



國立中央大學  
*National Central University*



**NATIONAL CENTRAL UNIVERSITY - COLLEGE OF EARTH SCIENCES  
GRADUATE INSTITUTE OF APPLIED GEOLOGY**

**Scale effect of the spatial correlation factor  
used in Markov random fields: A case study  
in Taipei Basin**

Presenter: Le Hoai Han  
Advisor: Prof. Jia-Jyun Dong  
Co-Advisor: Dr. Yu-Chen Lu  
Date: 2023/03/03

# CONTENT

**1. INTRODUCTION**

**2. METHODOLOGY**

**3. RESULTS AND DISCUSSION**

**4. CONCLUSIONS**

**5. FUTURE WORKS**

# 1. INTRODUCTION

## Geological model

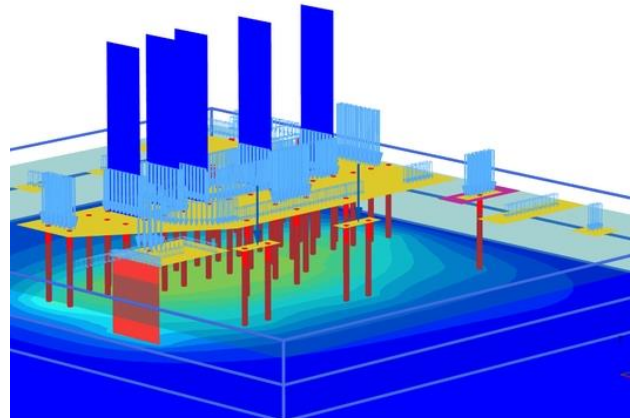
Important in **geotechnical systems** (foundations, tunnels, slopes, hydrological..)

Basis for the **analysis, simulation, evaluation**

The derived geological model and geo-material properties could be **highly uncertain**, especially at a **complex deposits site and limit data**

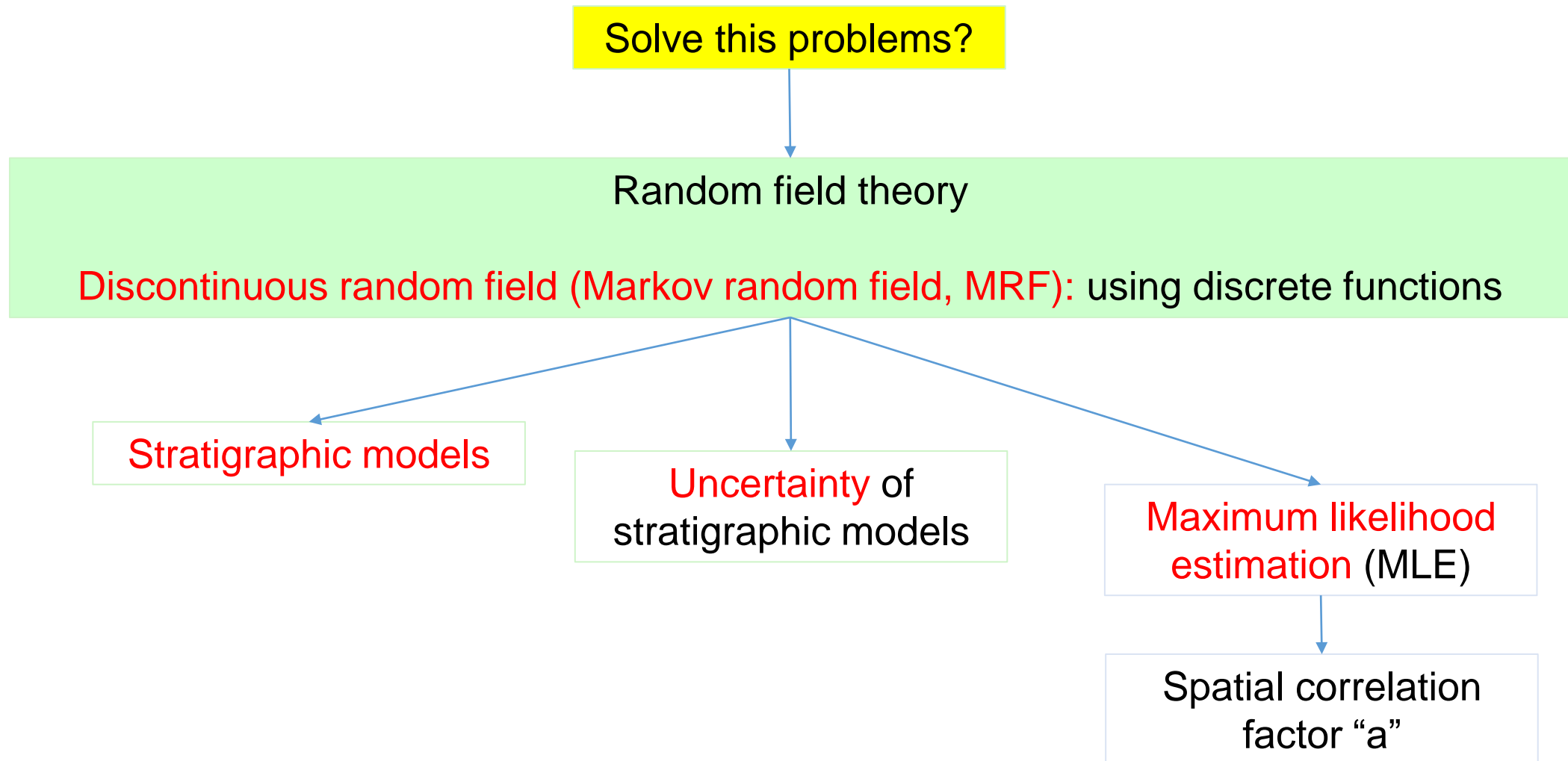


Source: World Construction Today

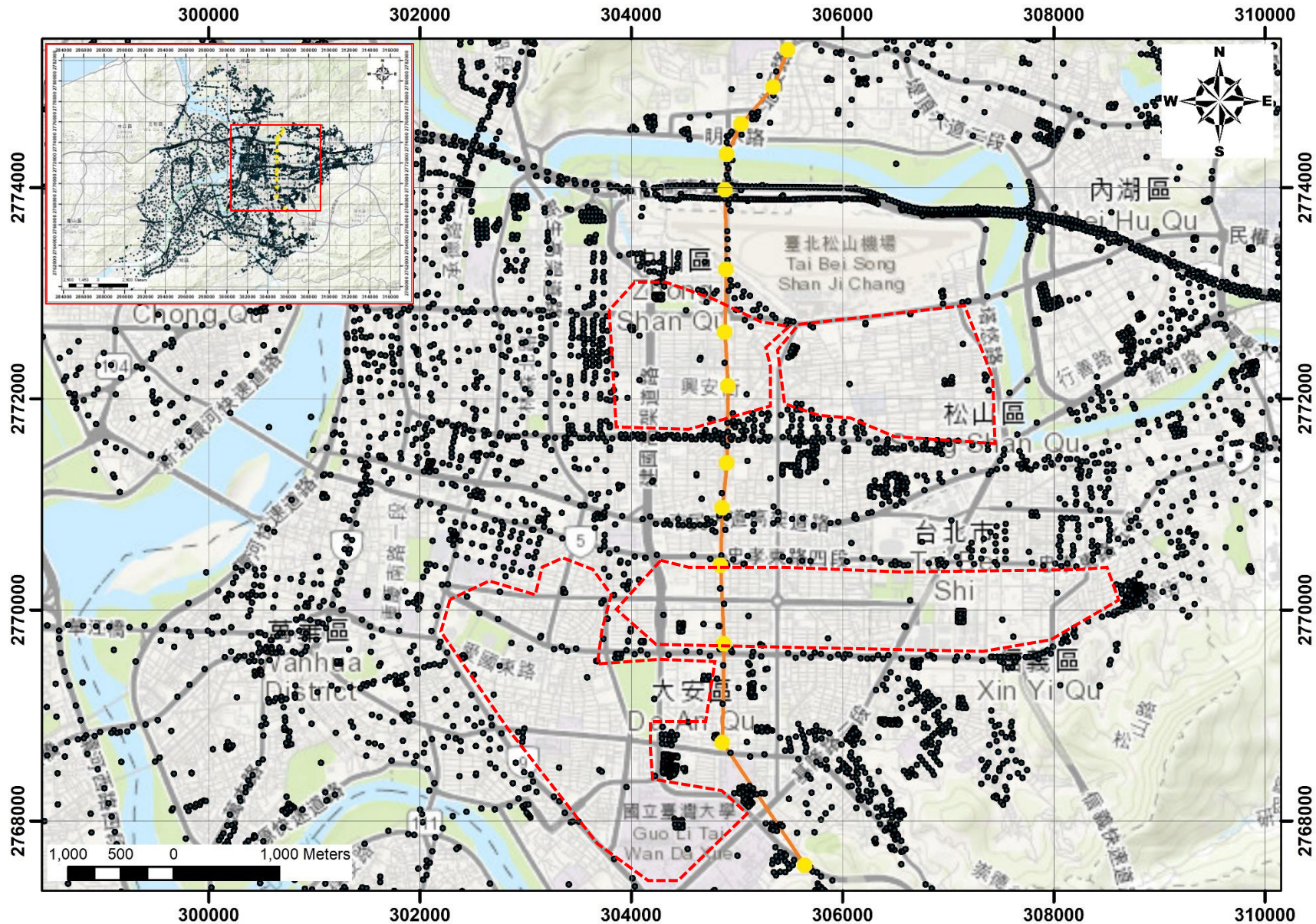


Source: CMW Geosciences

# 1. INTRODUCTION



# 1. INTRODUCTION



## LEGEND

- NS-1 section (CGS, 2011)
- Boreholes in NS-1 section
- Boreholes from CGS databank

How to use MRF in an area of empty boreholes data?

...considering the sedimentary environment & calibrate the spatial correlation factor related to the scale effect

