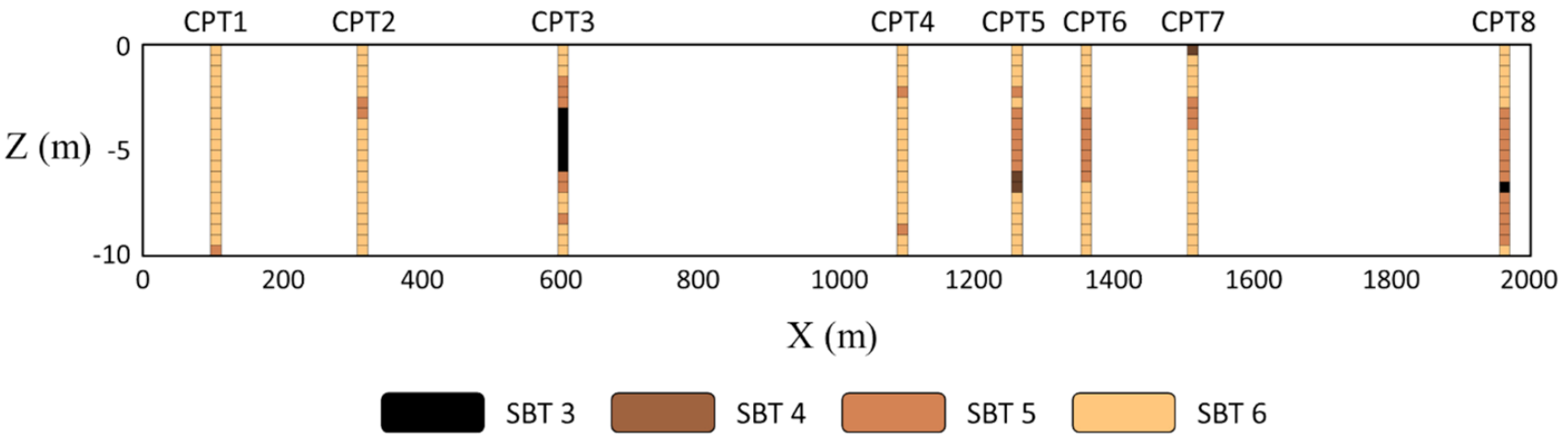

Fig. SBT distribution at CPT locations

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CRF

Step 1: De-trending

Step 2: Using *Maximum Likelihood Estimation* to find the statistical parameters of the site I_c

Step 3: Using *Cholesky decomposition* to simulate the unconditional random field of I_c

Step 4: Using *Kriging method* to establish a conditional random field of I_c

Step 5: Establishing an *SBT random field*

MRF

Step 1: Establishing the *neighborhood system* and determining the sampling order

Step 2: *Calculating the probability* of occurrence of each SBT

Step 3: Determination of SBT using *Markov-chain Monte Carlo (MCMC)* simulation

CONTENT

3. RESULTS AND DISCUSSION

3.1. Comparison of stratigraphic models

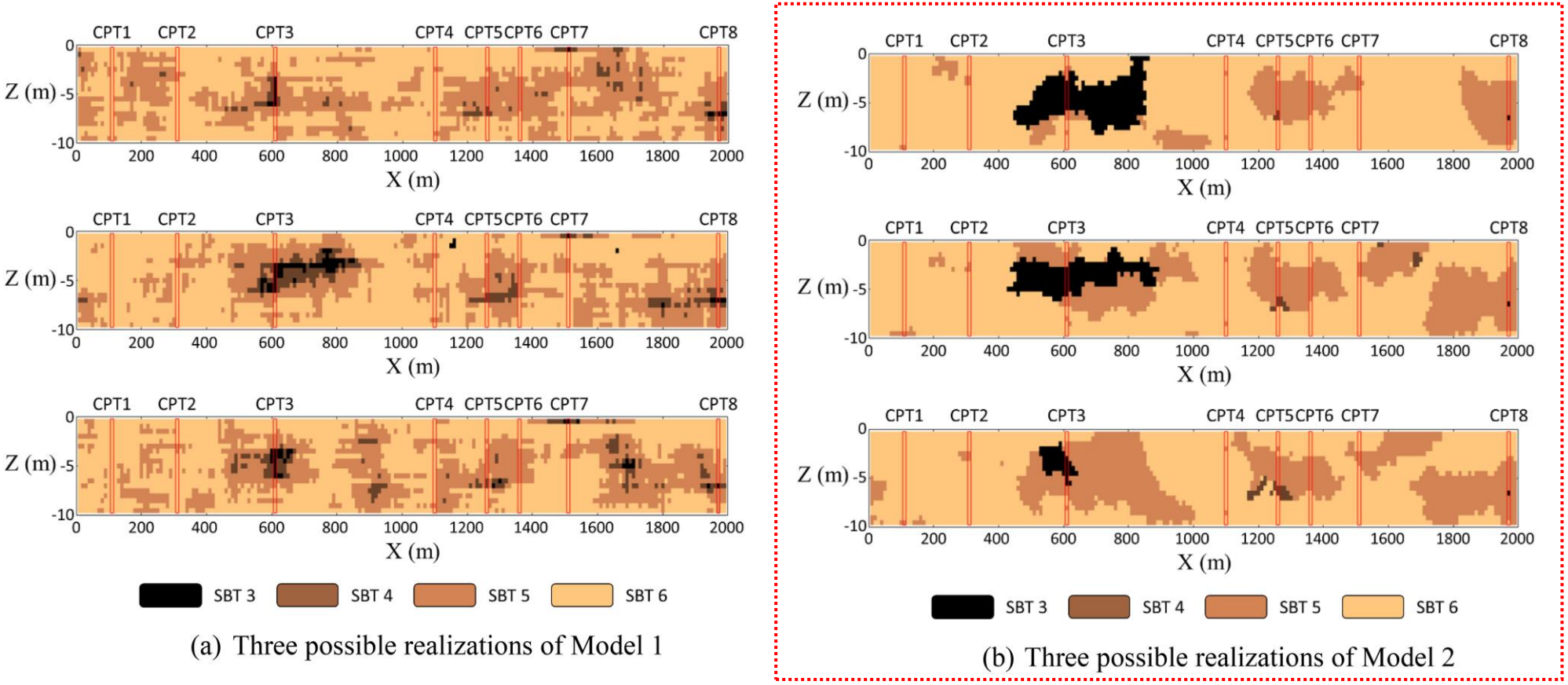
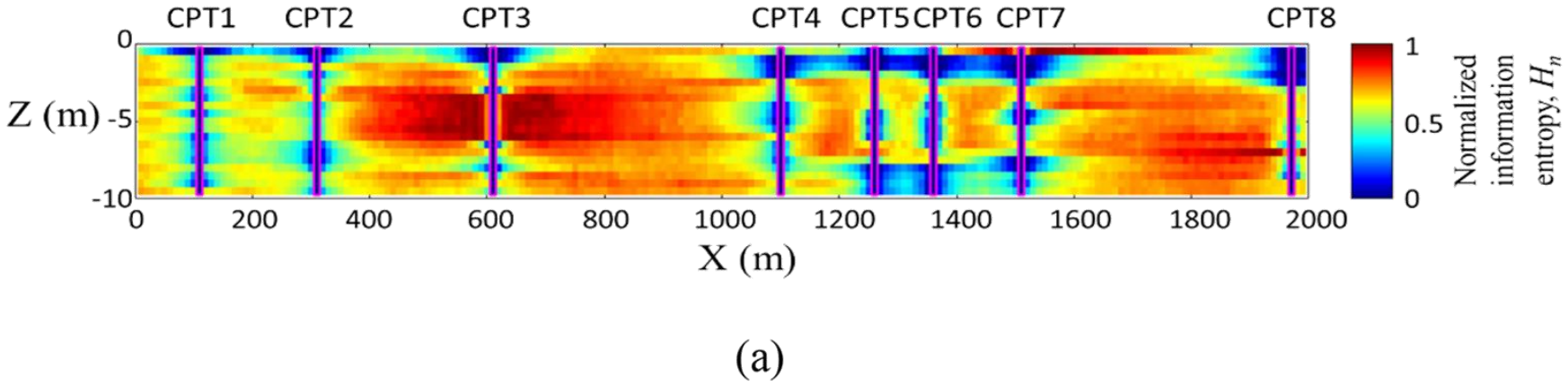


