Analytical solutions of tidal groundwater flow in coastal two-aquifer system

Hailong Li,2002, Advances in Water Resources

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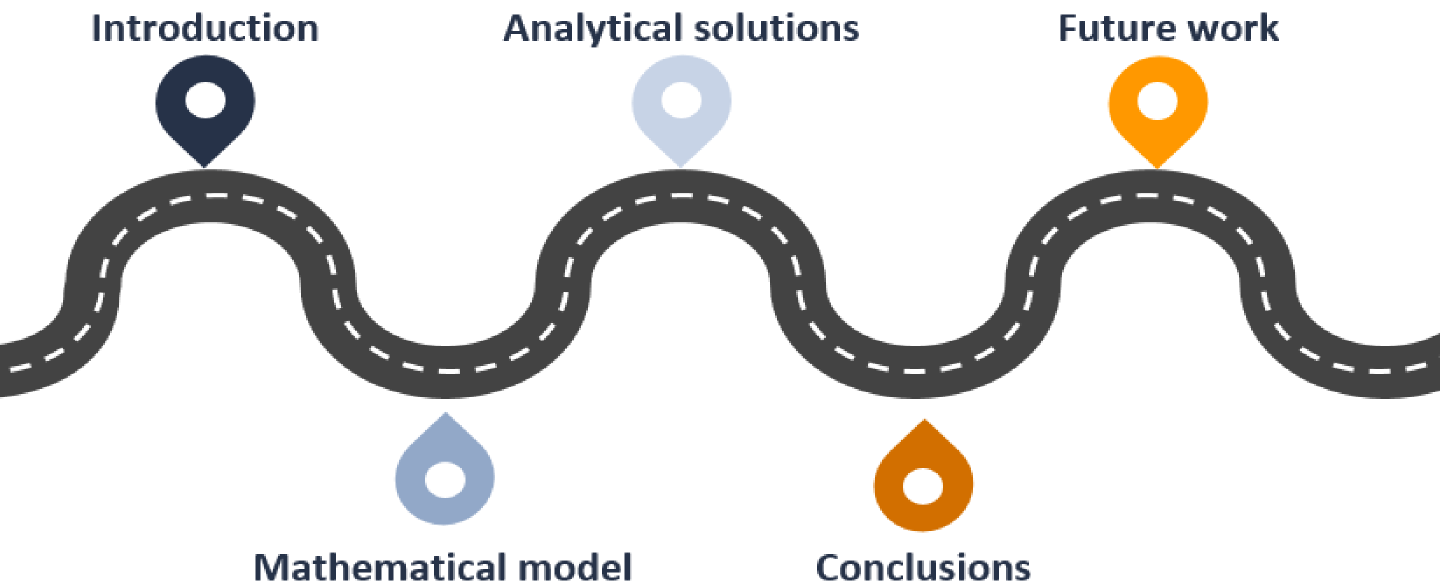
Table of contents