# CONTENT

1. INTRODUCTION

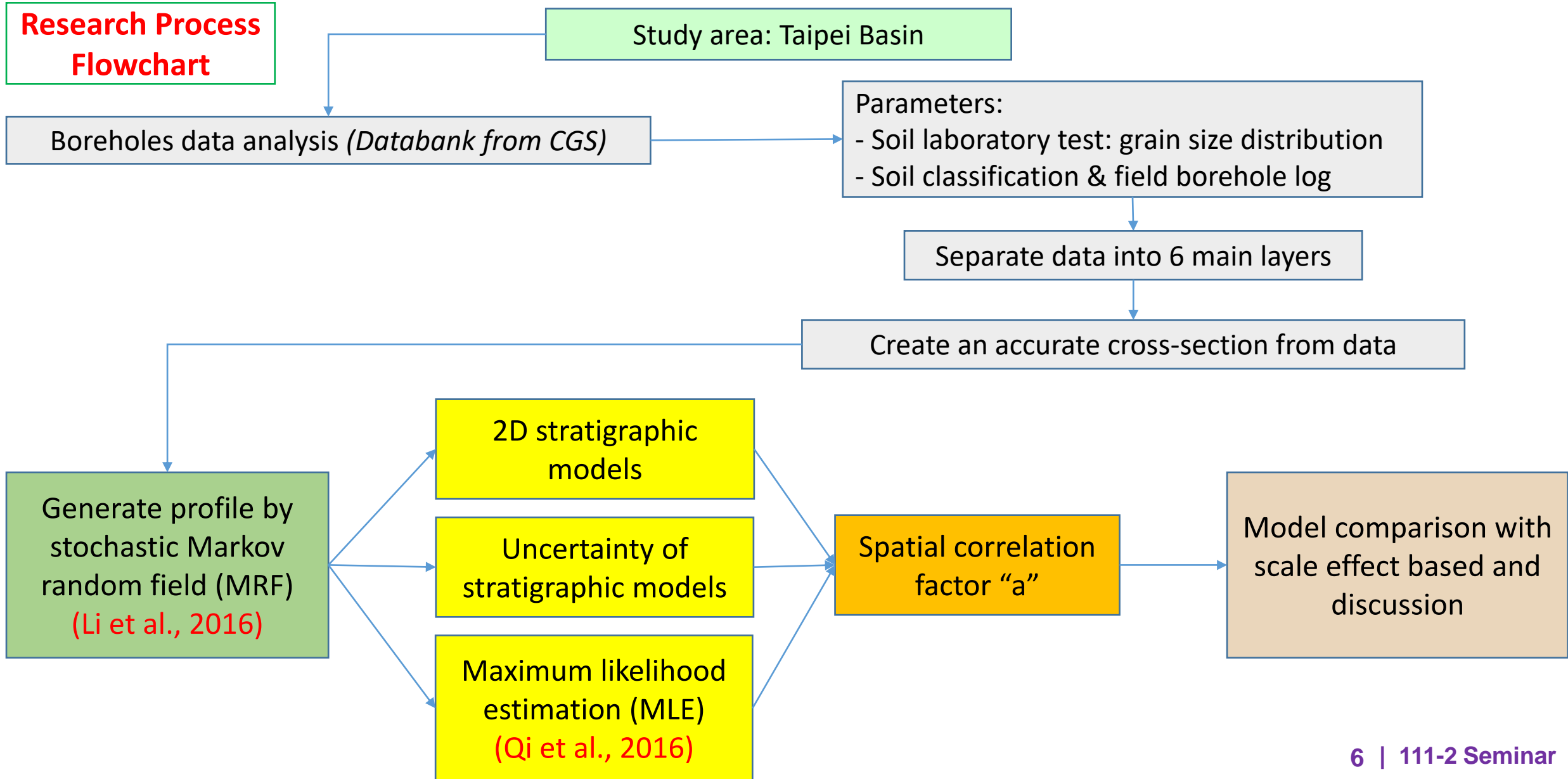
**2. METHODOLOGY**

3. RESULTS AND DISCUSSION

4. CONCLUSIONS

5. FUTURE WORKS

## 2. METHODOLOGY

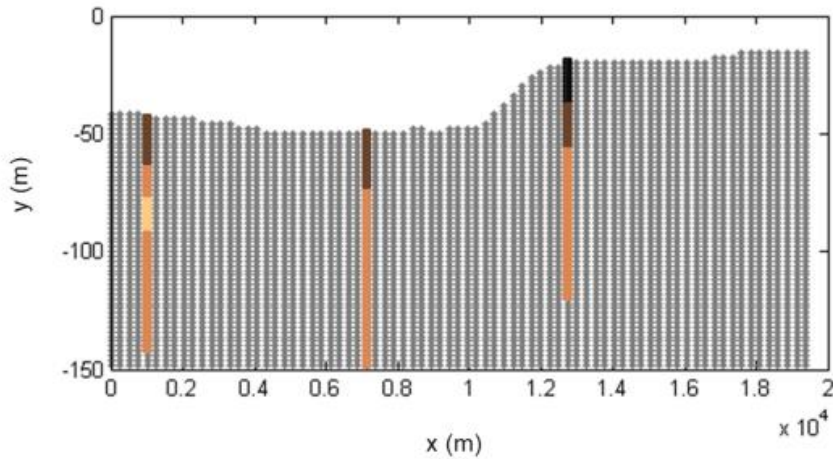


# 2. METHODOLOGY

## The process of MRF

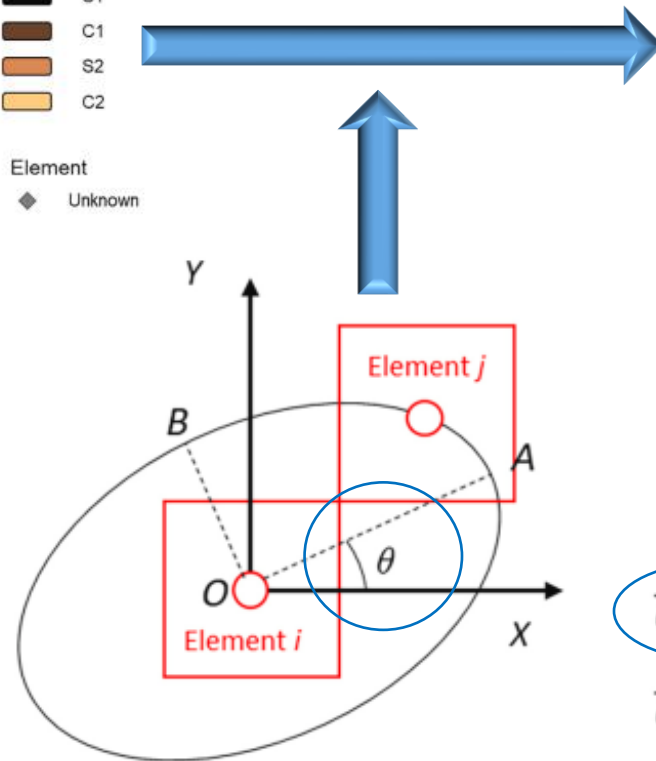
### 2.1 Spatial Correlation Model Used in MRF

Discretized mesh of cross-section



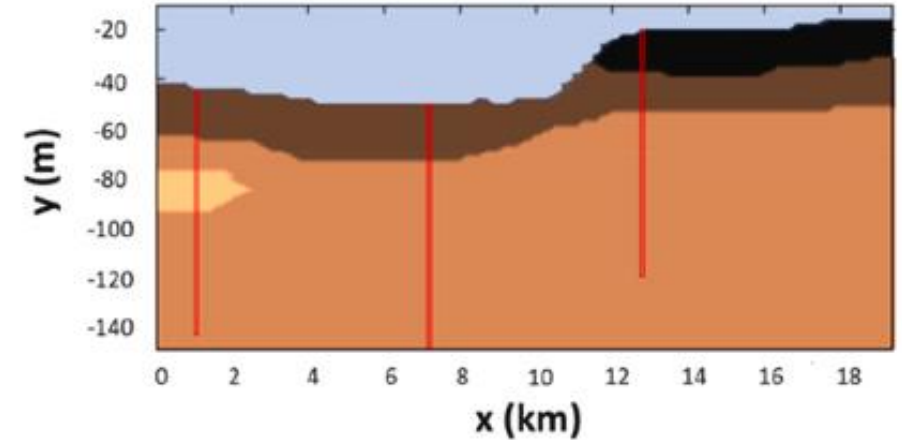
(Hsu et al., 2022)

## MRF simulation



Spatial Correlation Model  
(after Li et al., 2016)

1 realization generated by MRF



(Hsu et al., 2022)

## Two parameters should be determined

$$\overline{OA} = a$$

$$\overline{OB} = 1$$

spatial correlation factor,  $a$

spatial correlation dip,  $\theta$

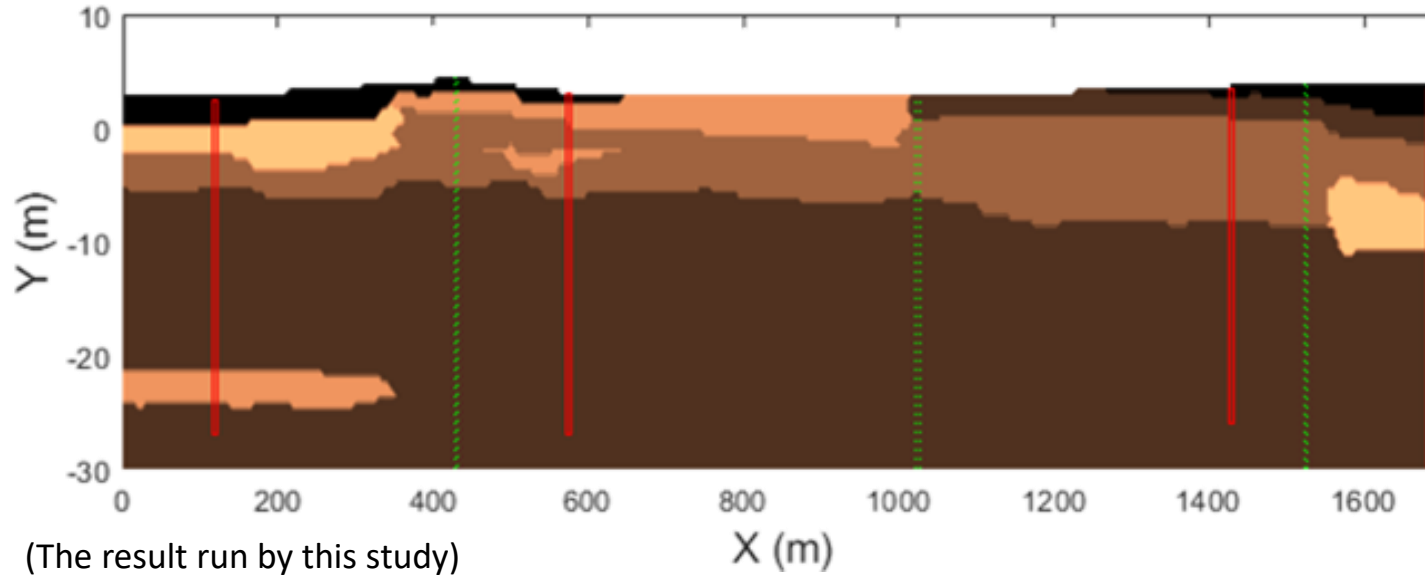


## 2. METHODOLOGY

### The process of MRF

### 2D Stratigraphic Model: Stochastic MRF Simulation

a = 12 (1000 realizations)



#### LEGEND



Backfilling



Borehole for MRF simulation



Clay



Observation borehole



Sand



Silt with plasticity



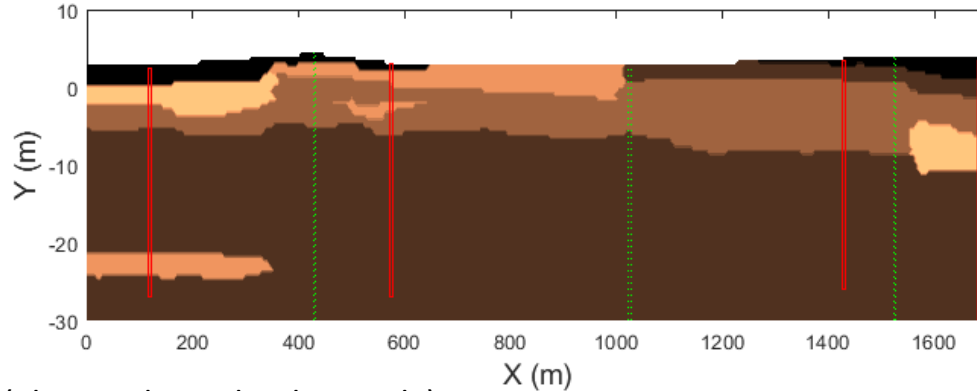
Silt without plasticity

# 2. METHODOLOGY

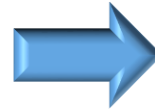
## The process of MRF

### 2.2 The Uncertainty of Stratigraphic model

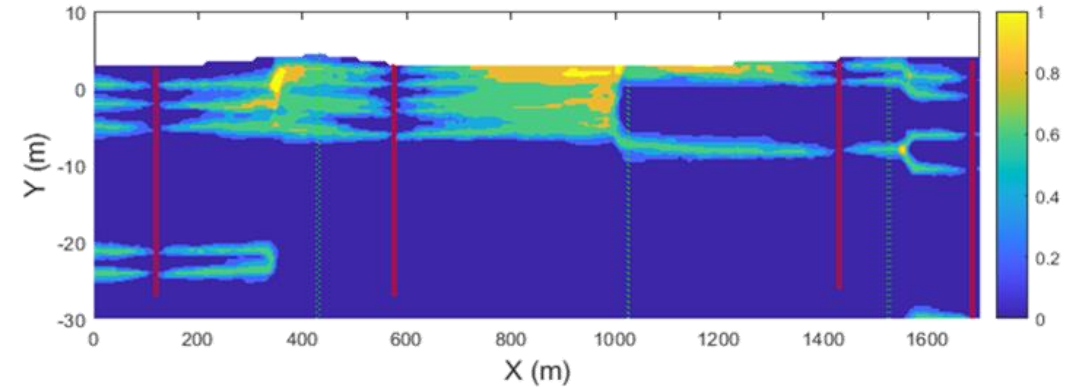
$a = 12$



(The result run by this study)