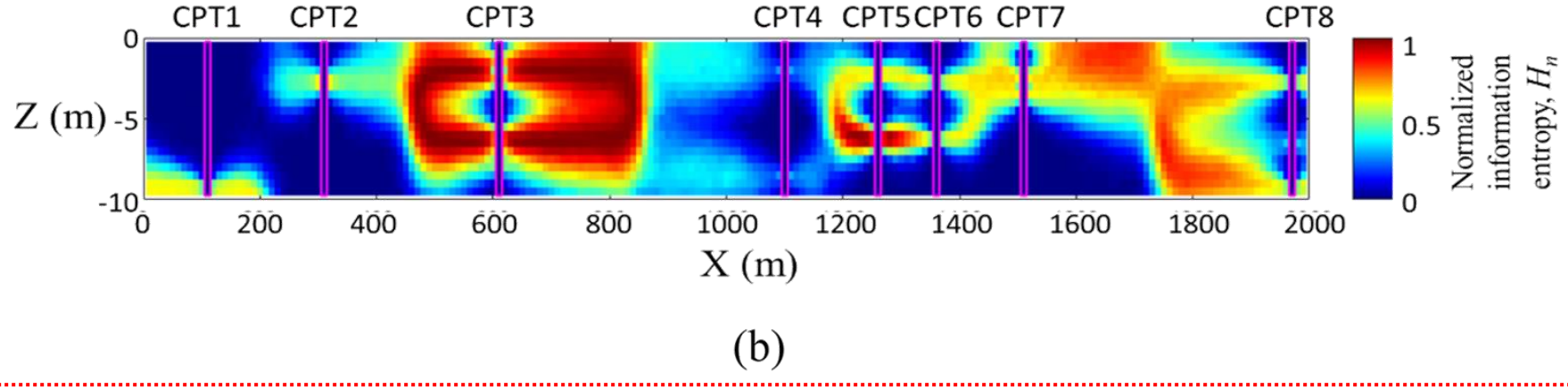
Fig. SBT stratigraphic models established: conditional random field (CRF, Model 1), Markov random field (MRF, Model 2)

3. RESULTS AND DISCUSSION

3.2. Comparison of stratigraphic model uncertainty (1000 realizations)



(a)



(b)

	E_A	Mean of LPI*	COV of LPI*
Model 1A	0.59	25.34	0.14
Model 1B		25.87	0.17
Model 2A	0.21	21.80	0.18
Model 2B		22.85	0.22

Fig. Average information entropy, the mean and coefficient of variation of LPI of the entire study site analyzed with four models

Fig. Simulation the uncertainty of the SBT stratigraphic model (information entropy)
(a) conditional random field (CRF, Model 1)
(b) Markov random field (MRF, Model 2)

3. RESULTS AND DISCUSSION

3.2. Comparison of stratigraphic model uncertainty

	E_A	Mean of LPI*	COV of LPI*
Model 1A	0.59	25.34	0.14
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Model 2A	0.21	21.80	0.18
Model 2B		22.85	0.22

* *Based on 200 LPI data corresponding to the grid cell number in the horizontal direction*

Fig. Average information entropy, the mean and coefficient of variation of LPI of the entire study site analyzed with four models

CONTENT

4. CONCLUSIONS

By use of two methods (conditional random field, CRF, and Markov random field, MRF) to establish soil behavior type (SBT) stratigraphic models, the uncertainty of these stratigraphic models:

- The SBT stratigraphic model:

  + CRF: SBT was distributed more evenly throughout, but some SBT clusters were less continuous.

  + MRF: SBT was more concentrated in specific zones, and several SBT clusters were continuous.

- The uncertainty (information entropy):

  + CRF: presented a relatively uniform distribution and high information entropy ($E_A = 0.59$).

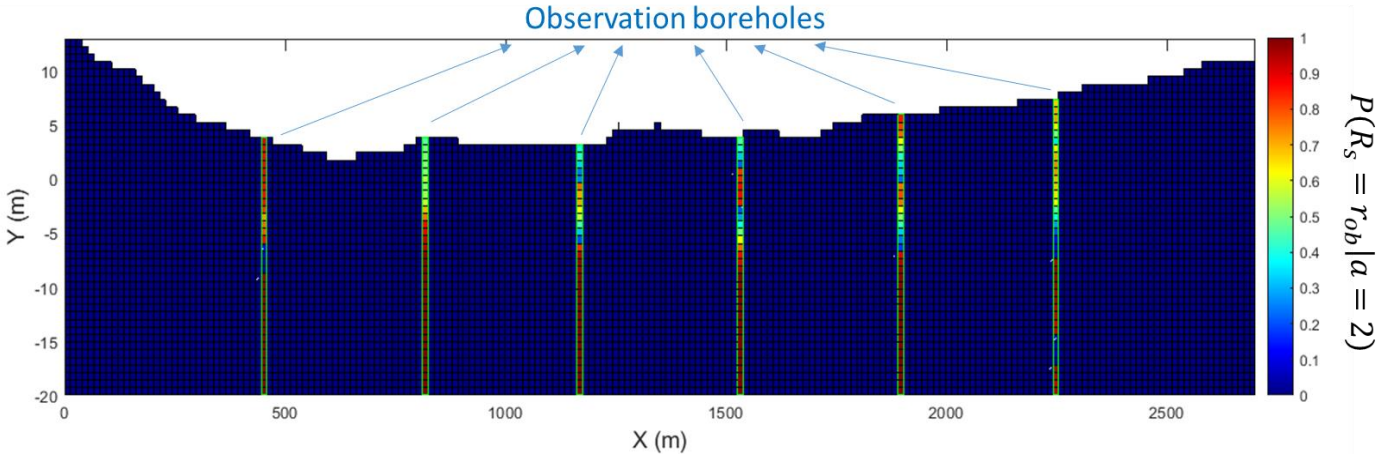
  + MRF: presented a relatively uneven distribution of information entropy, many locations exhibited low and high information entropy and smaller average information entropy ($E_A = 0.21$).

CONTENT

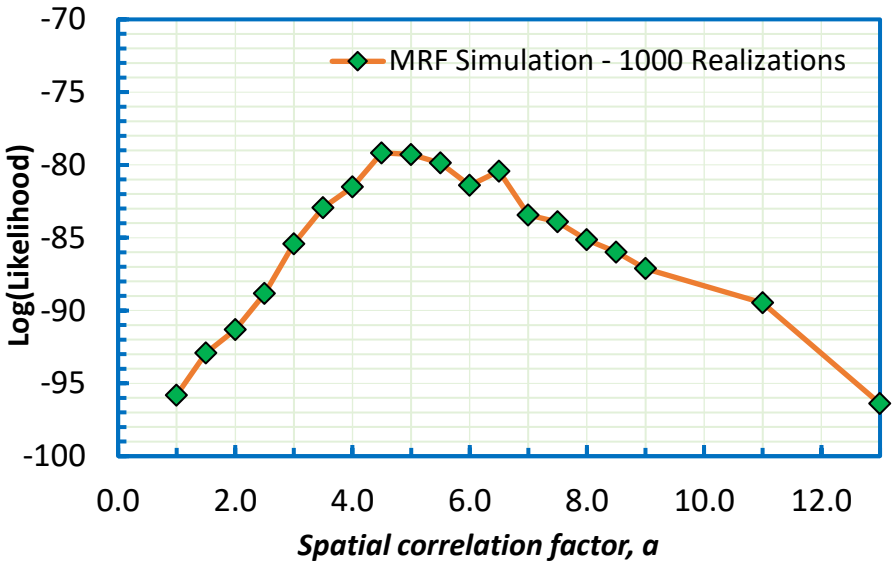
5. FUTURE WORKS

"CALIBRATION OF SPATIAL CORRELATION FACTOR IN MARKOV RANDOM FIELD"

- **Stochastic Markov random field** (Li et al., 2016) was employed for geological model generations.
- **Calibration approach (maximum likelihood estimation, MLE)** proposed by Qi et al. (2016) was used.



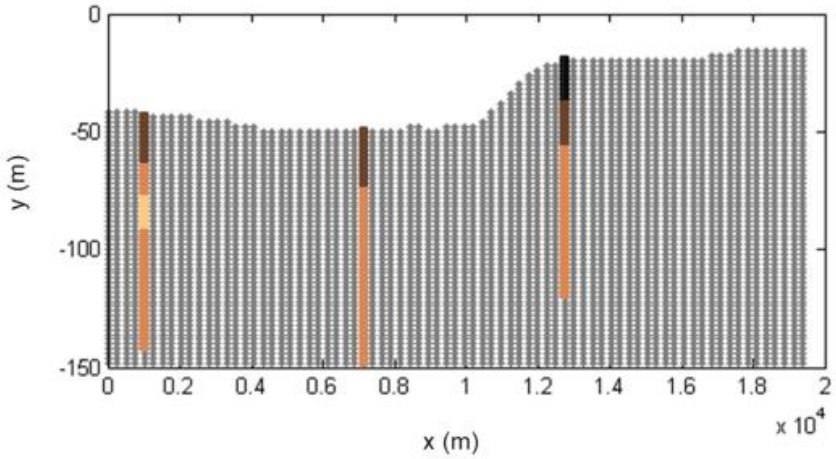
(Result was run by reference NS-1 section CGS, 2011)



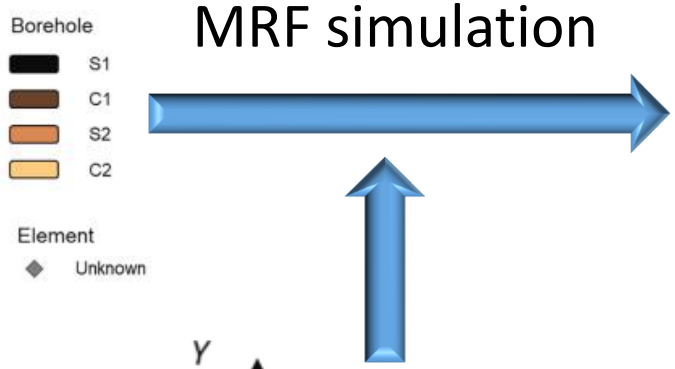
Calculate the probability of corrected simulation (= observation) for each *a value* (spatial correlation factor)