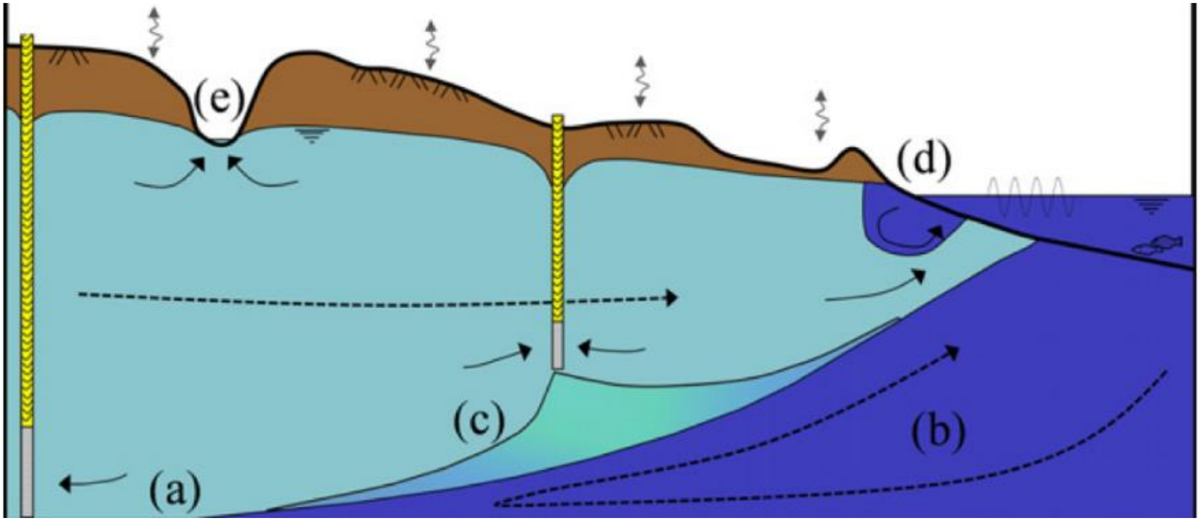
Introduction

- With the social and economic development in coastal areas, various problems arise

seawater intrusion → deterioration of marine environment → stability of coastal structures

- To solve these problems, it is necessary to identify the coastal hydrogeological conditions

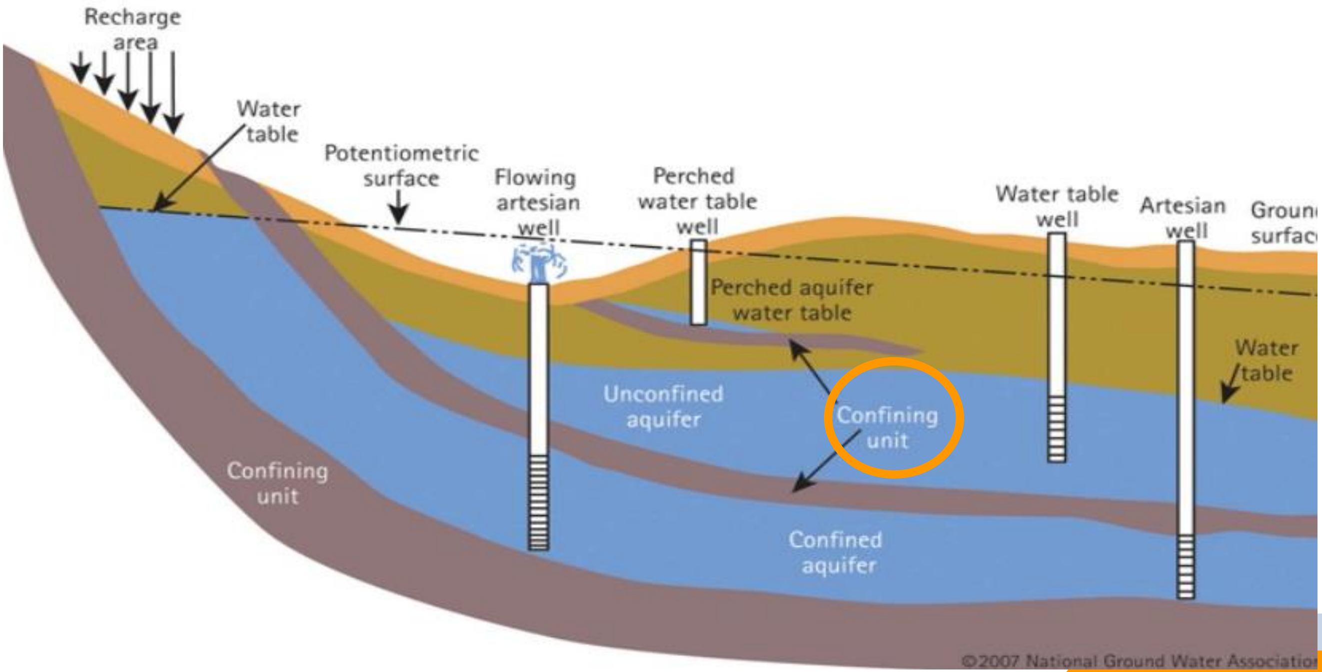
Coastal aquifer diagram



(Werner et al.,2013)

Introduction

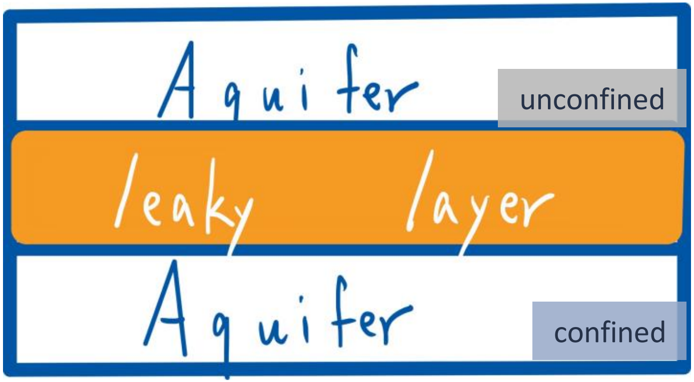
- This paper consider the **confining unit** is semipermeable, hereafter it will be referred to as **leaky layer** or **semipermeable layer**