## Information entropy



### LEGEND

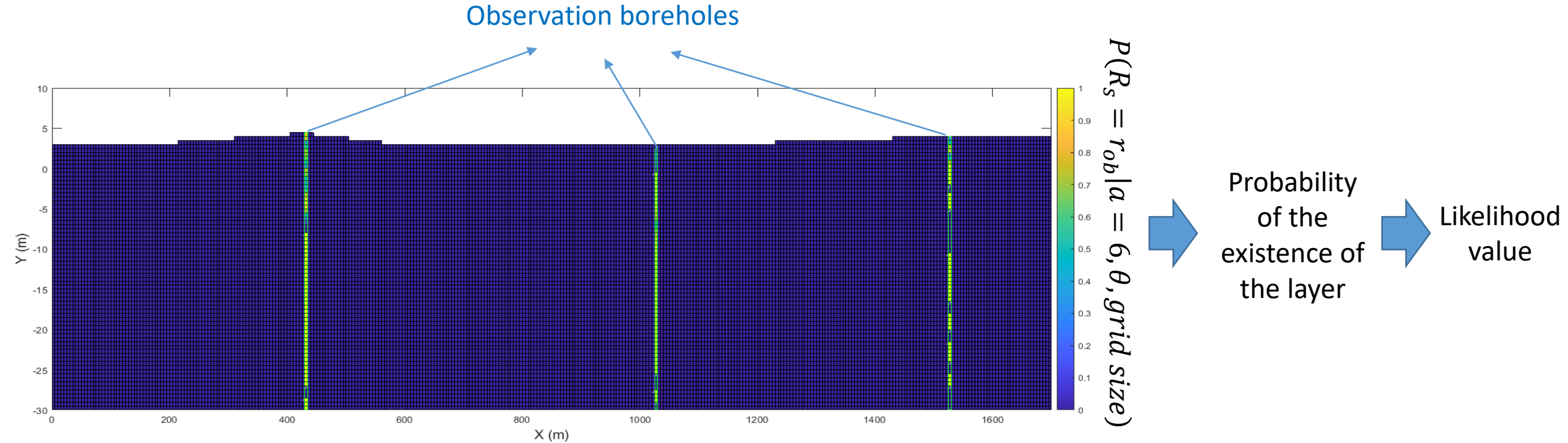
- Backfilling 1000 realizations
- Clay
- Sand
- Silt with plasticity
- Silt without plasticity
- Borehole for MRF simulation
- Observation borehole

- Calculate the probability of soil type existence
- Substitute the probability into equation of information entropy

## 2. METHODOLOGY

### The process of MRF

#### 2.3 Likelihood Value at the Observation Borehole

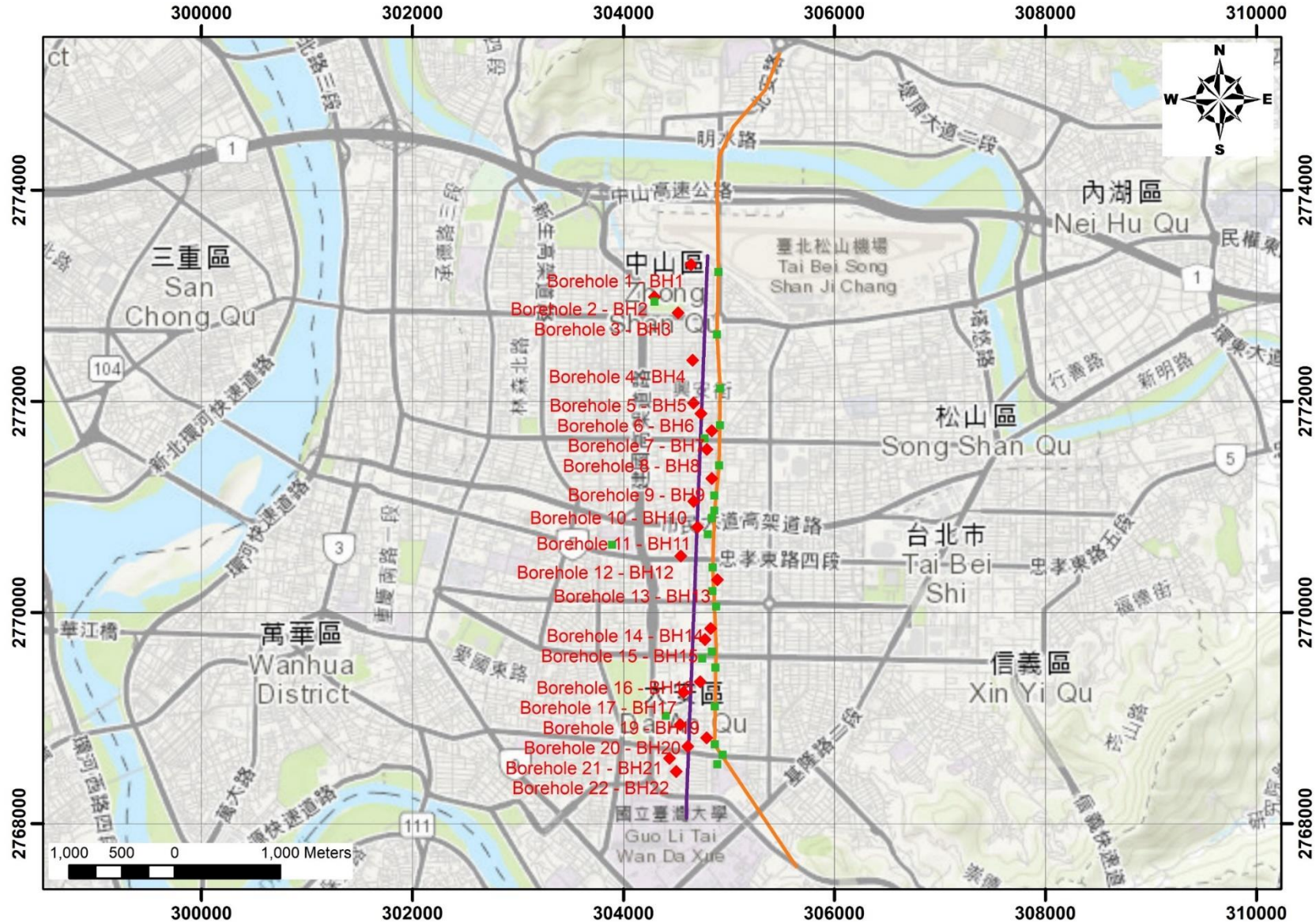


(The result run by this study)





The probability of **simulation equal to observation** for spatial correlation,  $\alpha$  equal to 12 (only compared with observation borehole)

# 2. METHODOLOGY

## The study site

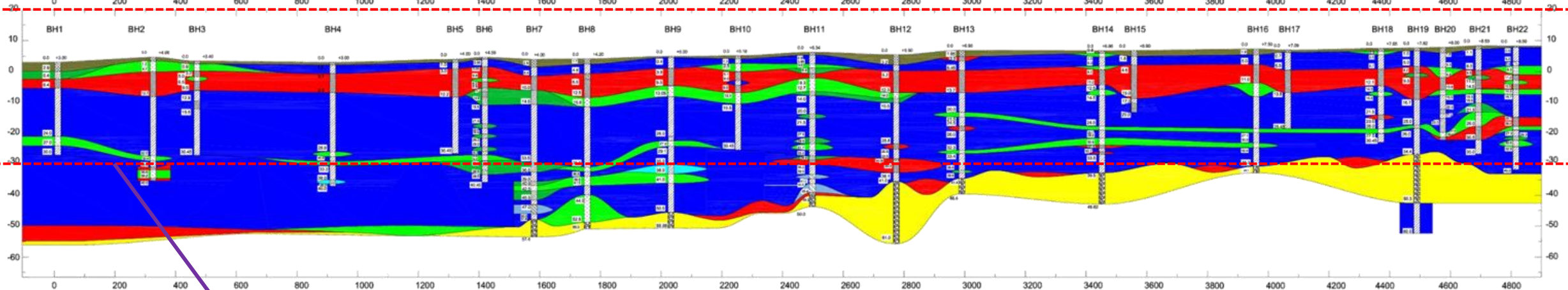
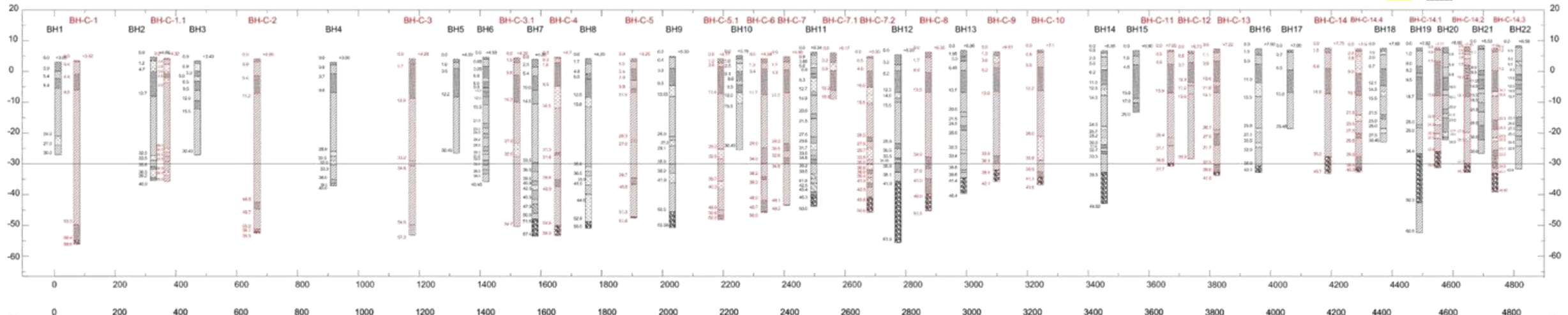


## LEGEND

-  NS-1 section (CGS, 2011)
-  Boreholes used for analyzed (22 BHs)
-  The line use for projected boreholes
-  Boreholes used for checking information (23 BHs)

# 2. METHODOLOGY

Create a cross-section from borehole data



Simulation area

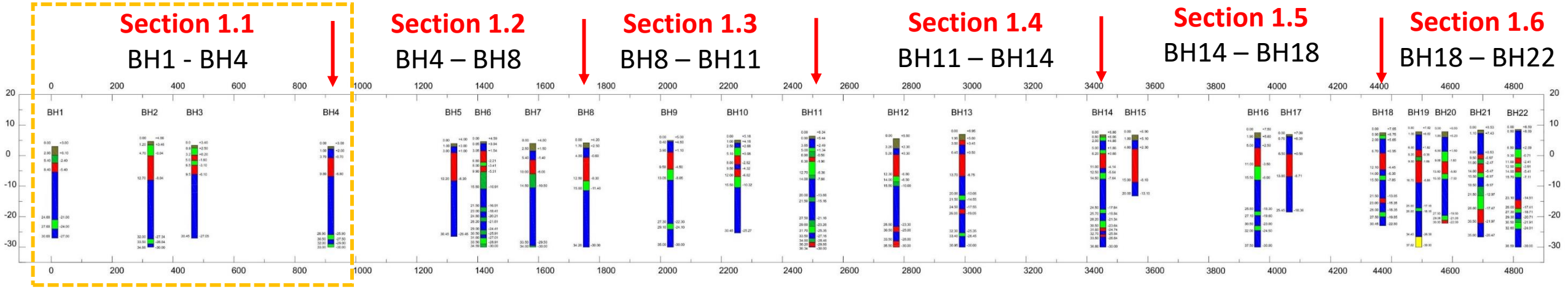
# 2. METHODOLOGY

## Scale effect based

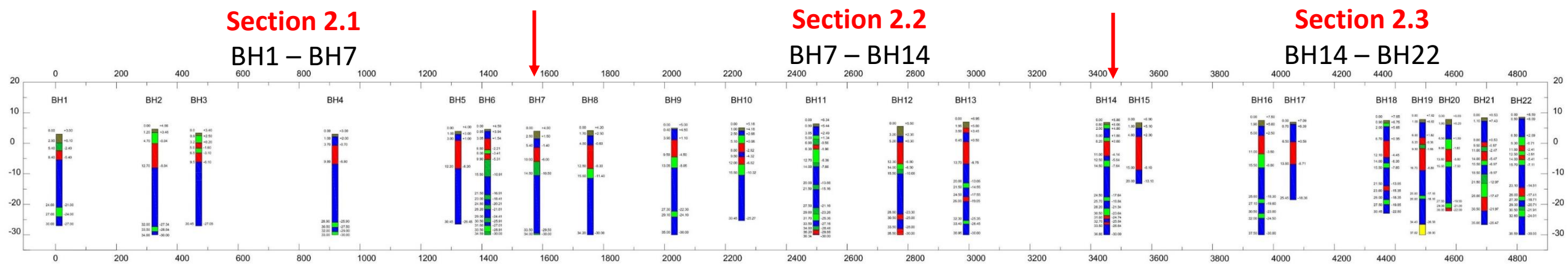
### LEGEND

- Backfilling
- CL: Lean clay
- SM: Silty sand
- ML: Silts without plasticity
- ML-1: Silts with plasticity
- Jingmei formation

