國立中央大學
National Central University

COLLEGE OF EARTH SCIENCES
Graduate Institute of Applied Geology

Seminar of Geotechnical Engineering | 大地工程專題討論

Scale effect on the Determination of the Spatial Correlation Factor Used in Markov Random Field

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OUTLINE

1. Introduction
2. Study Sites
3. Methodology
4. Simulation Results
5. Concluding Remarks

Introduction

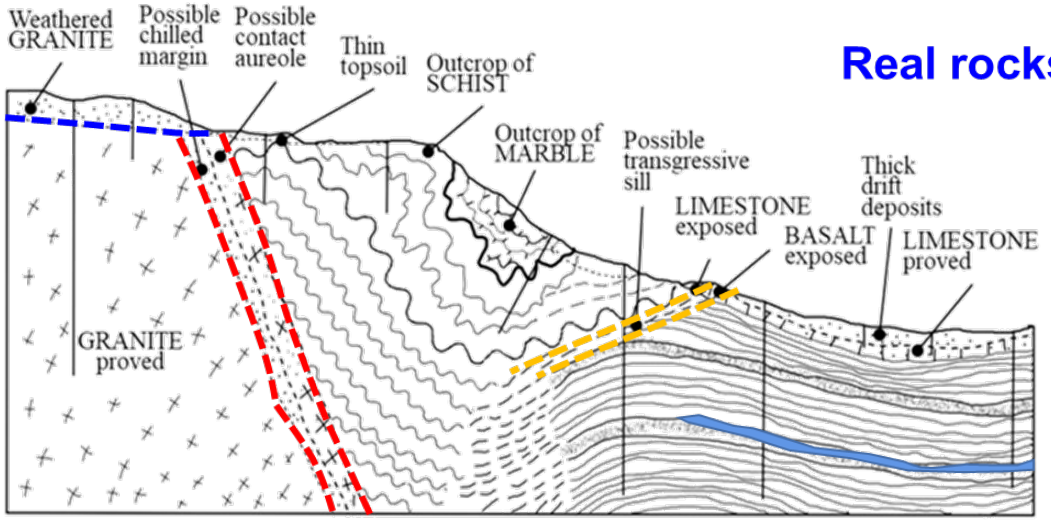
The Role of the Geological Model and Why It Has Uncertainty

- The geological model **is crucial** in geotechnical engineering analysis and design, geological hazard assessment, and socio-economic risk analysis (*Fookes, 1997; Keaton, 2013; Juang et al., 2019a, 2019b; Yeh et al., 2021*)

- The complete geological model is virtually impossible to obtain by limited geological survey, leading to **geological model uncertainty**.