Mathematical model

Basic assumption

- Each layer is **homogeneous** and horizontal
- This model have a **clear-cut vertical** water-land boundary
- Consider the vertical leakage between two aquifers



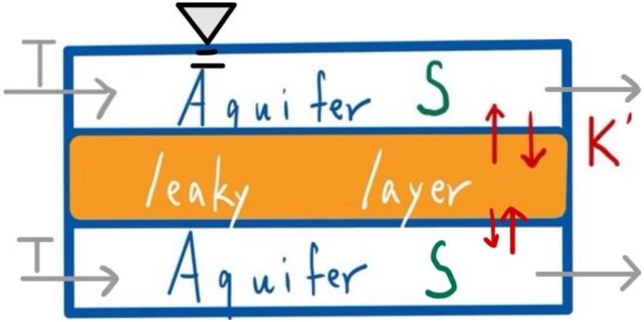
Groundwater flow governing equation :

Two aquifer / Leaky layer

$$S_j \frac{\partial H_j}{\partial t} = T_j \frac{\partial^2 H_j}{\partial x^2} \pm K' \frac{\partial h}{\partial z}\bigg|_{Z=Z_i} \quad (-\infty < t < \infty, x > 0, j = 2,1)$$

Leaky layer's governing equation:

$$s_s' \frac{\partial h}{\partial t} = K' \frac{\partial^2 h}{\partial z^2}$$



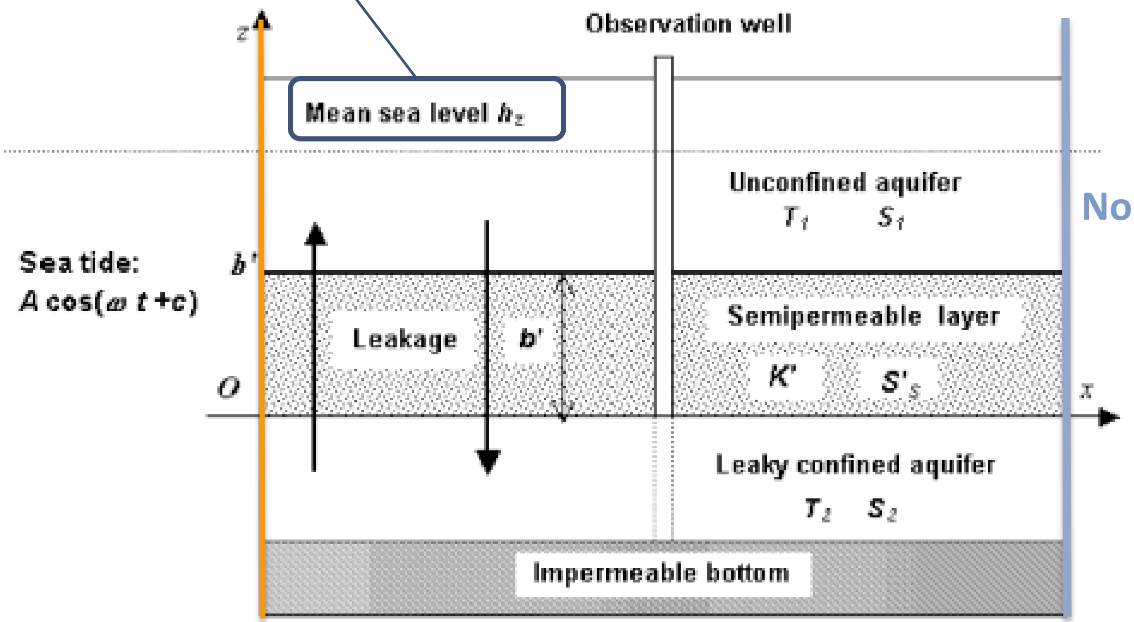
h:groundwater head
K':vertical permeability
Z:thickness
j=1:upper aquifer
j=2:lower aquifer