Model 1: Scale effect with the length ~800m



Model 2: Scale effect with the length ~1600m



Model 3: Scale effect with whole section (~5000m)

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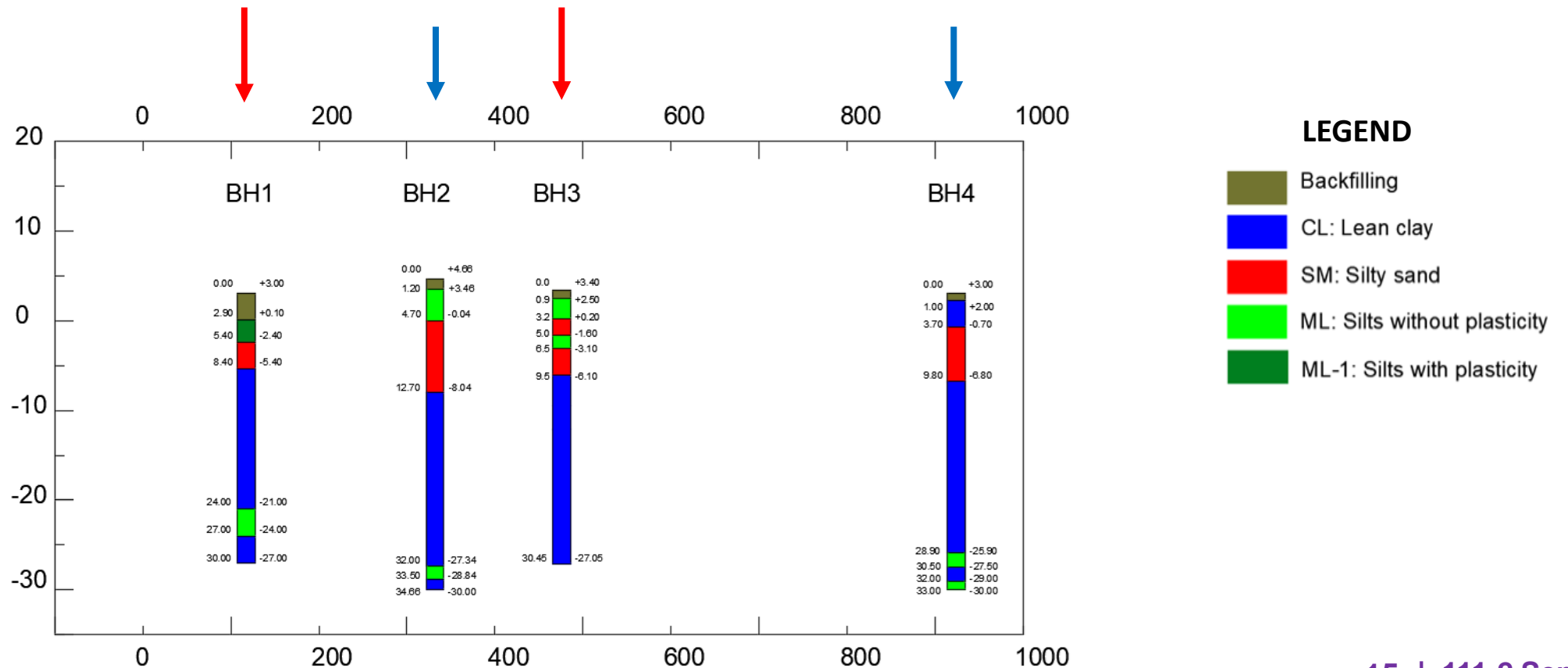
# 3. RESULTS AND DISCUSSION

## Simulation results in Section 1.1 – Model 1

### SECTION 1.1: BH1-BH4

Simulation borehole (2 boreholes)

Observation borehole (2 boreholes)

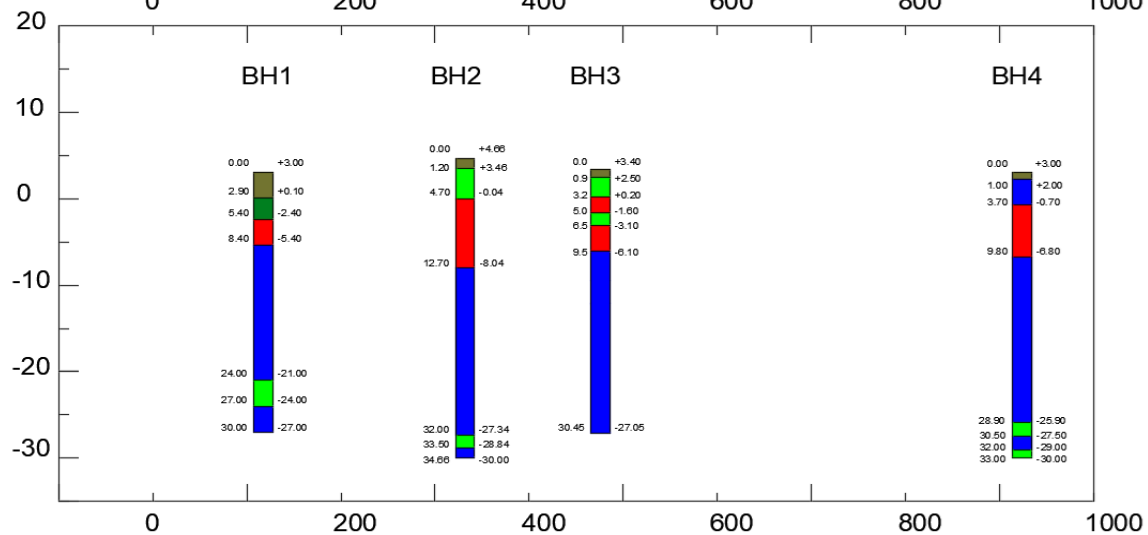
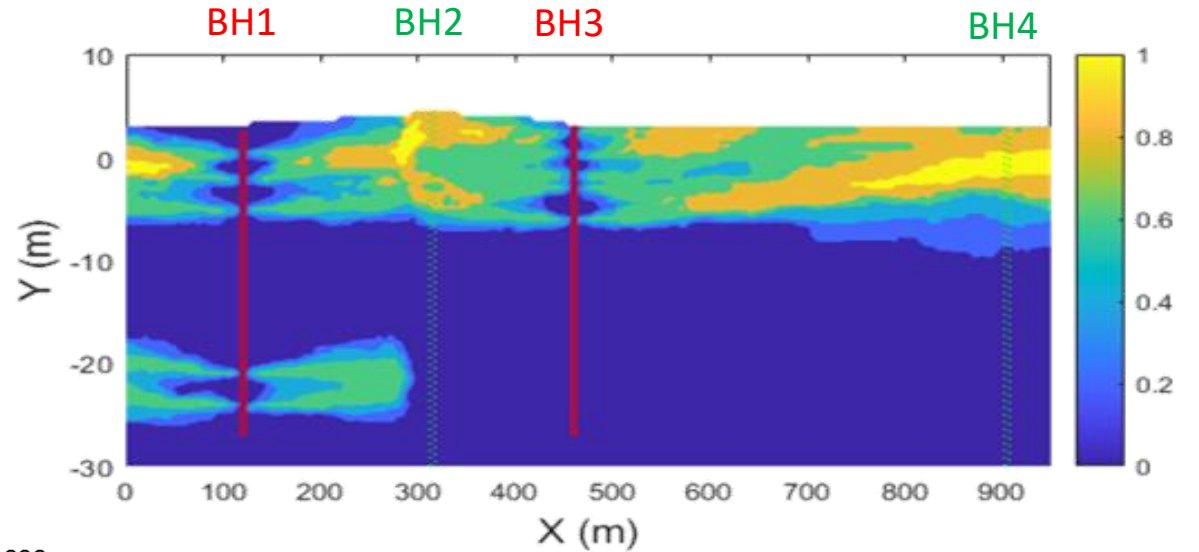
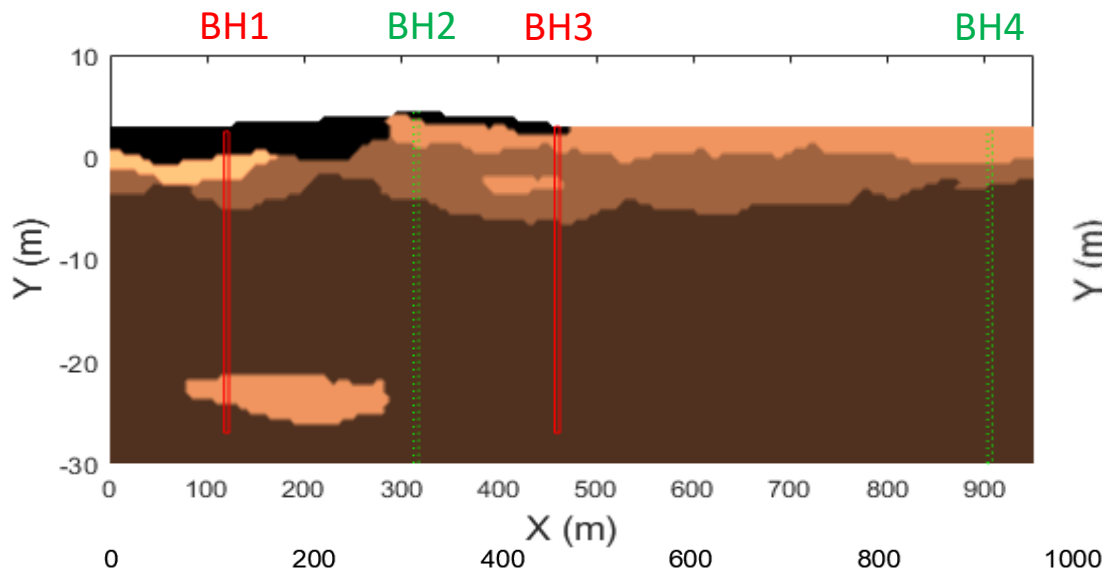




# 3. RESULTS AND DISCUSSION

## Simulation results in Section 1.1 – Model 1: 2D stratigraphic model & Uncertainty of stratigraphic model

a = 7 (1000 realizations)