The Geological Model Uncertainty

School rocks from limited information

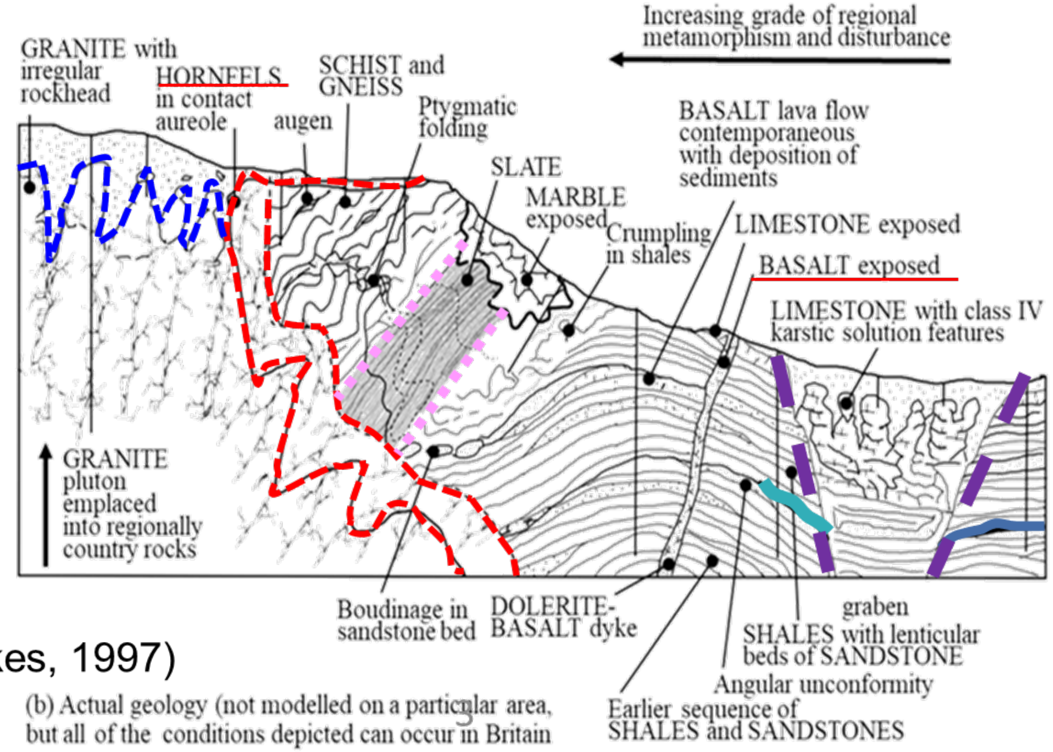


(a) Some potential misinterpretations of geology of (b) from borehole evidence

"Geological model"

Real rocks

(after Fookes, 1997)



(b) Actual geology (not modelled on a particular area, but all of the conditions depicted can occur in Britain

"True geological condition"

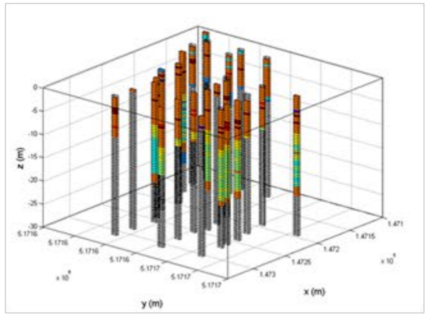
The Probabilistic Method for Estimating the Geological Model and Characterizing Its Uncertainty

- **Random fields** could simulate a series of potential geological models by **considering spatial variabilities of stratigraphic distribution** and, through a series of simulations, could **evaluate the geological model uncertainty**.

- The **Markov random field (MRF)**, one of the probabilistic approaches for quantifying the geological model uncertainty, was **widely employed for the geological model uncertainty simulation** (*Qi et al., 2016; Li et al., 2016; Wang et al., 2016, 2017; Gong et al., 2019; Hsu et al., 2022; Chien et al., 2022, 2023a, 2023b; Wei and Wang, 2022; Lu et al., 2023a, 2023b*)

The Geological Model and Its Uncertainty Simulated by MRF

Borehole data



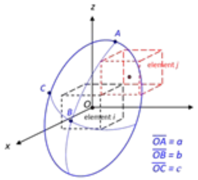
MRF

Probability model:

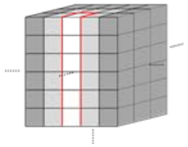
$$P(R_i = r_i | r_{N_i}) = Z_i^{-1} exp\left[-\sum_{j \in N_i} V_c(\mathbf{r})/T\right], r_i \in \Lambda$$

Gibbs sampling:

$$h(u | r_{N_i}{}^k, T = k) = \begin{cases} 1, & for\ 0 < u \le P(R_i = r_i | r_{N_i}{}^k, T = k) \\ r_i, & for \sum_{r_i=1}^{r_i=m-1} P(R_i = r_i | r_{N_i}{}^k, T = k) < u \le \sum_{r_i=1}^{r_i=m} P(R_i = r_i | r_{N_i}{}^k, T = k),\ r_i = 2, \ldots, m, \ldots, M \end{cases}$$