S:storativity
T:transmissivity
H:hydraulic head

h:groundwater head
K':vertical permeability
z:thickness
s_s' :specific storativity of leaky layer

hydraulic heads of two aquifer

Land side



No flow boundary

$$\lim_{x\to\infty} \frac{\partial H_j}{\partial x} = 0$$

Water-land boundary

$$H_j(0,t) = A\cos(\omega t), j = 2,1$$

ω=frequency
A=amplitude
H:hydraulic head
x:landside distance

Analytical solution

Definition of basic parameters:

- Buffer capacity θ of the leaky layer:

Tidal

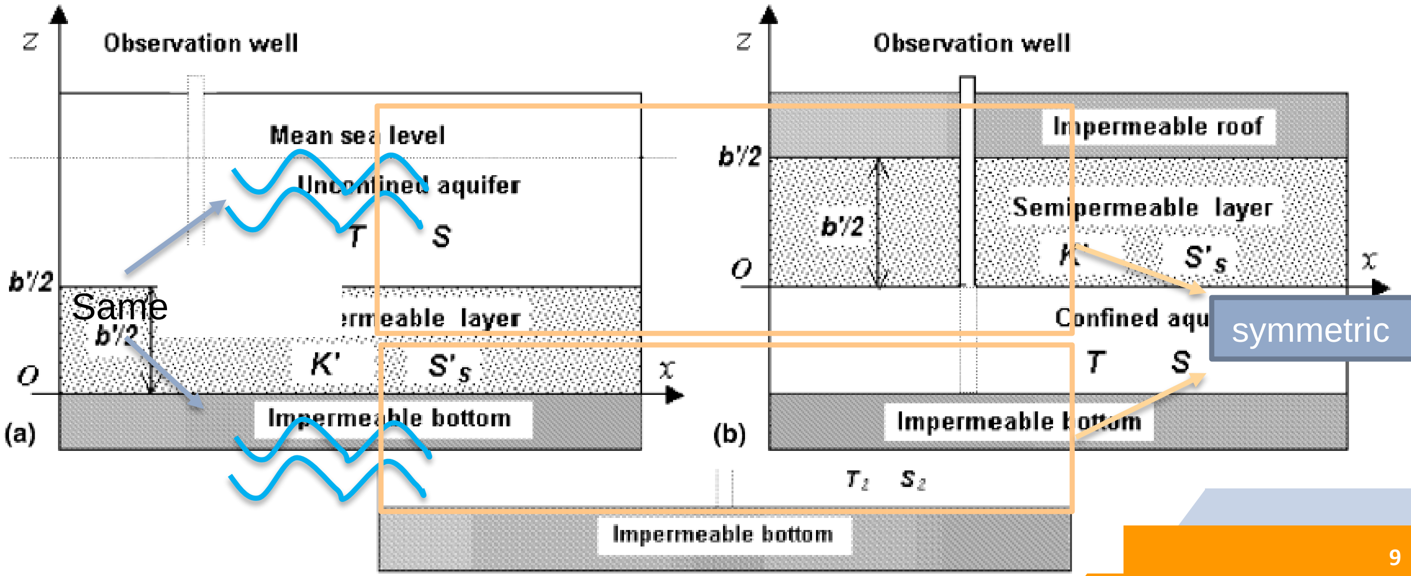
$$\theta = b' \sqrt{\frac{\omega s_s'}{2K'}}$$

K':vertical permeability of leaky layer
s_s' :specific storativity of leaky layer
b′: thickeness of leaky layer
ω: tidal frequency