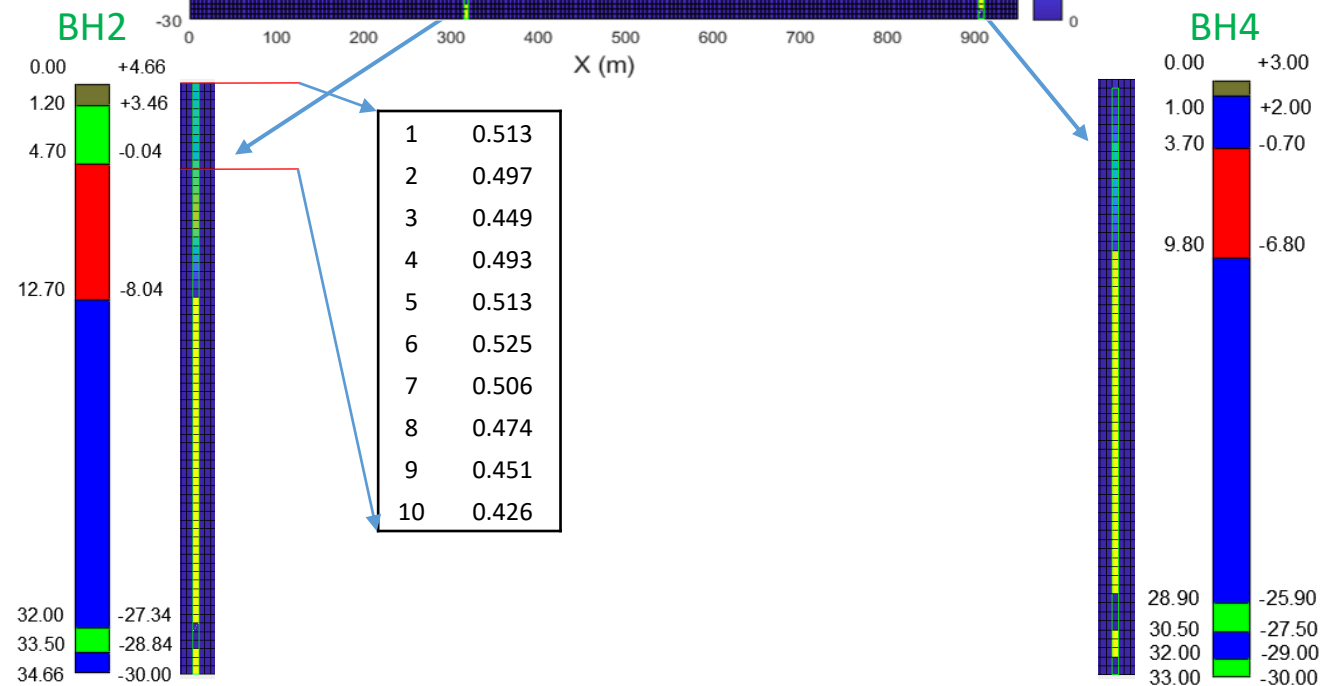
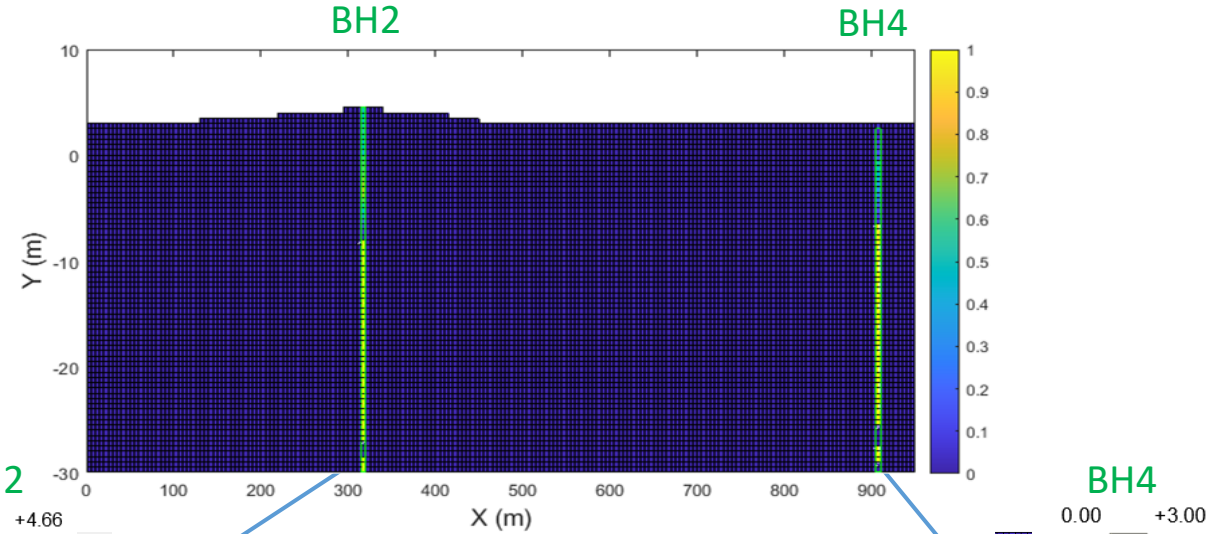
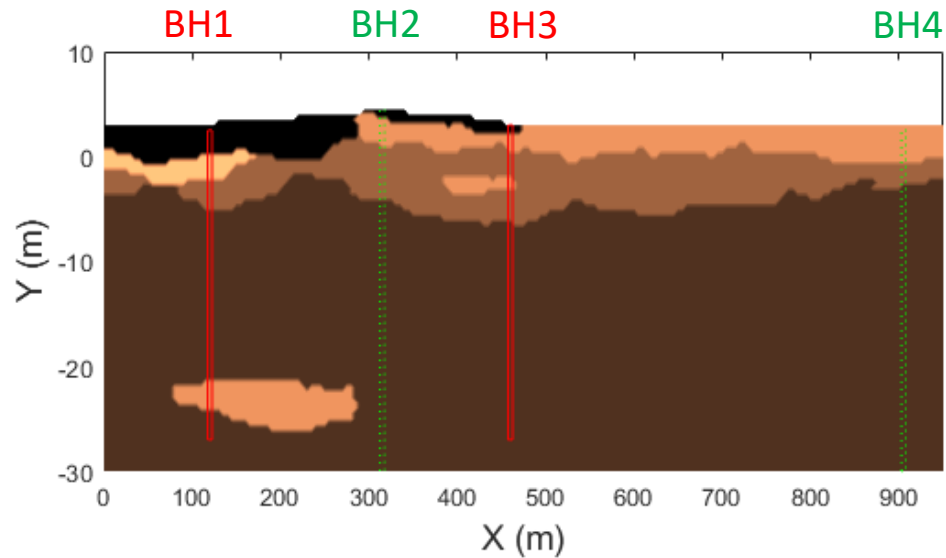


### LEGEND

- Backfilling
- Clay
- Sand
- Silt with plasticity
- Silt without plasticity
- Borehole for MRF simulation
- Observation borehole
- Backfilling
- CL: Lean clay
- SM: Silty sand
- ML: Silts without plasticity
- ML-1: Silts with plasticity

# 3. RESULTS AND DISCUSSION

## Simulation results in Section 1.1 – Model 1: 2D stratigraphic model & Maximum likelihood estimation



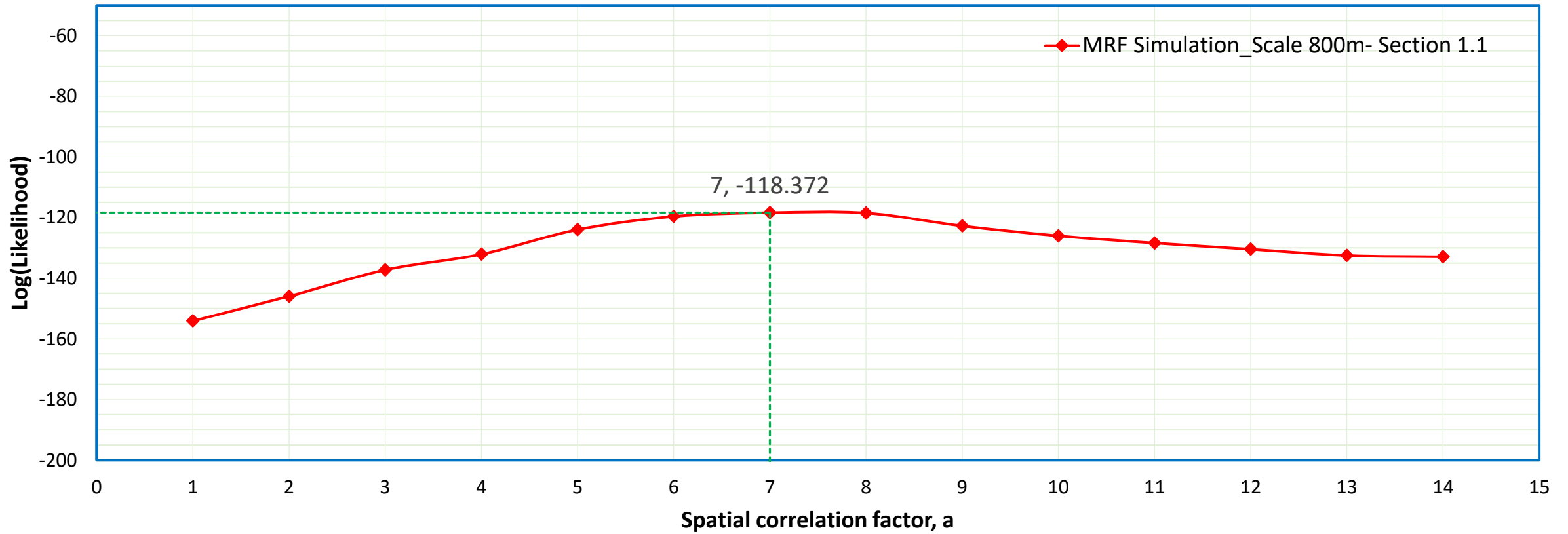
### LEGEND

- Backfilling
- Clay
- Sand
- Silt with plasticity
- Silt without plasticity
- Borehole for MRF simulation
- Observation borehole
- Backfilling
- CL: Lean clay
- SM: Silty sand
- ML: Silts without plasticity
- ML-1: Silts with plasticity

# 3. RESULTS AND DISCUSSION

## Simulation results in Section 1.1 – Model 1

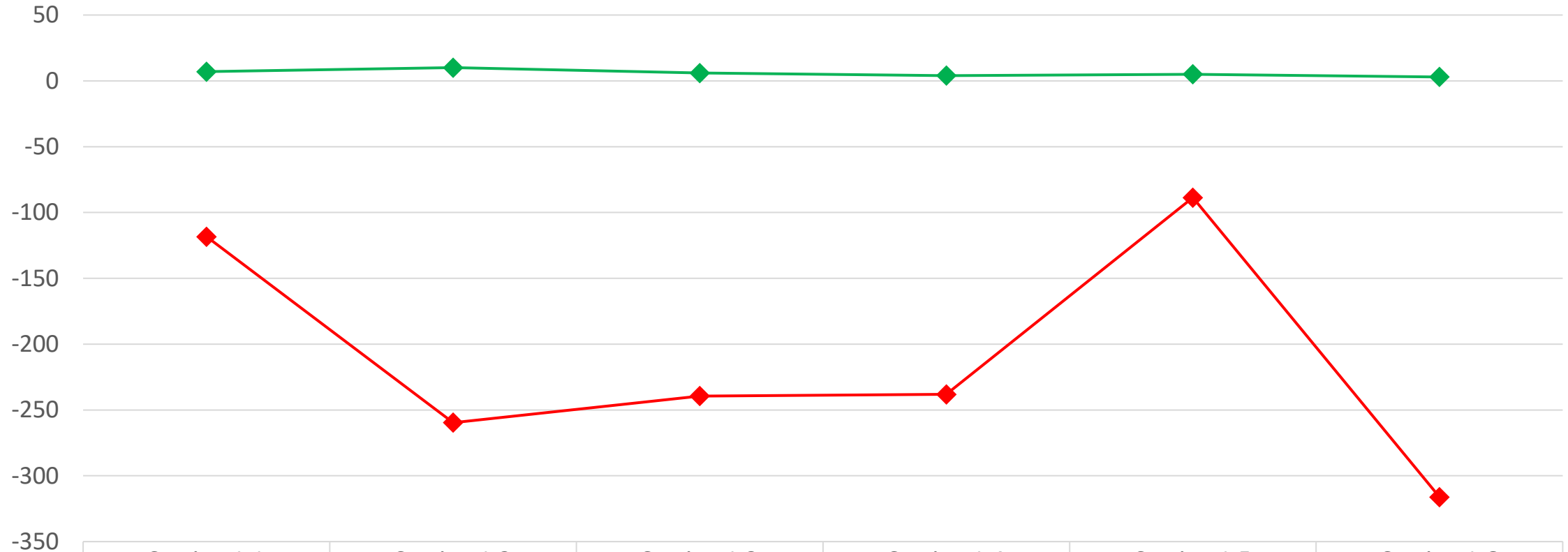
Relationship between Log(Likelihood) and Spatial correlation factor



# 3. RESULTS AND DISCUSSION

## Simulation results in Model 1

LOG(LIKELIHOOD) AND SPATIAL CORRELATION FACTOR VALUE - MODEL 1



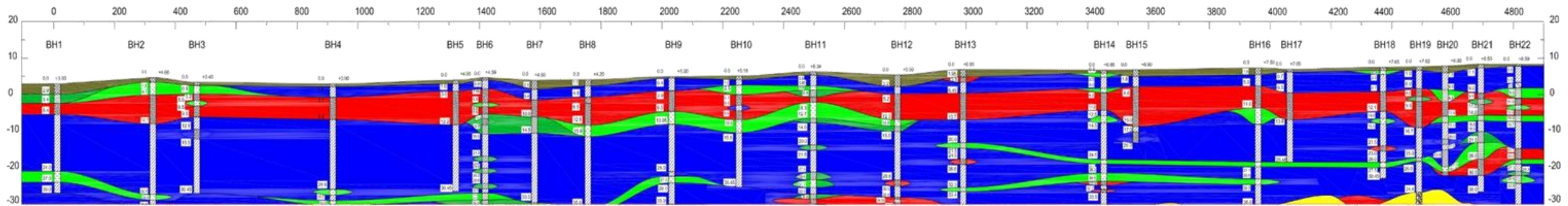
◆ a value  
◆ Log(Likelihood)