1. Spatial Correlation Model Used in MRF

Discritized mesh of cross-section

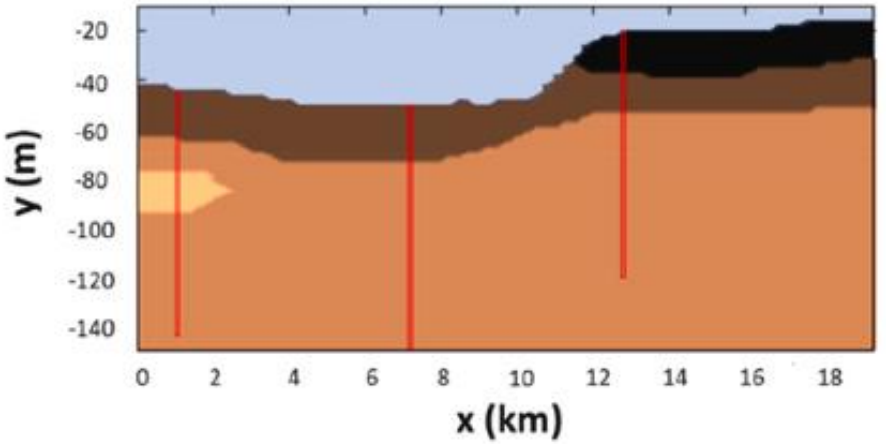


(Hsu et al., 2022)

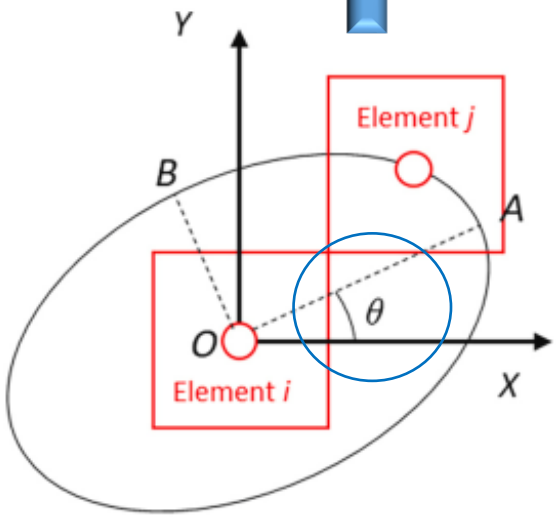


MRF simulation

1 realization generated by MRF



(Hsu et al., 2022)



Spatial Correlation Model
(after Li et al., 2016)

Two parameters should be determined

$\overline{OA} = a$

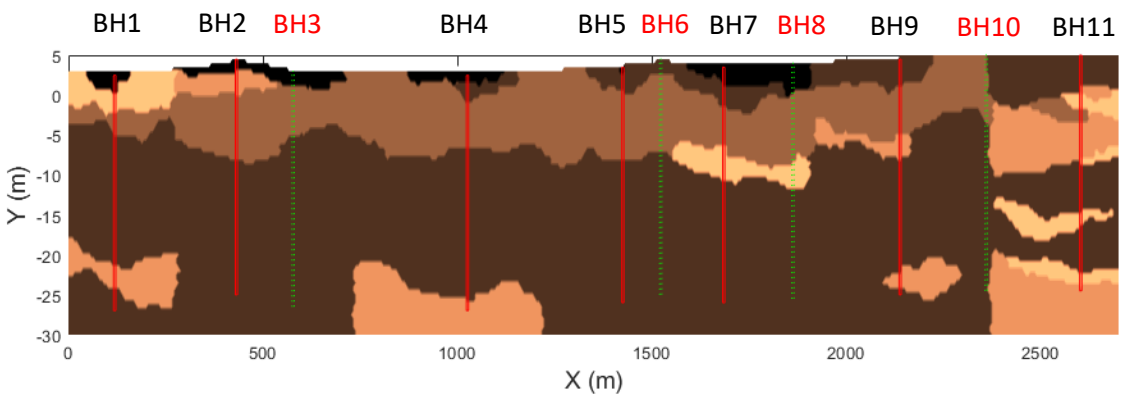
$\overline{OB} = 1$

spatial correlation factor, a

spatial correlation dip, θ

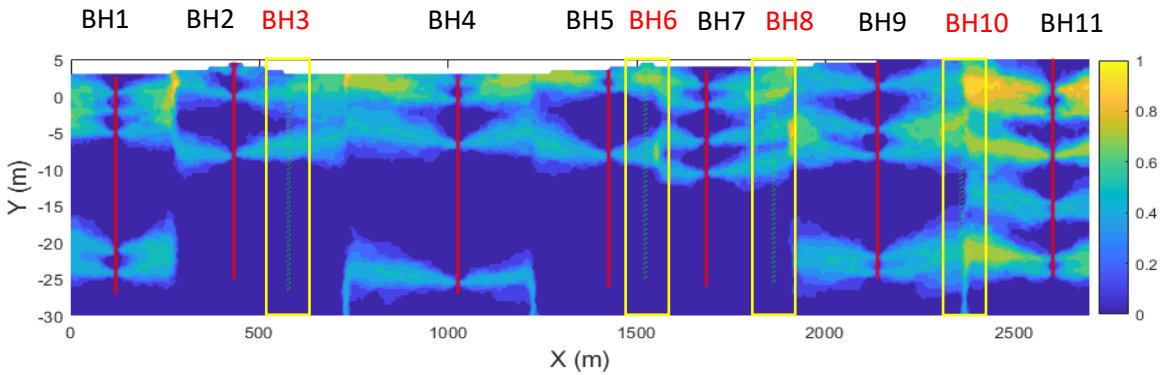
2. The Uncertainty of Geological Model

$\alpha = 6$



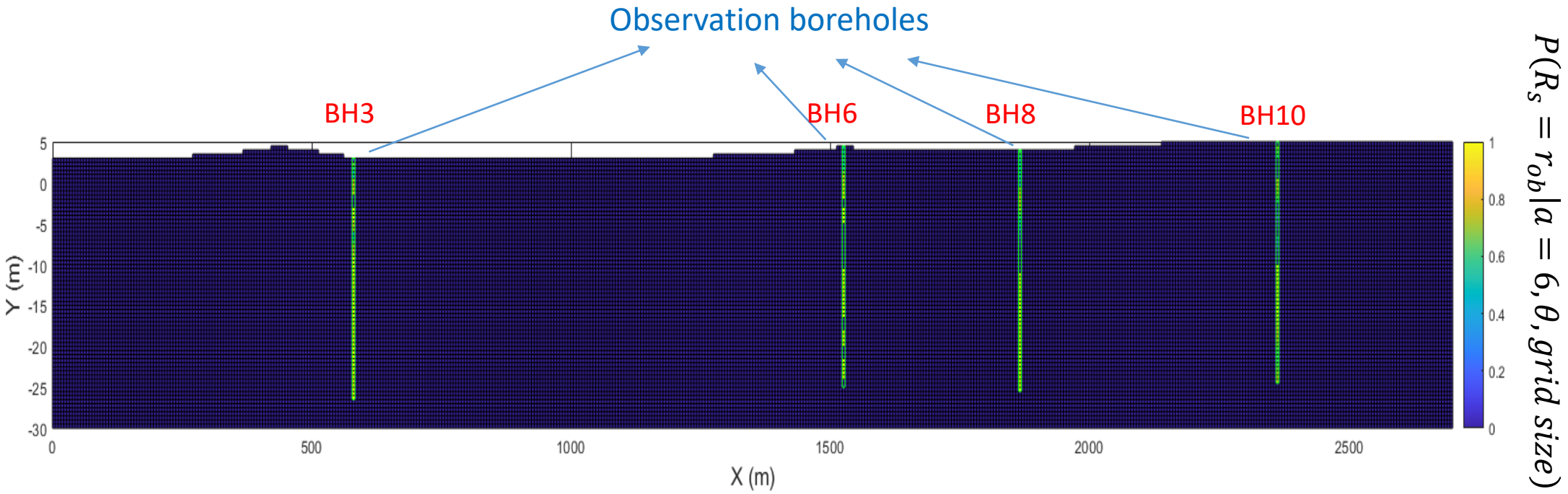
1000 realizations

Information entropy



- Calculate the probability of soil type existence
- Substitute the probability into equation of information entropy