B1=
B2=

Solutions when B1=B2=B

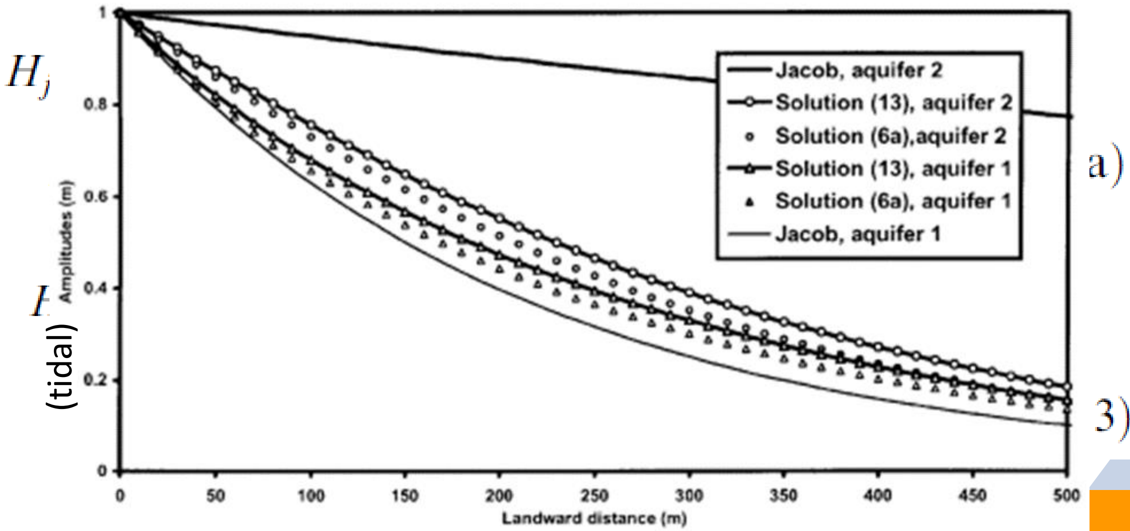
- When B1=B2, it's means that the upper and lower aquifers received the same tidal interference



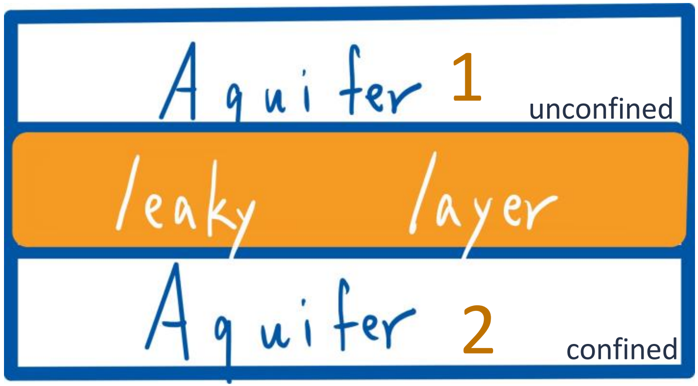
Solutions when $\lambda_1 = \lambda_2 = \lambda$

- λ= the wavelength of the tide

- The middle layer
  - semipermeable
    - 6a$\Rightarrow$ $\lambda_1 \neq \lambda_2$ (θ>0)
    - 13=> $\lambda_1 = \lambda_2$ (θ=0)
  - impermeable — Jacob



Li and Jiao solution



Basic assuLarge

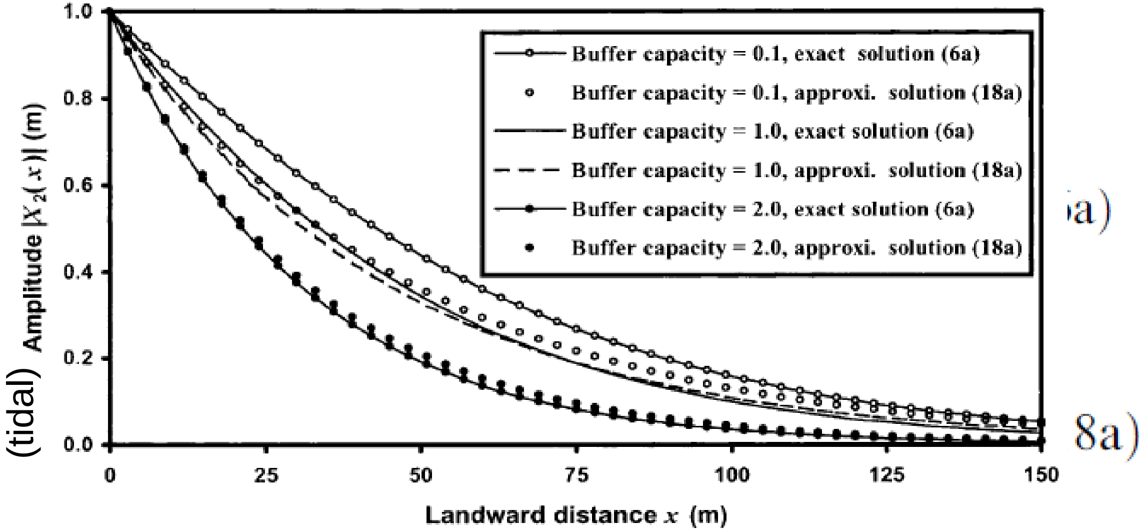
1has very large specific yield → Cause → The tidal fluctuation:**2>>1**