	Section 1.1	Section 1.2	Section 1.3	Section 1.4	Section 1.5	Section 1.6
a value	7	10	6	4	5	3
Log(Likelihood)	-118.372	-259.5801	-239.3648	-238.1021	-88.6695	-316.1605

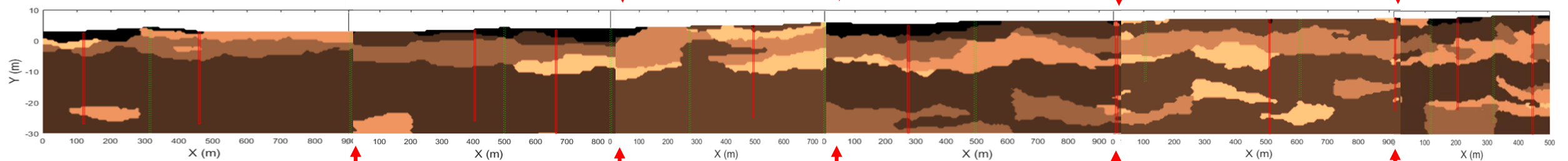
# 3. RESULTS AND DISCUSSION

## Model 1 – Scale 800m

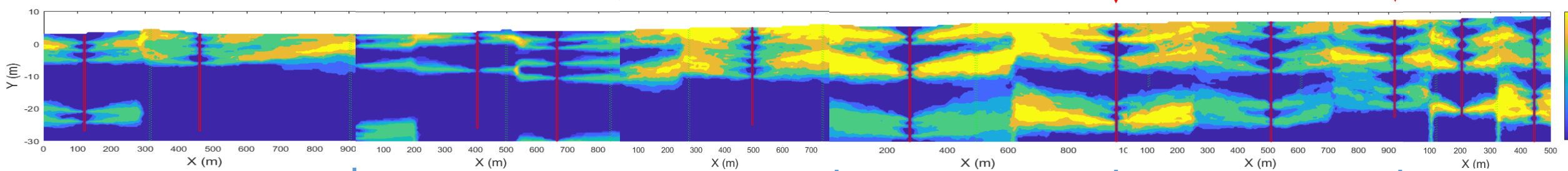
### Original cross - section



### Stochastic Markov Random Field Simulation



### Information entropy



$a = 7$

$a = 10$

$a = 6$

$a = 4$

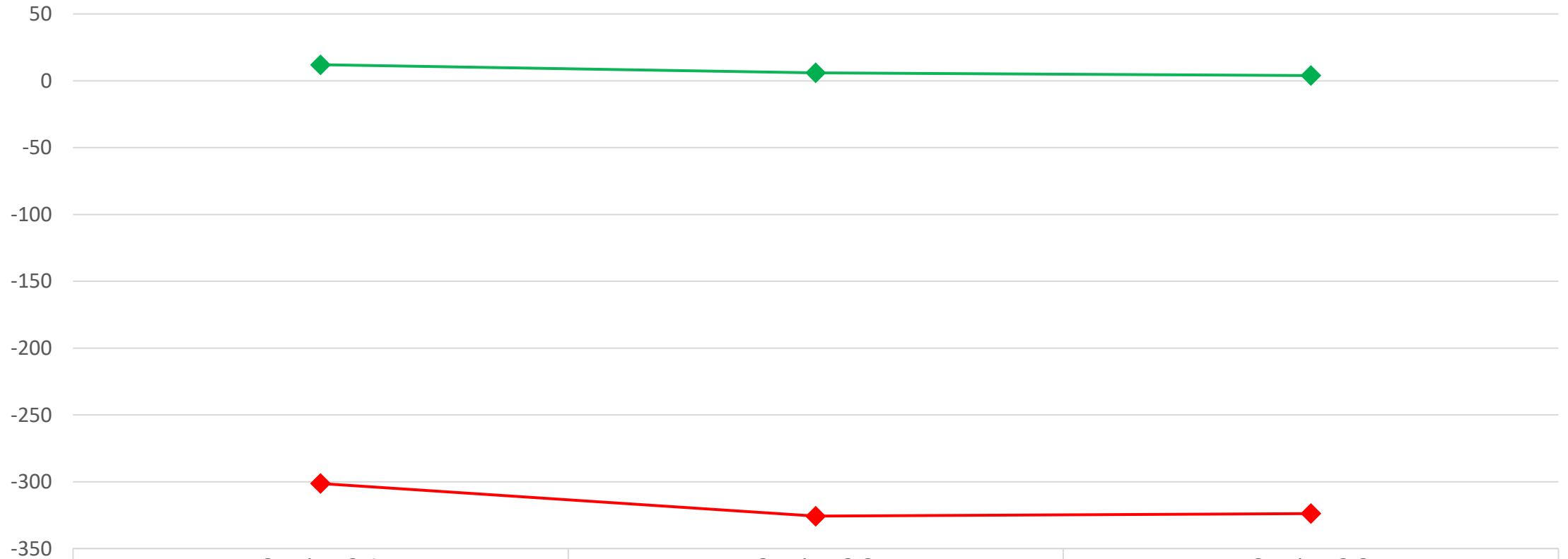
$a = 5$

$a = 3$

# 3. RESULTS AND DISCUSSION

## Model 2 – Scale 1600m

**LOG(LIKELIHOOD) AND SPATIAL CORRELATION FACTOR VALUE - MODEL 2**



◆ a value  
◆ Log(Likelihood)

Section 2.1

12

-301.2073

Section 2.2

6

-325.7202

Section 2.3

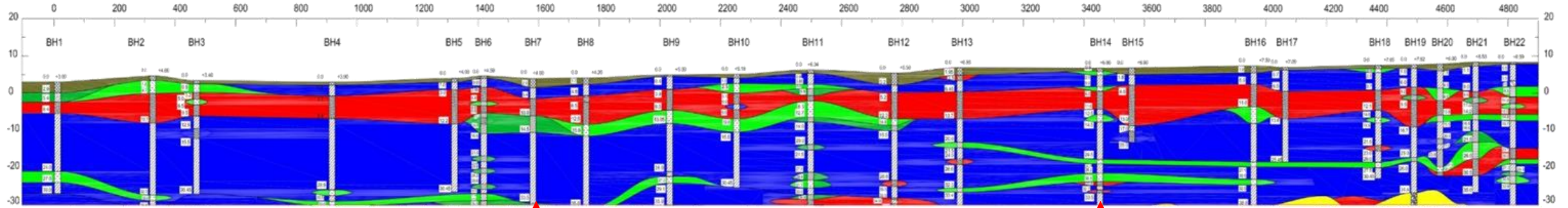
4

-323.7728

# 3. RESULTS AND DISCUSSION

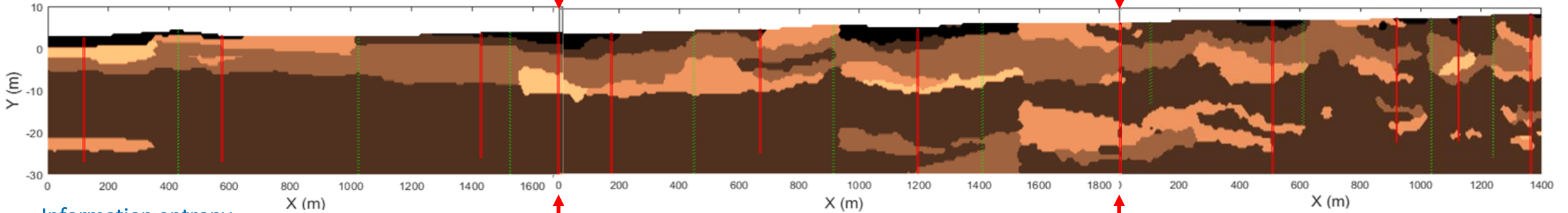
## Model 2 – Scale 1600m

Original cross - section

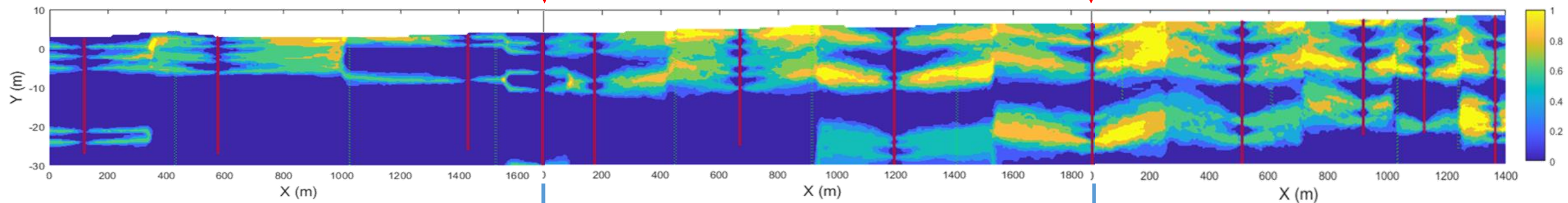


- |  |  |
|--|--|
| <b>LEGEND</b>  | <b>LEGEND</b>  |
| <span style="display:inline-block; width:15px; height:15px; background-color:darkgreen;"></span> Backfilling                       | <span style="display:inline-block; width:15px; height:15px; background-color:black;"></span> Backfilling                   |
| <span style="display:inline-block; width:15px; height:15px; background-color:blue;"></span> CL: Lean clay                          | <span style="display:inline-block; width:15px; height:15px; background-color:darkbrown;"></span> Clay                      |
| <span style="display:inline-block; width:15px; height:15px; background-color:red;"></span> SM: Silty sand                          | <span style="display:inline-block; width:15px; height:15px; background-color:brown;"></span> Sand                          |
| <span style="display:inline-block; width:15px; height:15px; background-color:limegreen;"></span> ML: Silts without plasticity (NP) | <span style="display:inline-block; width:15px; height:15px; background-color:orange;"></span> Silt with plasticity         |
| <span style="display:inline-block; width:15px; height:15px; background-color:darkgreen;"></span> ML-1: Silts with plasticity       | <span style="display:inline-block; width:15px; height:15px; background-color:lightorange;"></span> Silt without plasticity |
| <span style="display:inline-block; width:15px; height:15px; background-color:yellow;"></span> Jingmei formation                    | <span style="display:inline-block; width:15px; height:15px; border:1px solid red;"></span> Borehole for MRF simulation     |
|  | <span style="display:inline-block; width:15px; height:15px; border:1px dashed green;"></span> Observation borehole         |