3. Likelihood Value at the Observation Borehole

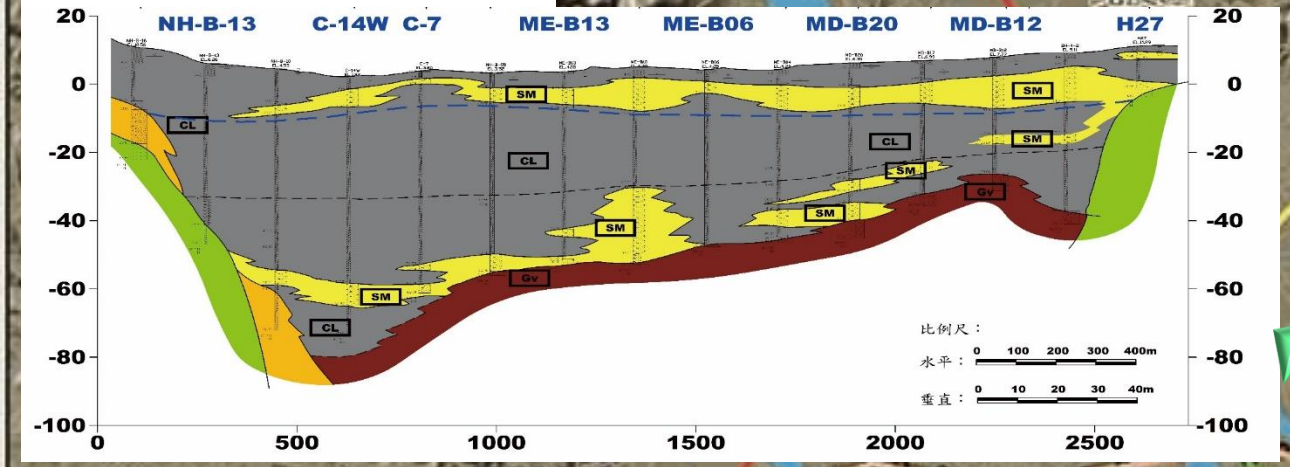


$P(R_s = r_{ob}|a = 6, \theta, grid\ size)$

Likelihood value

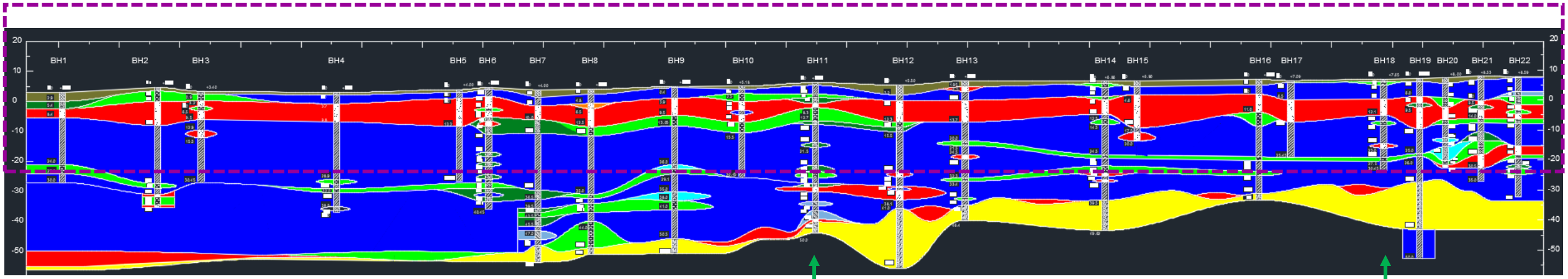
The probability of prediction equal to observation for spatial correlation, *a* equal to 6 (only compared with observation borehole)

The study site



(CGS, 2011)

Cross – section using for simulation



Backfilling
CL: Lean clay
CL-ML
SM: Silty sand
ML: Silts without plasticity
CH: Fat clay
GM: Gravel
ML-1: Silts with plasticity
Jingmei gravel formation

Section 1
BH1 – 11
Obs: BH3-6-8-10

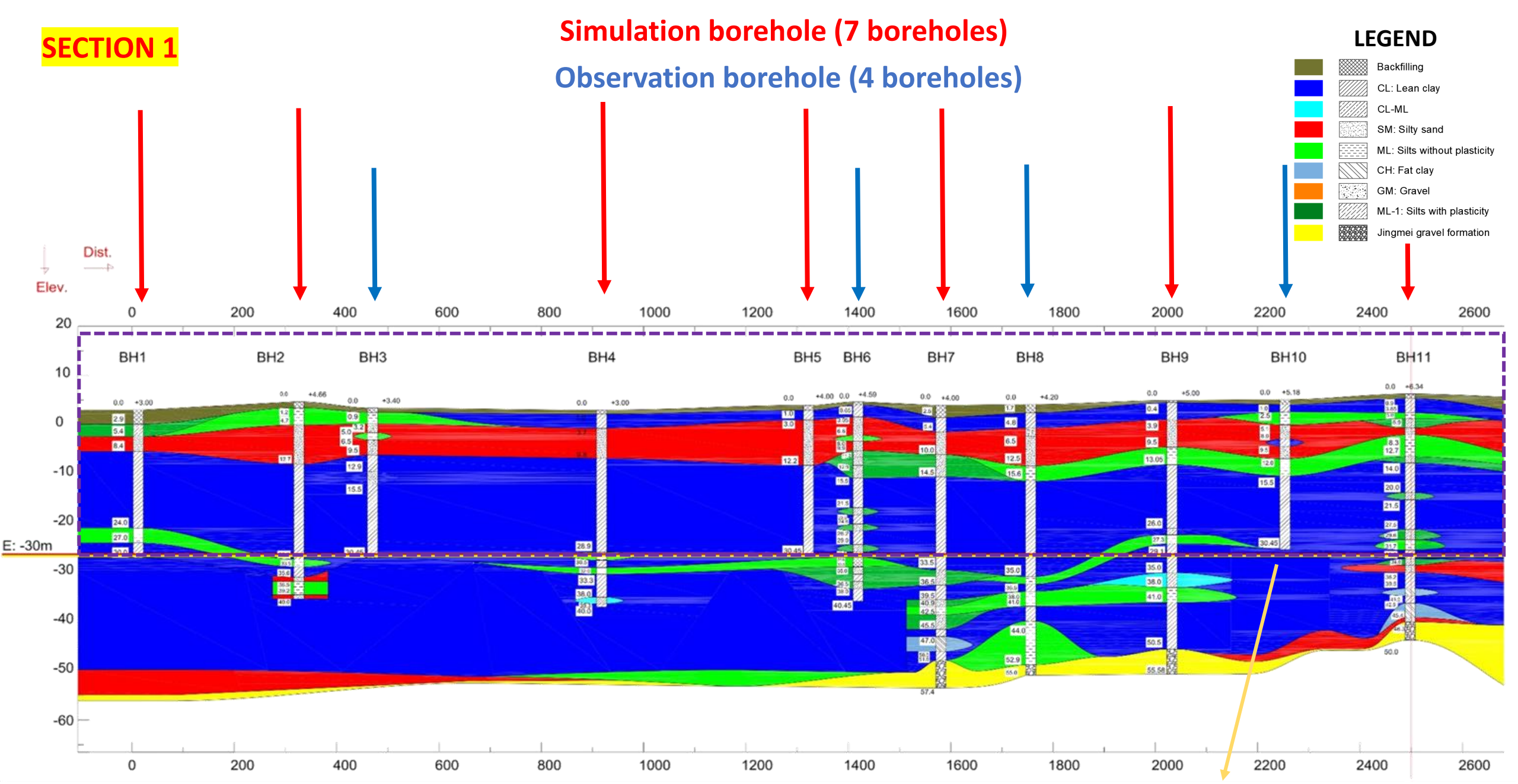
Section 2
BH11 – 18
Obs: BH12-15-17

Section 3
BH18 – 22
Obs: BH19-21

SECTION 1

Simulation borehole (7 boreholes)

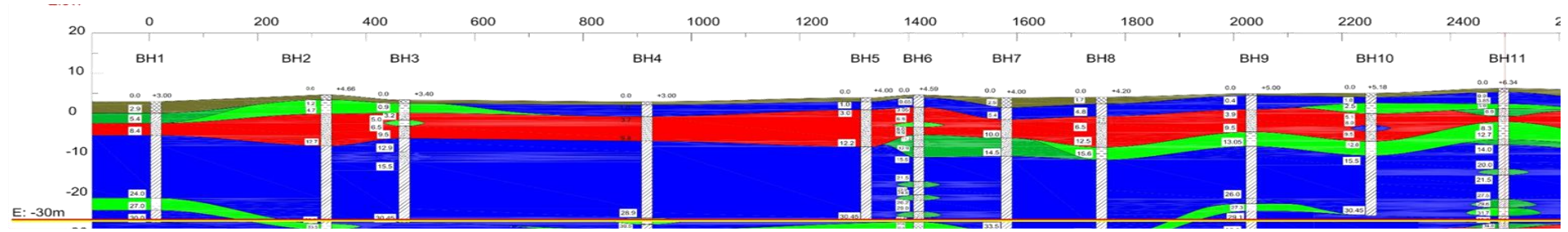
Observation borehole (4 boreholes)