↓

Can damp effectively the tidal effect

Comparison with Li and Jiao solution

- Prior solution → use → 6a $\lambda_1 \neq \lambda_2$ (θ>0)
- Li and Jiao's solution → use → 18a



Lower aquifer

Conclusions

- This paper presents an **analytical** solution
- The system can be simplified into an equivalent aquifer–aquitard **double-layered system** bounded by impermeable layers from up and down
- The buffer capacity **increases** with the leaky layer's thickness, specific storage, and **decreases** with the leaky layer's vertical permeability.
- If the buffer capacity **increase,** the tidal wave interference between the two aquifers will **decline quickly**

Future work

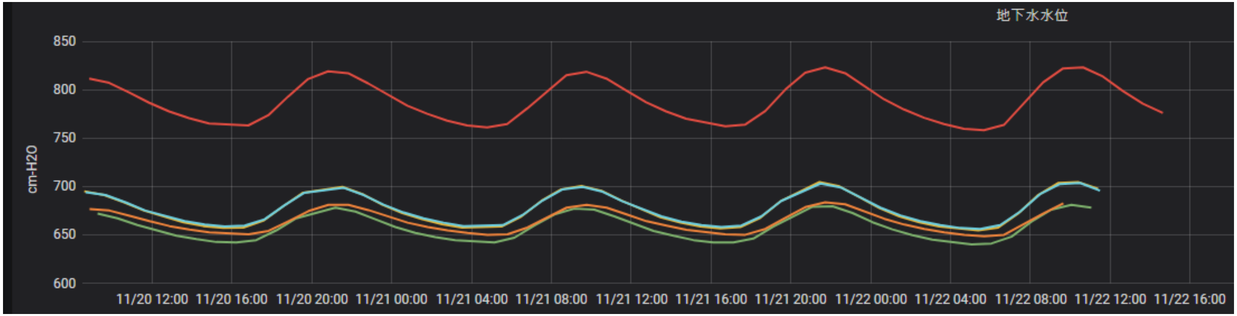
- Because of the deterioration of marine environment, it is necessary to identify the groundwater flow’s direction

1\.

2\. s

3\. n

Future work


地下水水位
850
800
750
700
650
600
cm-H2O
11/20 12:00
11/20 16:00
11/20 20:00
11/21 00:00
11/21 04:00
11/21 08:00
11/21 12:00
11/21 16:00
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11/22 12:00
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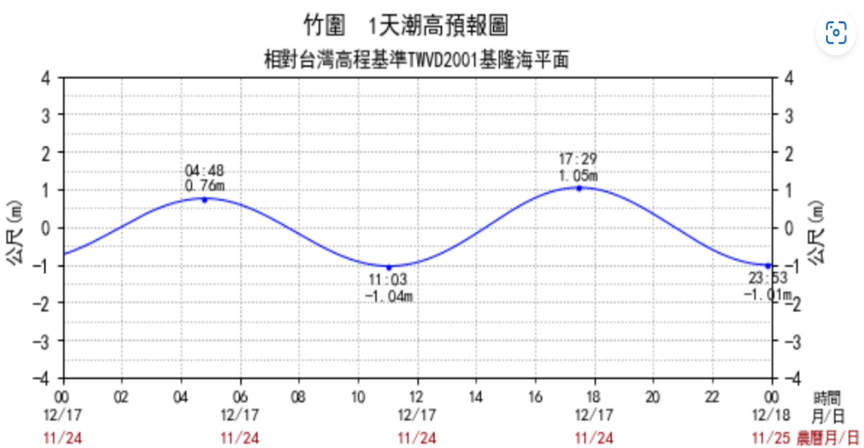
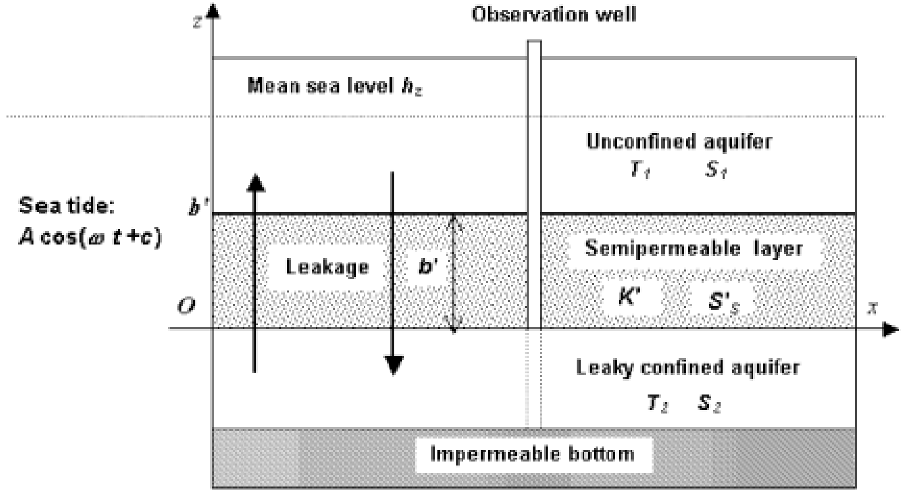
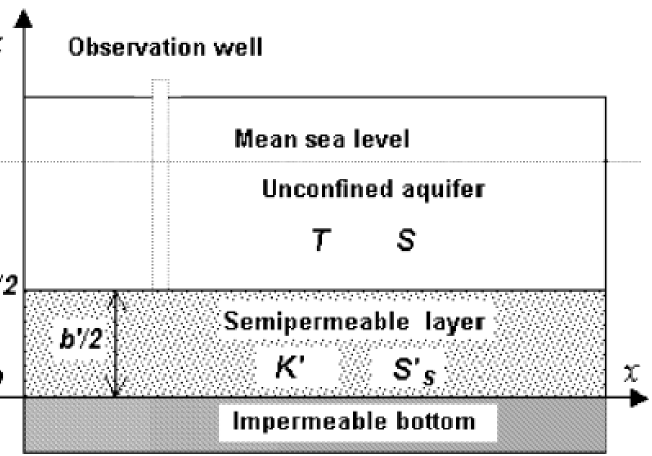