Stochastic Markov Random Field Simulation



Information entropy



$a = 12$

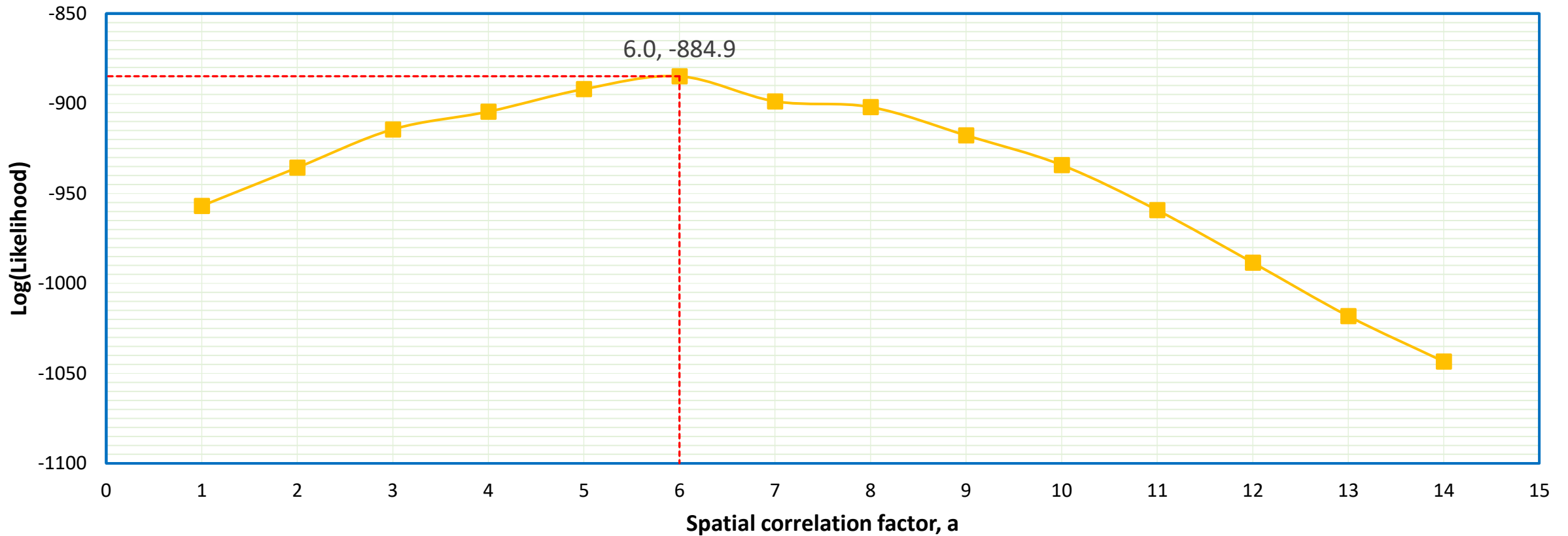
$a = 6$

$a = 4$

# 3. RESULTS AND DISCUSSION

## Model 3 – Scale 5000m

Relationship between Log(Likelihood) and Spatial correlation factor

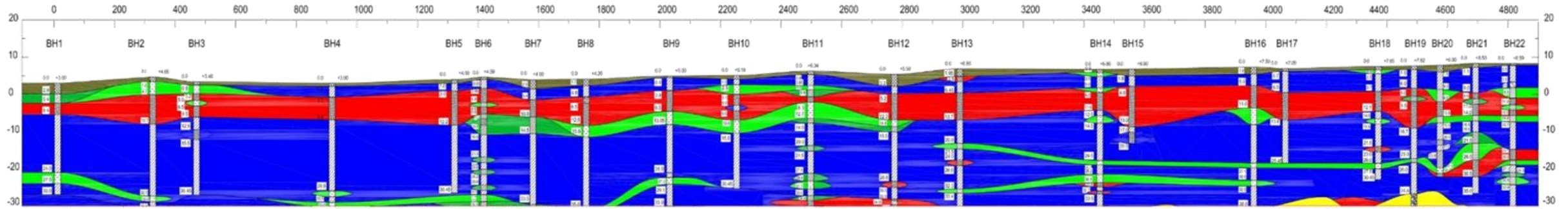




# 3. RESULTS AND DISCUSSION

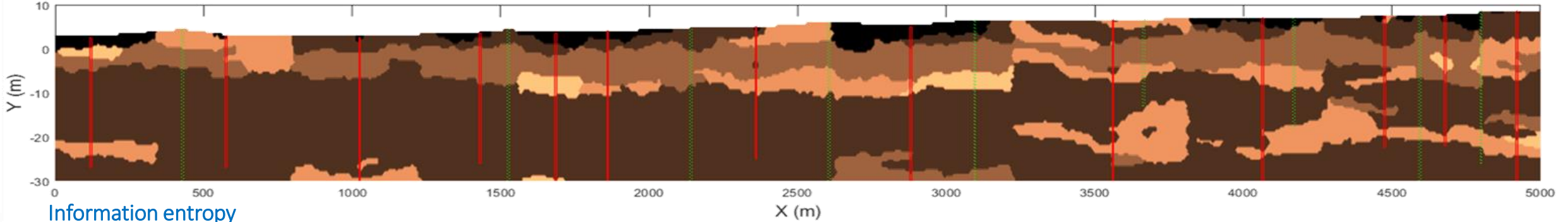
## Model 3 – Scale 5000m

Original cross - section

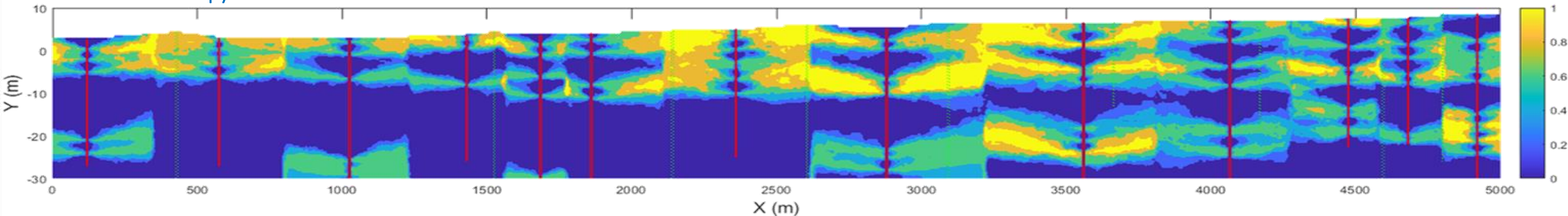


- |   |  |
|---|--|
| <b>LEGEND</b>   | <b>LEGEND</b>  |
| <span style="display:inline-block; width:15px; height:15px; background-color:darkbrown;"></span> Backfilling                        | <span style="display:inline-block; width:15px; height:15px; background-color:black;"></span> Backfilling                   |
| <span style="display:inline-block; width:15px; height:15px; background-color:blue;"></span> CL: Lean clay                           | <span style="display:inline-block; width:15px; height:15px; background-color:darkbrown;"></span> Clay                      |
| <span style="display:inline-block; width:15px; height:15px; background-color:red;"></span> SM: Silty sand                           | <span style="display:inline-block; width:15px; height:15px; background-color:lightbrown;"></span> Sand                     |
| <span style="display:inline-block; width:15px; height:15px; background-color:lightgreen;"></span> ML: Silts without plasticity (NP) | <span style="display:inline-block; width:15px; height:15px; background-color:orange;"></span> Silt with plasticity         |
| <span style="display:inline-block; width:15px; height:15px; background-color:darkgreen;"></span> ML-1: Silts with plasticity        | <span style="display:inline-block; width:15px; height:15px; background-color:lightorange;"></span> Silt without plasticity |
| <span style="display:inline-block; width:15px; height:15px; background-color:yellow;"></span> Jingmei formation                     | <span style="display:inline-block; width:15px; height:15px; border:1px solid red;"></span> Borehole for MRF simulation     |
|   | <span style="display:inline-block; width:15px; height:15px; border:1px dashed green;"></span> Observation borehole         |

Stochastic Markov Random Field Simulation



Information entropy



$a = 6$

# CONTENT

1. INTRODUCTION

2. METHODOLOGY

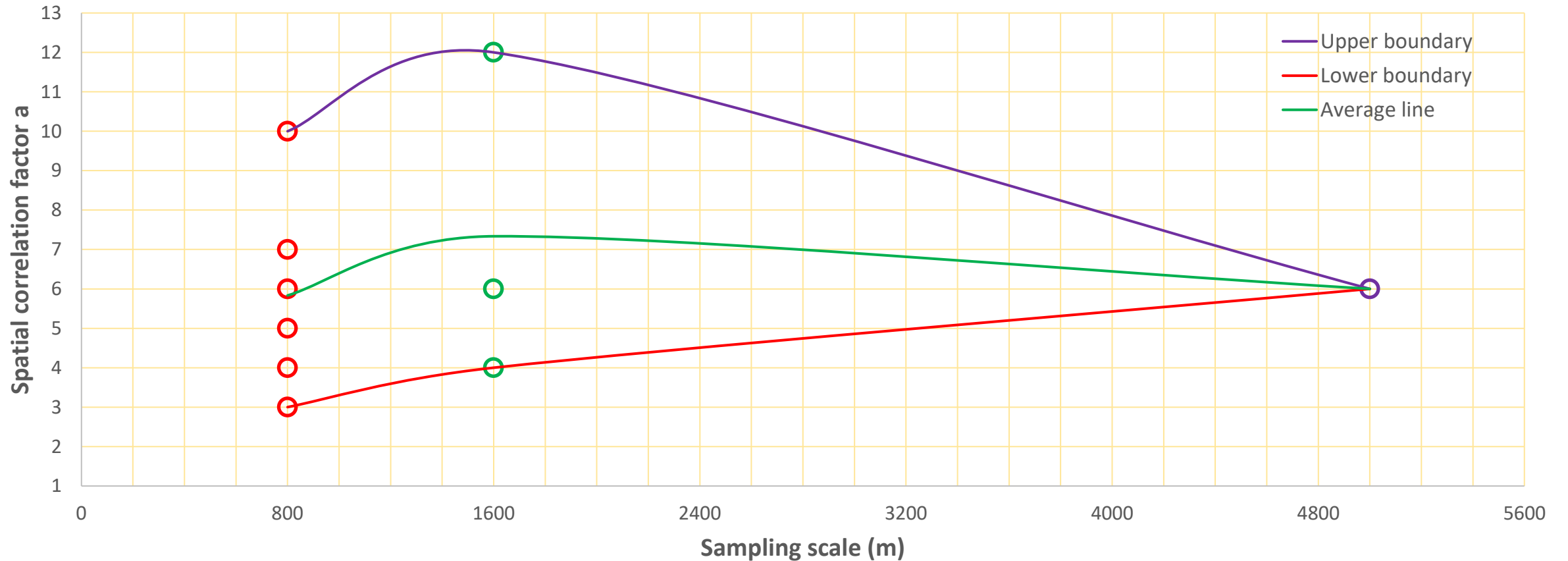
3. RESULTS AND DISCUSSION

**4. CONCLUSIONS**

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# 4. CONCLUSIONS

## Evaluation scale effect of sample



Model 1: scale 800m – 6 points

Model 2: scale 1600m – 3 points

Model 3: scale 5000m – 1 points

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1. INTRODUCTION

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## 5. FUTURE WORKS

1. Choose another cross-section in the E-W direction to see the scale effect;
2. Apply 3D simulation with considering the spatial orientation distribution.  
Then, comparing with 2D simulation results.

# 5. FUTURE WORKS

## The study site 2 (EW-2)



(CGS, 2011)



**THANK  
— YOU**

**To transfer the geological data to  
numerical models**

**It's an interesting process..**



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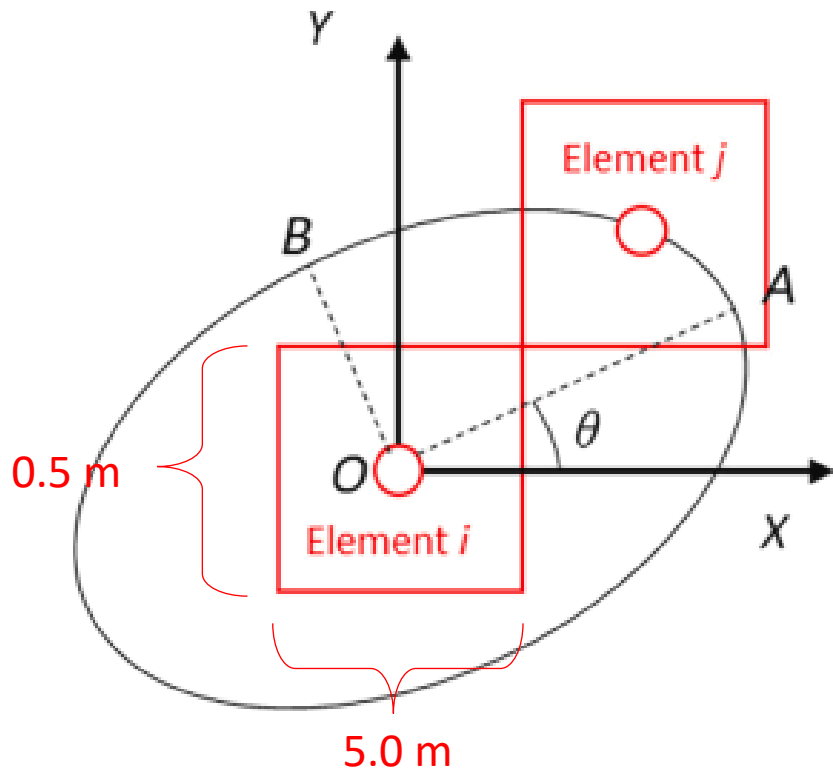
# **Scale effect of the spatial correlation factor used in Markov random fields: A case study in Taipei Basin**

Presenter: Le Hoai Han  
Advisor: Prof. Jia-Jyun Dong  
Co-Advisor: Dr. Yu-Chen Lu  
Date: 2023/03/03



# Grid Size Used in This Study

element\_number\_x = ....; model\_length\_x = ....;  
element\_number\_y = ....; model\_length\_y = ....;



