徑向發散流場移流-延散方程之全解析解

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摘要

徑向流場中溶質傳輸的解析解在抽注/發散/收斂流示踪試驗中的傳輸研究、通過抽水修復含水層以及含水層儲存回抽中具有多種實際應用。然而，徑向移流延散傳輸的解析解已被證明難以開發。過去研究提出的大多數徑向移流延散輸運的解通常為半解析解，最終濃度值是借助數值 Laplace 逆轉換獲得。本研究採用了較新穎的方式來求解徑向發散流場移流-延散傳輸問題。首先執行關於時間變數的 Laplace 轉換和關於空間變數的廣義積分轉換技術，以將暫態控制偏微分方程轉換為代數方程。隨後，使用簡單的代數操作來求解代數方程，從而在轉換域中容易地求得解。通過 Laplace 逆轉換和相應的廣義積分逆轉換連續應用，最終求得原始域中的解。使用發散流示蹤劑試驗證明了所提出的方法的穩定性，該方法用於推導徑向移流延-散傳輸問題的全解析解。將本研究的解析解模式的計算結果與 Laplace 轉換有限差分法（LTFD）進行比較，以驗證解的正確性。通過 LTFD 驗證了所開發的解析解。結果表明，本研究的全解析解與 LTFD 非常吻合。

關鍵字： 全解析解, 徑向移流-延散傳輸, Laplace 轉換, 發散示踪劑試驗, 廣義型積分轉換

Exact analytical solutions to the advection-dispersion equation in a radially divergent flow field

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Abstract

An analytical solution for solute transport in a radial flow field has a variety of practical applications in the study of the transport in push-pull/divergent/convergent flow tracer tests, aquifer remediation by pumping and aquifer storage and recovery. However, an analytical solution for radial advective-dispersive transport has been proven very difficult to develop. Most of the solutions for radial advective-dispersive transport presented in the past have generally been solved semi-analytically with the final concentration values being obtained with the help of a numerical Laplace inversion. This study uses a novel solution strategy for analytically solving the radial divergent flow field advective-dispersive transport problem. A Laplace transform with respect to the time variable and a generalized integral transform technique with respect to the spatial variable are first performed to convert the transient governing partial differential equations into an algebraic equation. Subsequently, the algebraic equation is solved using simple algebraic manipulations, easily yielding the solution in the transformed domain. The solution in the original domain is ultimately obtained by successive applications of the Laplace and corresponding generalized integral transform inversions. A divergent flow tracer test is used to demonstrate the robustness of the proposed method for deriving an exact analytical solution to the radial advective-dispersive transport problem. The computational results of the analytical solution model of this study will be compared with the use of the modified Laplace transform finite difference method (LTFD) to verify the correctness of the solution. The developed analytical solution is verified against a LTFD. The results show perfect agreement between this study exact analytical solution and the LTFD.

Keywords: Exact analytical solution, Radial advection-dispersion transport, divergent flow tracer test, Laplace transform, Generalized integral transform