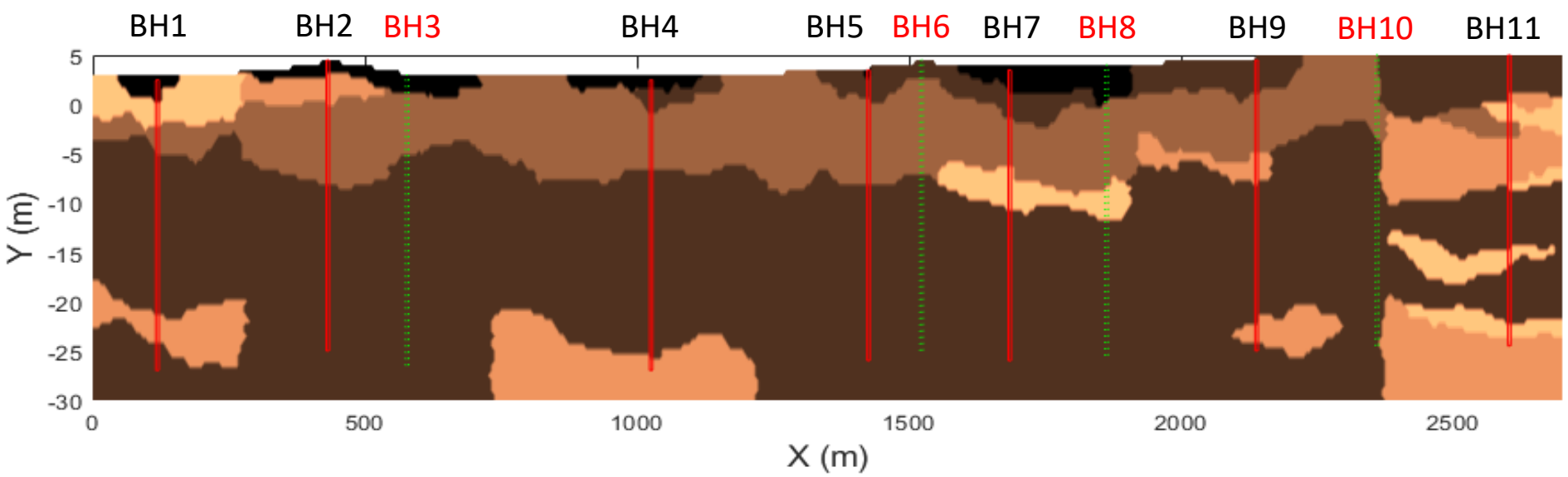


Distance between 2 boreholes from 96.8m to 447.5m

Stratigraphic model

a = 6 (1000 realizations)





LEGEND

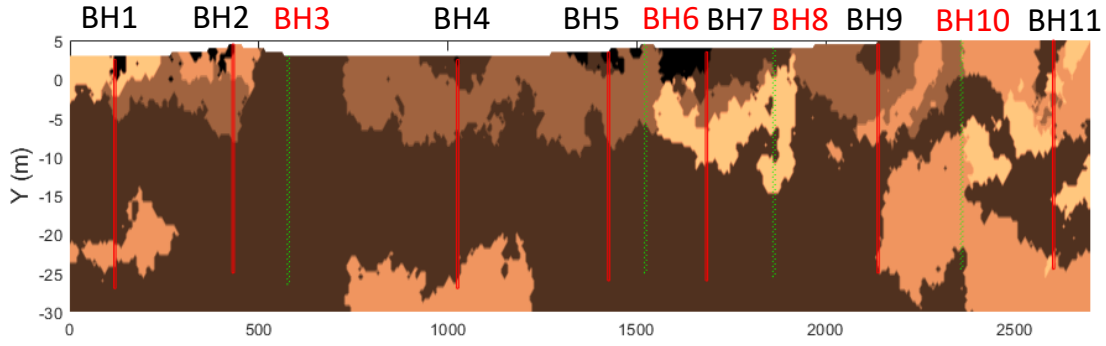
- Backfilling
- CL: Lean clay
- SM: Silty sand
- ML: Silts without plasticity
- ML-1: Silts with plasticity

LEGEND

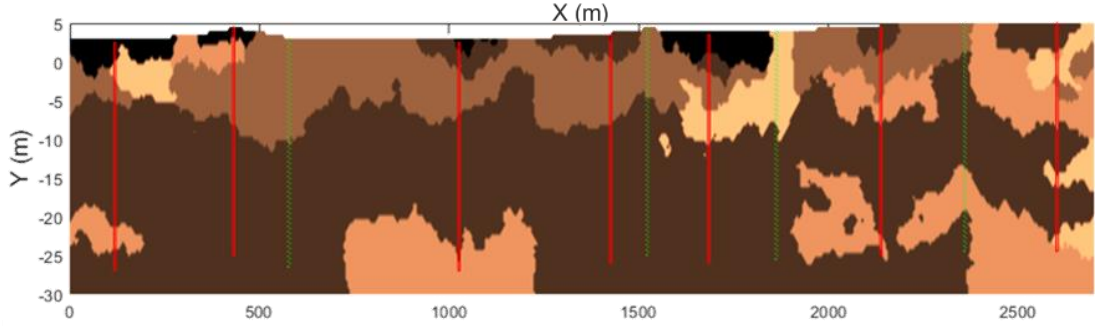
- Backfilling
- Clay
- Sand
- Silt with plasticity
- Silt without plasticity
- Borehole for MRF simulation
- Observation borehole