竹圍　1天潮高預報圖
相對台灣高程基準TWVD2001基隆海平面
公尺 (m)
04:48
0.76m
11:03
-1.04m
17:29
1.05m
23:53
-1.01m
00 02 04 06 08 10 12 14 16 18 20 22 00
12/17 12/17 12/17 12/17 12/18
時間
月/日
11/24 11/24 11/24 11/24 11/25
農曆月/日

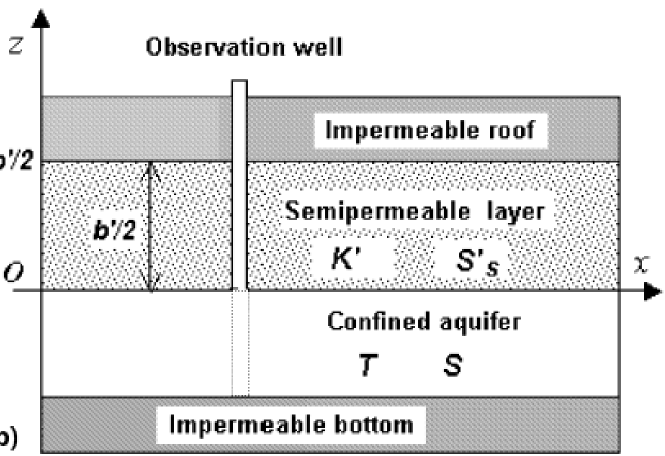
Thank you for listening

$$H_1(x,t) = H_2(x,t) = A\,\mathrm{Re}[\exp(-x\sqrt{B^2 - \varepsilon} + \mathrm{i}\omega t)]$$
$$\text{if } B_1 = B_2 = B. \tag{9}$$


Observation well
z
Mean sea level h_z
Unconfined aquifer
T_1 S_1
Sea tide:
$A \cos(\omega t + c)$
b'
Leakage
b'
Semipermeable layer
K' S'_s
O
x
Leaky confined aquifer
T_2 S_2
Impermeable bottom


z
Observation well
Mean sea level
Unconfined aquifer
T S
b'/2
b'/2
Semipermeable layer
K' S's
O
x
Impermeable bottom

(a)


z
Observation well
Impermeable roof
b'/2
b'/2
Semipermeable layer
K' S's
O
x
Confined aquifer
T S
Impermeable bottom

(b)