Grid size = 5.0 m \* 0.5 m

Horizontal grid size/vertical grid size =  $A_R = 10$

$$\overline{OA} = a$$

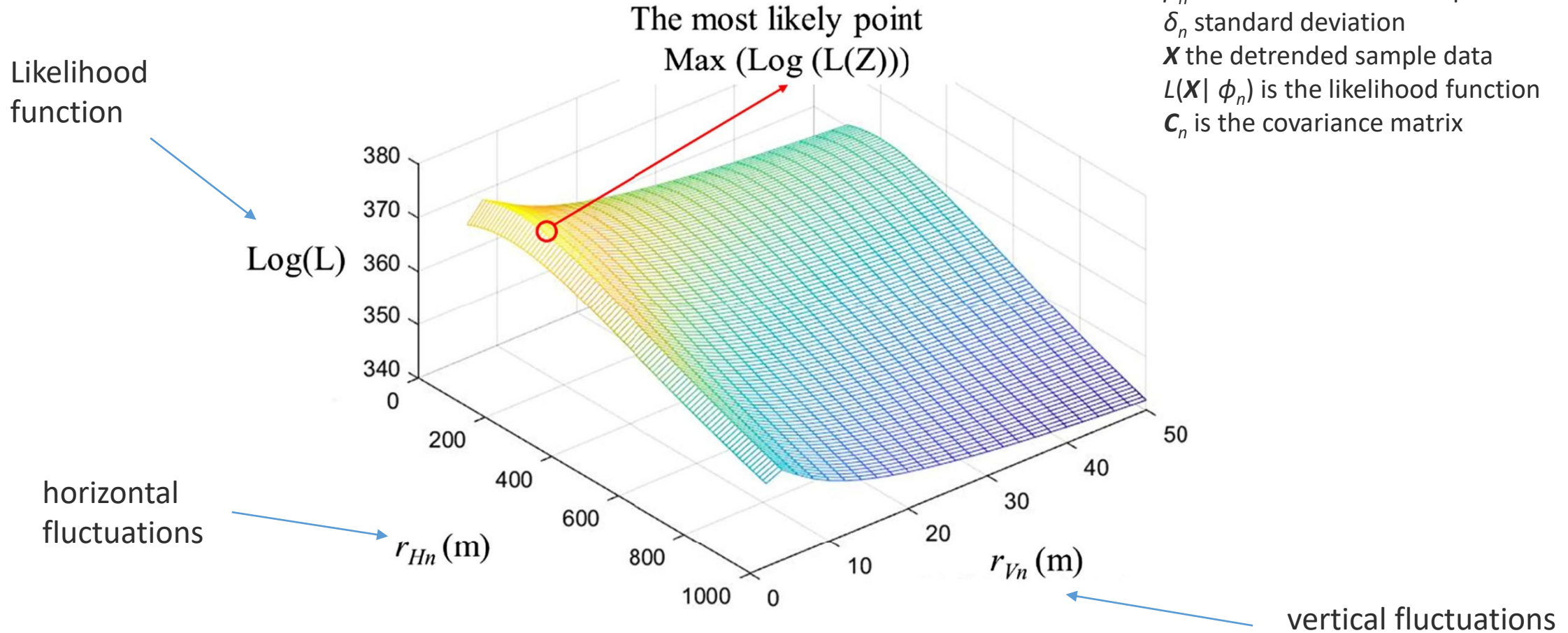
$$\overline{OB} = 1$$

$$a' = A_R \times a$$

# Maximum Likelihood Estimation

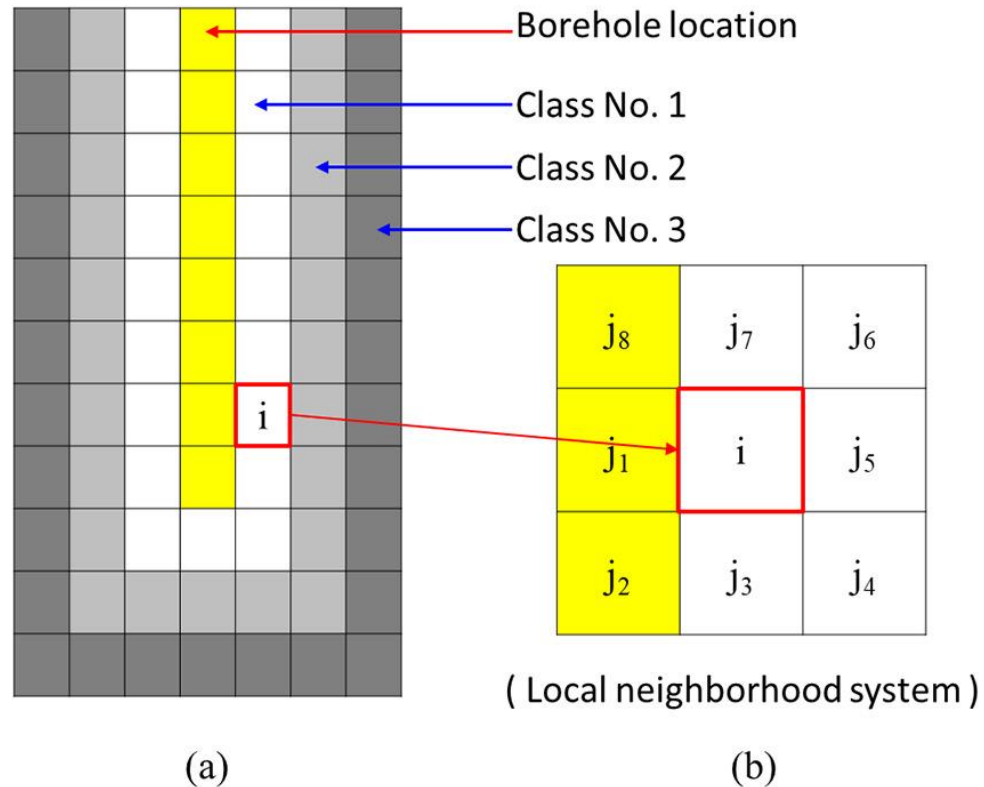
Find:  $\phi_n^T = \{\mu_n, \delta_n, r_{Vn}, r_{Hn}\}$   
 Subject to:  $\mathbf{X} = \{X_1, X_2, \dots, X_3\}$   
 Objective:  $L(\mathbf{X}|\phi_n) = \frac{1}{(2\pi)^{m/2} |\mathbf{C}_n|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{X} - \mu_n)^T \mathbf{C}_n^{-1} (\mathbf{X} - \mu_n)\right]$   
 Maximizing  $L(\mathbf{X}|\phi_n)$

$\phi_n^T$  is the site parameter to be determined  
 $\mu_n$  mean value of the site parameter  
 $\delta_n$  standard deviation  
 $\mathbf{X}$  the detrended sample data  
 $L(\mathbf{X}|\phi_n)$  is the likelihood function  
 $\mathbf{C}_n$  is the covariance matrix



Log-likelihood function of observational data  $I_c$  under different horizontal and vertical fluctuations

# Neighborhood system



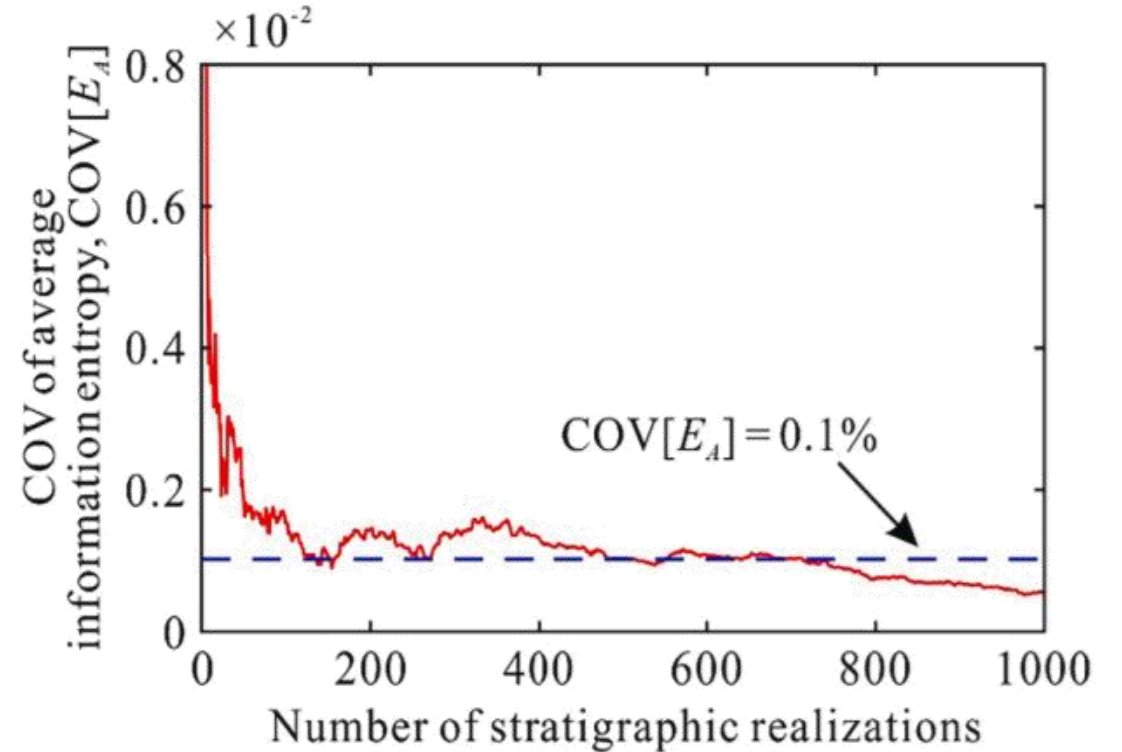
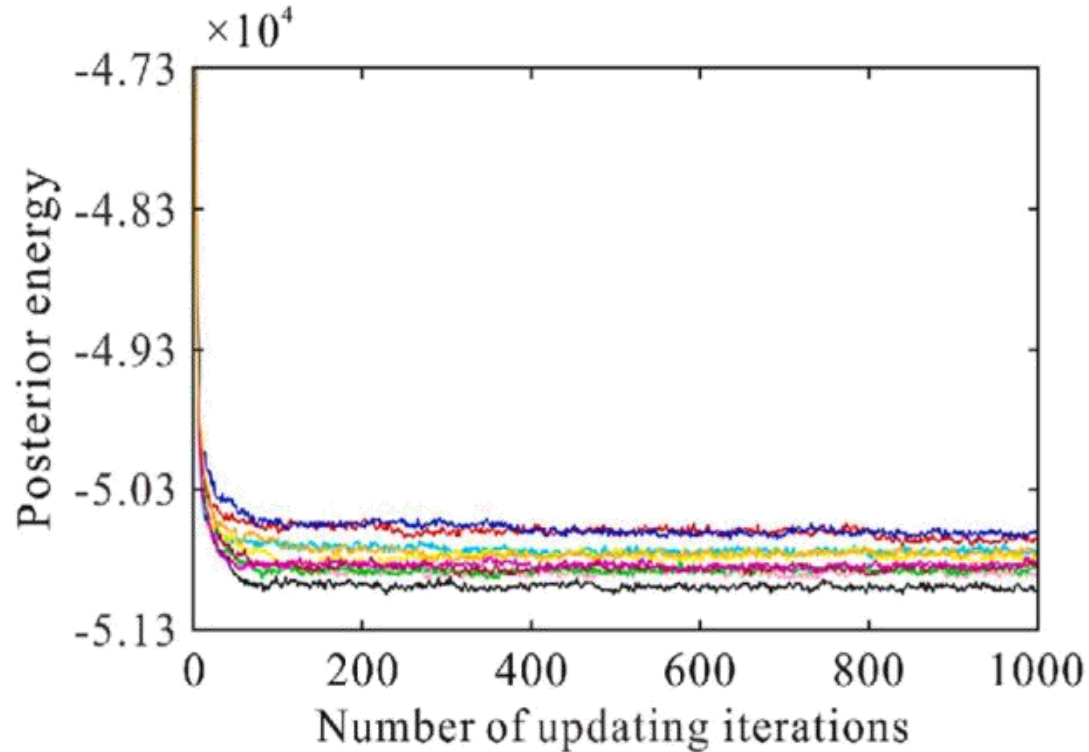
The analysis sequence starts by **determining the soil types of the nearest neighbors** and gradually expands outward.

The **grid cells immediately adjacent** to the borehole are defined as cells with the **highest priority**.

The grid cells adjacent to the **first priority** cells are defined as the cells with the **second priority**

Neighborhood system and sampling order (After [Gong et al., 2019](#))

## How to know how many realization was applied in the simulation?



Determination of the **number of iterations** adopted in the **MCMC updating** and the **number of sampled stratigraphic realizations** (Chao Zhao, 2021)