MRF 1000 Realizations with Various *α*

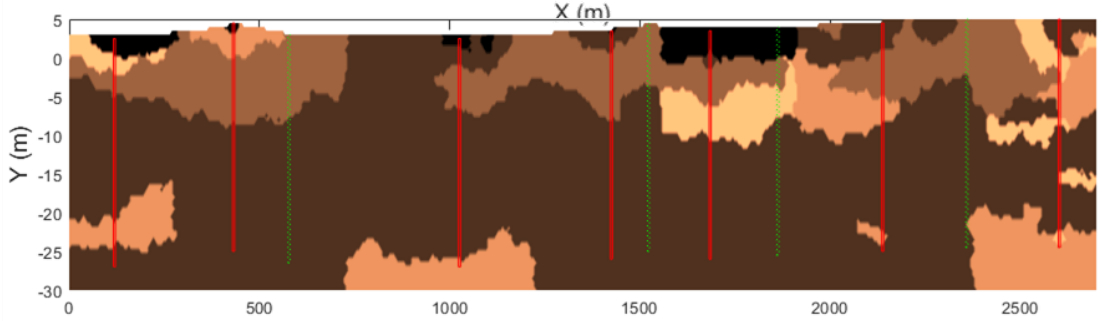
α=1



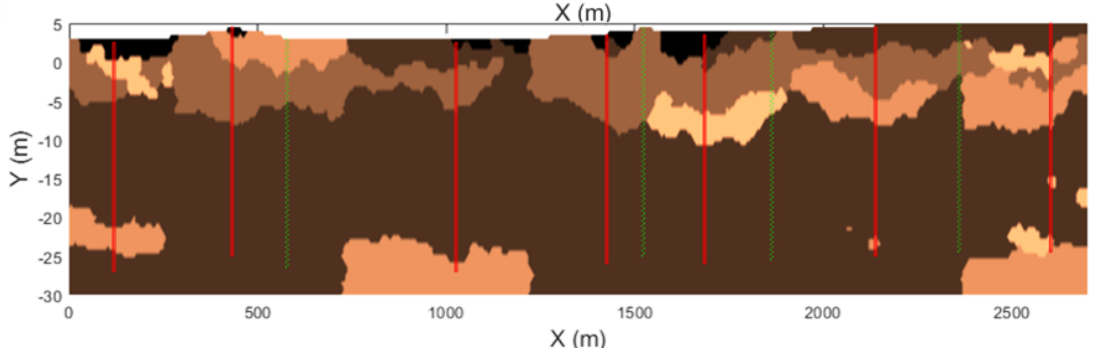
α=2



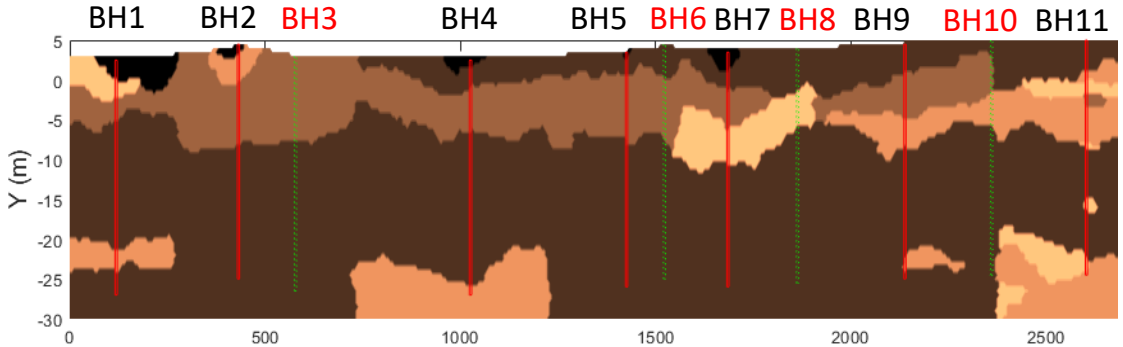
α=3



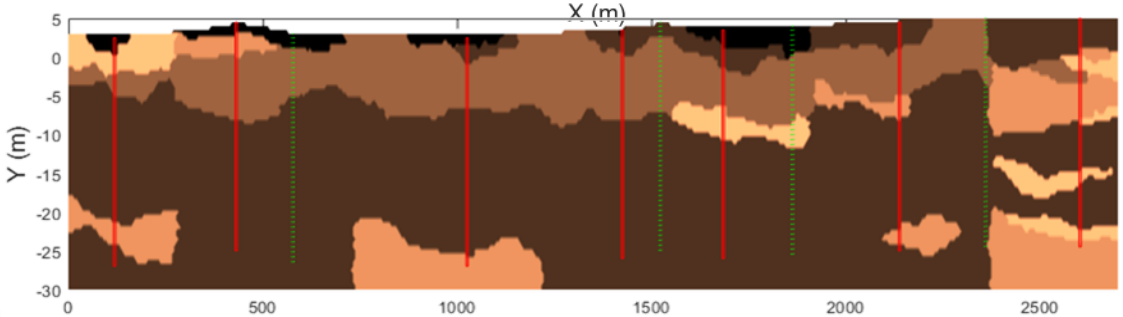
α=4



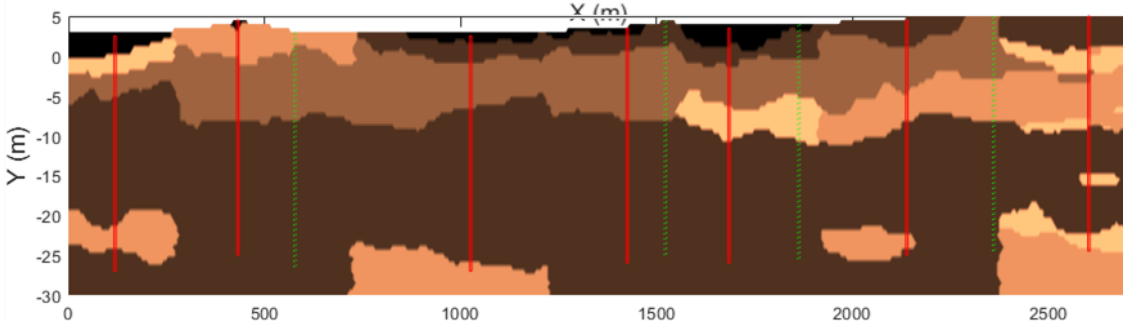
α=5



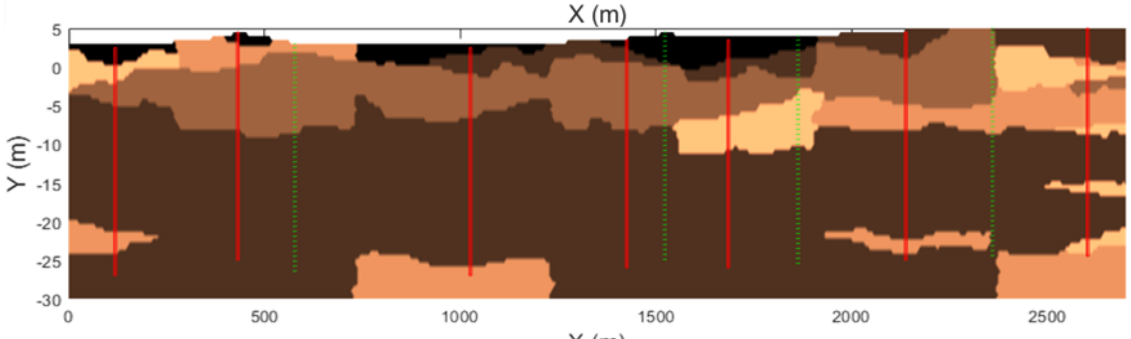
α=6



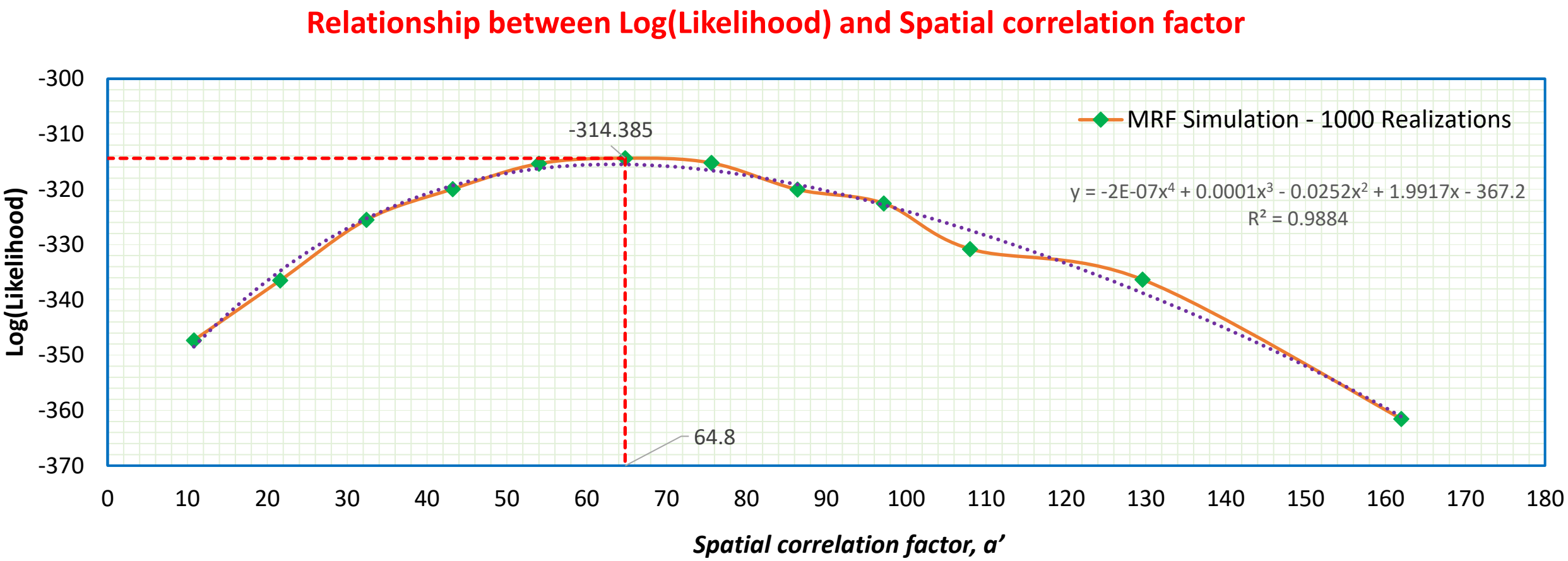
α=8



α=10



Calibration of Spatial Correlation Factor via MLE



THANK YOU

To learn more about how to write the code and simulation data

It’s an interesting process..

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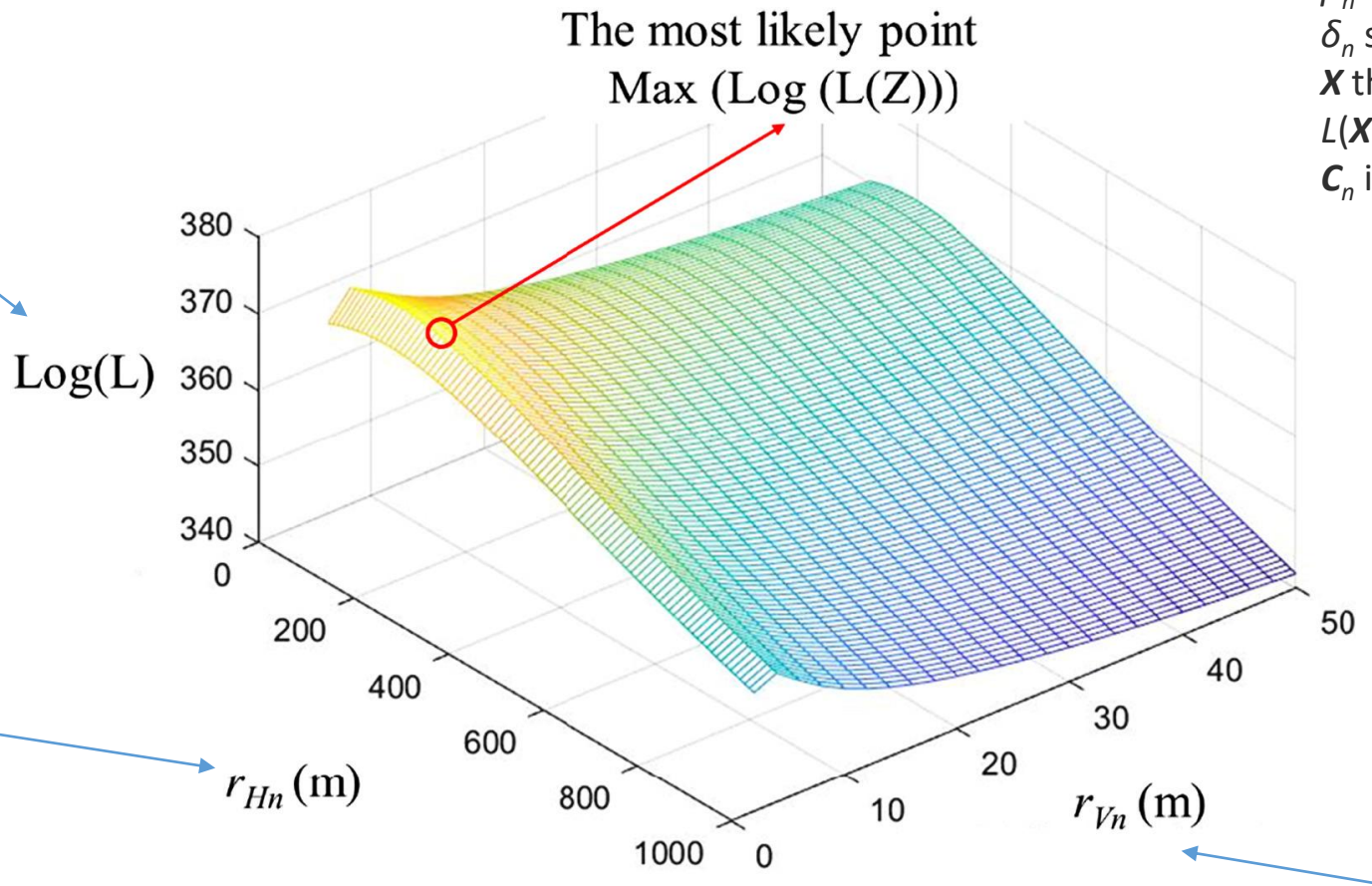
APPENDIX

Maximum Likelihood Estimation

$$\begin{aligned} &\text{Find:} && \phi_n^T = \{\mu_n, \delta_n, r_{Vn}, r_{Hn}\} \\ &\text{Subject to:} && \boldsymbol{X} = \{X_1, X_2, \ldots, X_3\} \\ &\text{Objective:} && L(\boldsymbol{X}|\phi_n) = \frac{1}{(2\pi)^{m/2}|\boldsymbol{C}_n|^{1/2}} exp\left[-\frac{1}{2}(\boldsymbol{X}-\boldsymbol{\mu}_n)^T \boldsymbol{C}_n^{-1}(\boldsymbol{X}-\boldsymbol{\mu}_n)\right] \\ & && \text{Maximizing } L(\boldsymbol{X}|\phi_n) \end{aligned}$$

ϕ_n^T is the site parameter to be determined
μ_n mean value of the site parameter
δ_n standard deviation
$\boldsymbol{X}$ the detrended sample data
$L(\boldsymbol{X}|\ \phi_n)$ is the likelihood function
$\boldsymbol{C}_n$ is the covariance matrix

Likelihood function

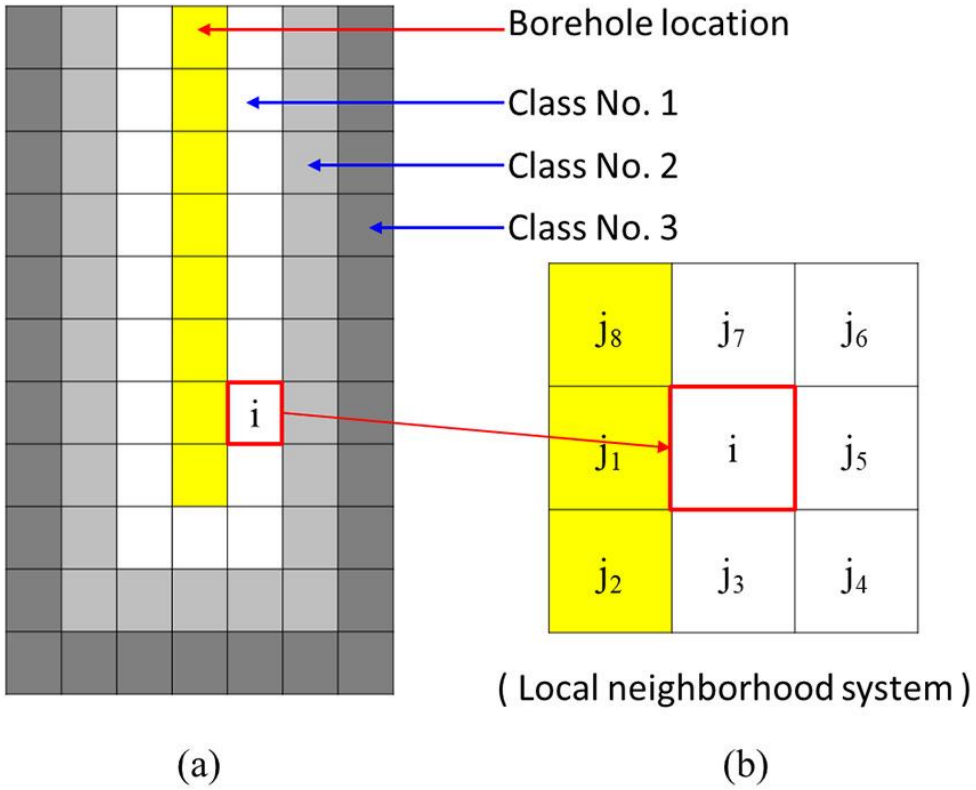


Horizontal fluctuations

Vertical fluctuations

Log-likelihood function of observational data I_c under different horizontal and vertical fluctuations

Neighborhood system



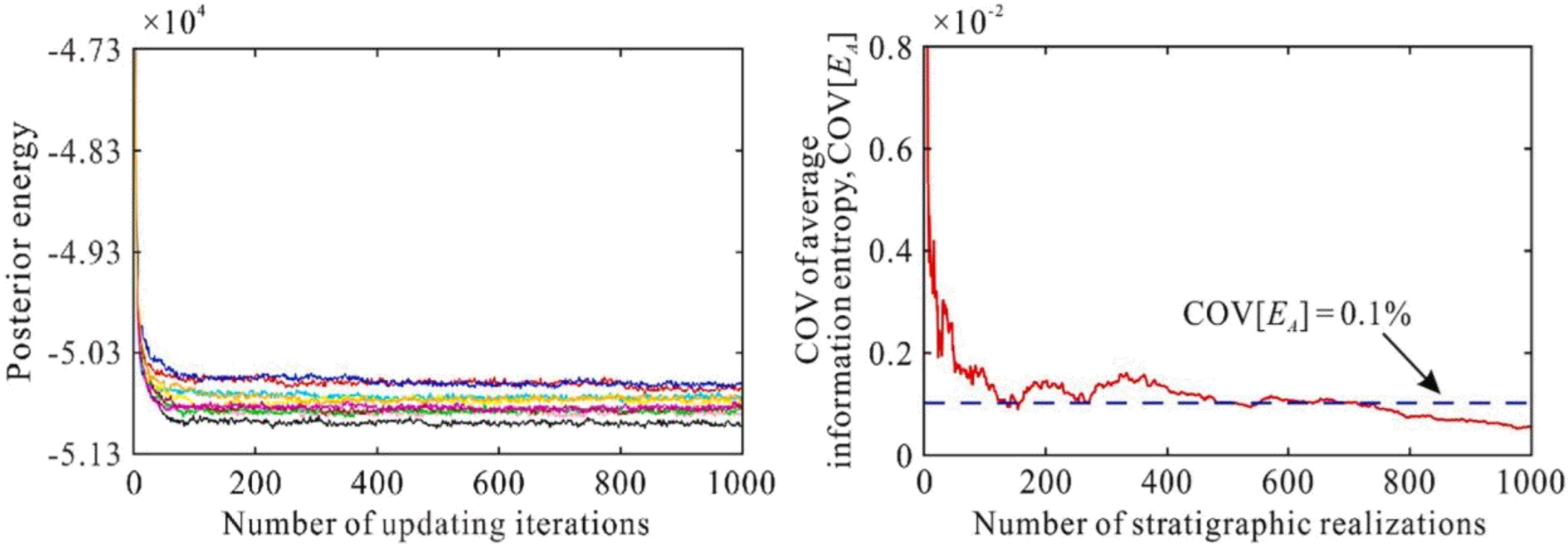
Neighborhood system and sampling order (After Gong et al., 2019)

The analysis sequence starts by determining the soil types of the nearest neighbors and gradually expands outward.

The grid cells immediately adjacent to the borehole are defined as cells with the highest priority.

The grid cells adjacent to the first priority cells are defined as the cells with the second priority

How to know how many realization was applied in the simulation?



Determination of the number of iterations adopted in the MCMC updating and the number of sampled stratigraphic realizations (Chao Zhao, 2021)

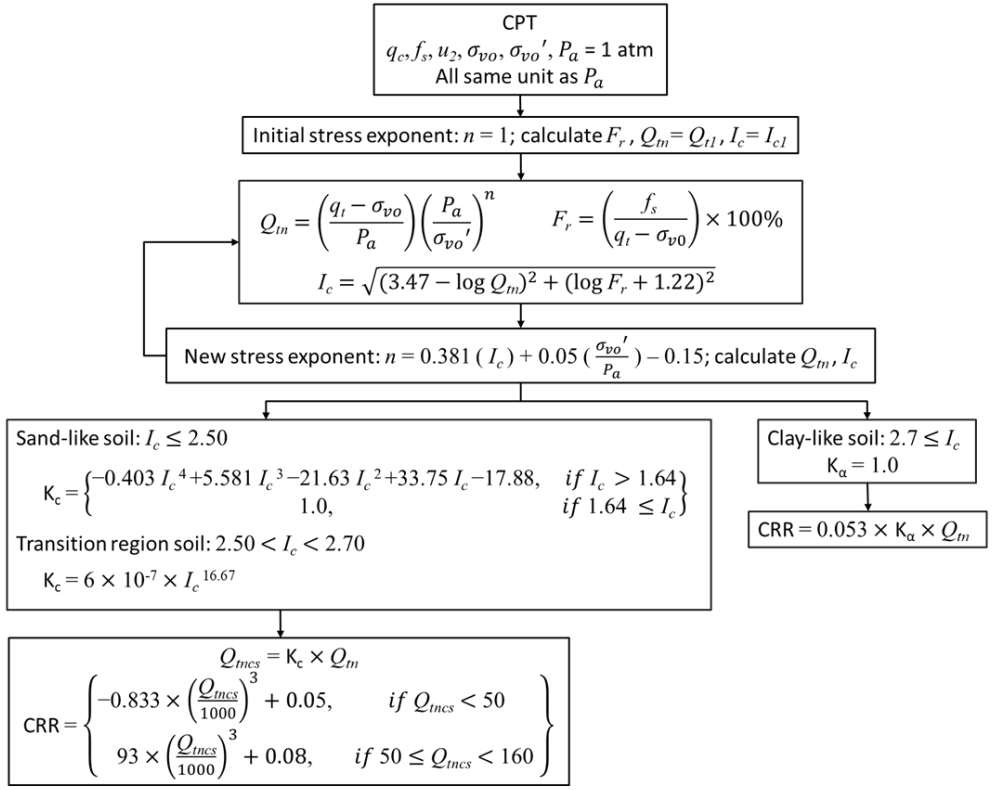
Liquefaction evaluation

$$FS = \frac{CRR}{CSR}$$

→ CRR (Cyclic resistance ratio)

(Robertson, 2009)

$$CRR = \begin{cases} if\ I_c \leq 2.7 \begin{cases} Q_{tncs} < 50, & 0.833 \times \left(\frac{Q_{tncs}}{1000}\right)^3 + 0.05 \\ 50 \leq Q_{tncs} < 160, & 93 \times \left(\frac{Q_{tncs}}{1000}\right)^3 + 0.08 \end{cases} \\ if\ I_c > 2.7, \quad 0.053 \times Q_{tncs} \end{cases}$$

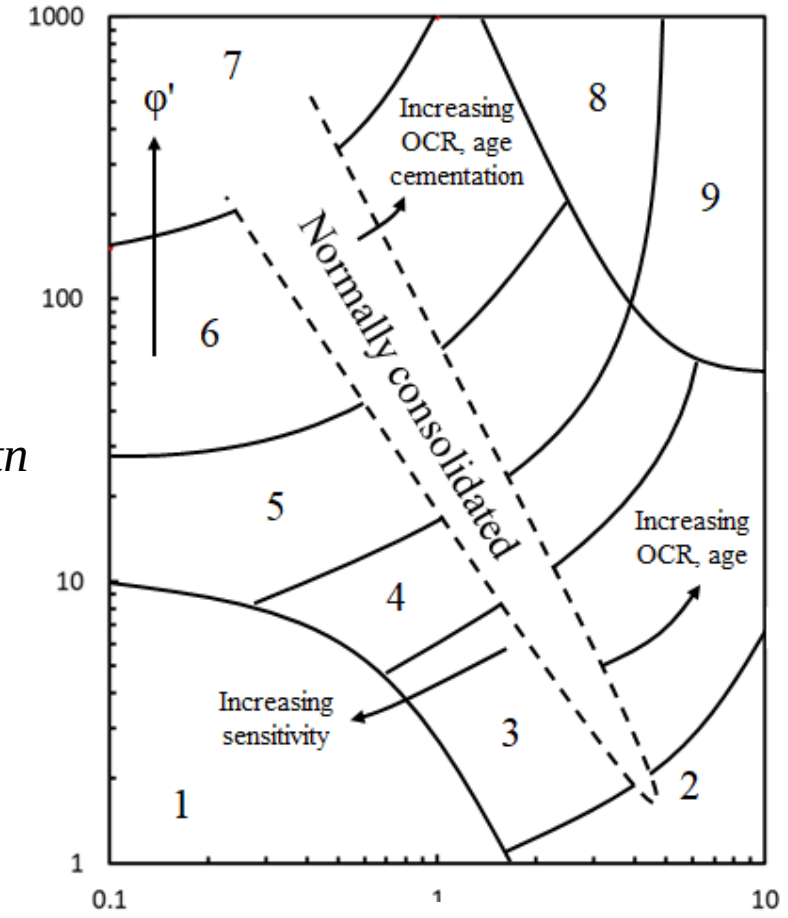


Liquefaction evaluation process
(Robertson, 2009)

→ CSR (Cyclic stress ratio)

$$CSR = 0.65 \times \frac{a_{max}}{g} \times \frac{\sigma_{v0}}{\sigma_{v0}'} \times \frac{\gamma_d}{MSF} \times \frac{1}{K_\sigma}$$ (Seed and Idriss, 1971)

Soil behavior types (SBT) & soil behavior types index (Ic)



Zone: Soil behavior types (SBT)

1. Sensitive, fine grained
2. Organic soils-peats
3. Clays-clay to silt clay
4. Silt mixtures clayey silt to silt clay
5. Sand mixtures; silt sand to sandy silt
6. Sands; clean sands to silt sands
7. Gravelly sand to sand
8. Very stiff sand to clayey sand
9. Very stiff fine grained

(Robertson, 1990)

$$I_c = \sqrt{(3.47 - \log Q_{tn})^2 + (\log F_r + 1.22)^2}$$

Soil behavior types (SBT) & soil behavior types index (I_c)

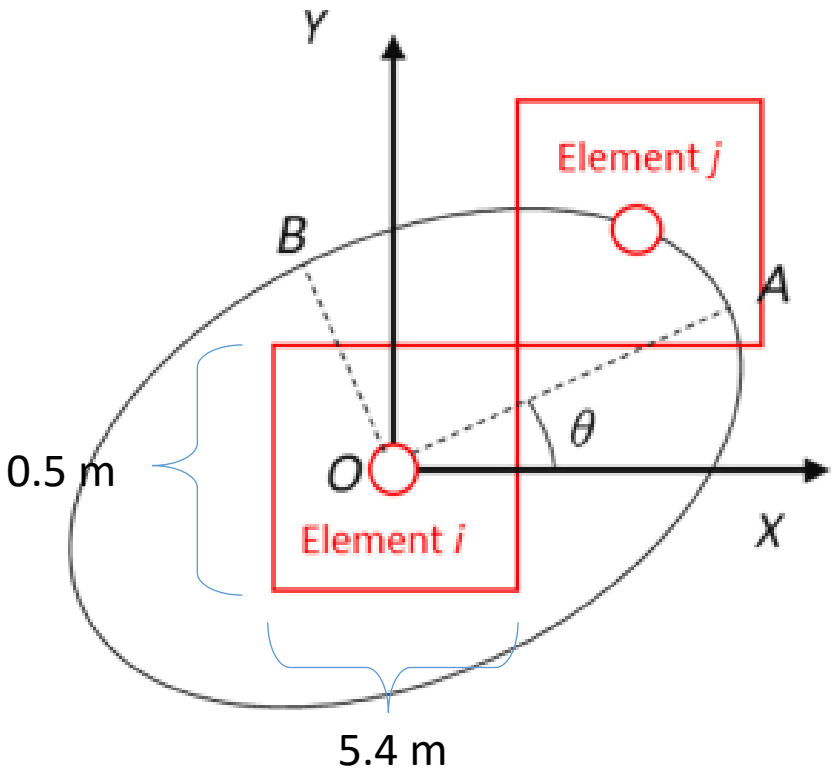
$$I_c = \sqrt{(3.47 - \log Q_{tn})^2 + (\log F_r + 1.22)^2}$$ (Robertson, 2009)

- SBT reflects mechanical properties.

Table 2. SBT & I_c & soil type (modified from Robertson, 2009)

Soil behavior type index（I_c）	Soil behavior type（SBT）	Description
$2.95 < I_c < 3.60$	3	Clay
$2.60 < I_c < 2.95$	4	Silt mixture: clayey silt to silty clay
$2.05 < I_c < 2.60$	5	Sand mixture: silty sand to sandy silt
$1.31 < I_c < 2.05$	6	Sands: clean sand to silty sand

Grid Size Conversion



element_number_x = 500; model_length_x = 2700;
element_number_y = 70; model_length_y = 35;

Grid size = 5.4 m * 0.5 m
Horizontal grid size/vertical grid size = A_R = 10.8

$\overline{OA} = a$

$\overline{OB} = 1$

$$a' = A_R \times a$$