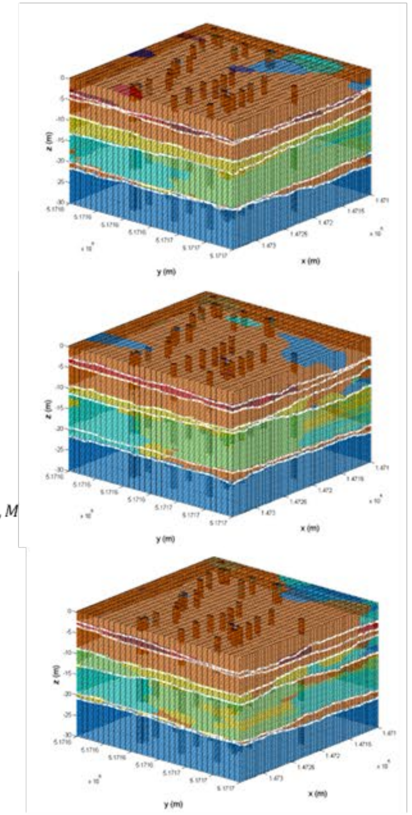
Neighborhood system



Scan order

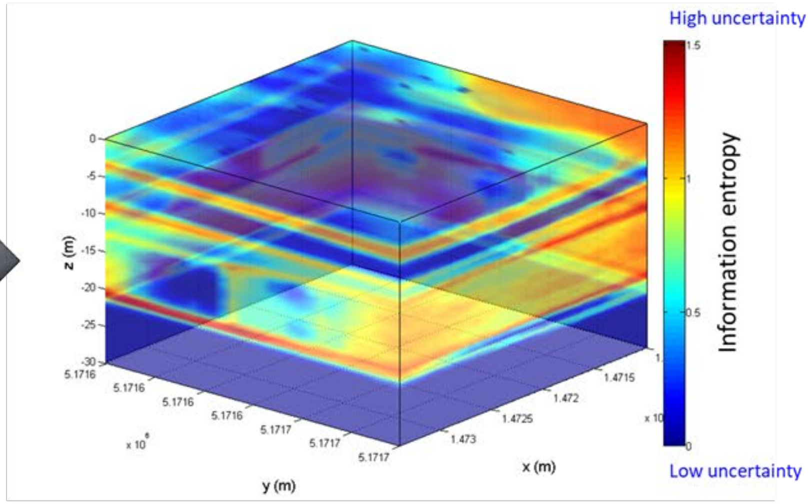
(after *Li et al., 2016*; *Lu et al., 2023*)

Potential geological models
simulated by **MRF**



⋮

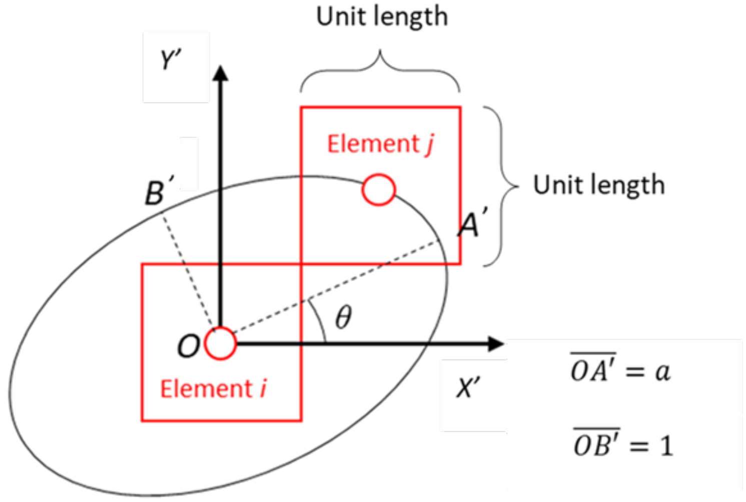
Geological model uncertainty
simulated by **MRF**



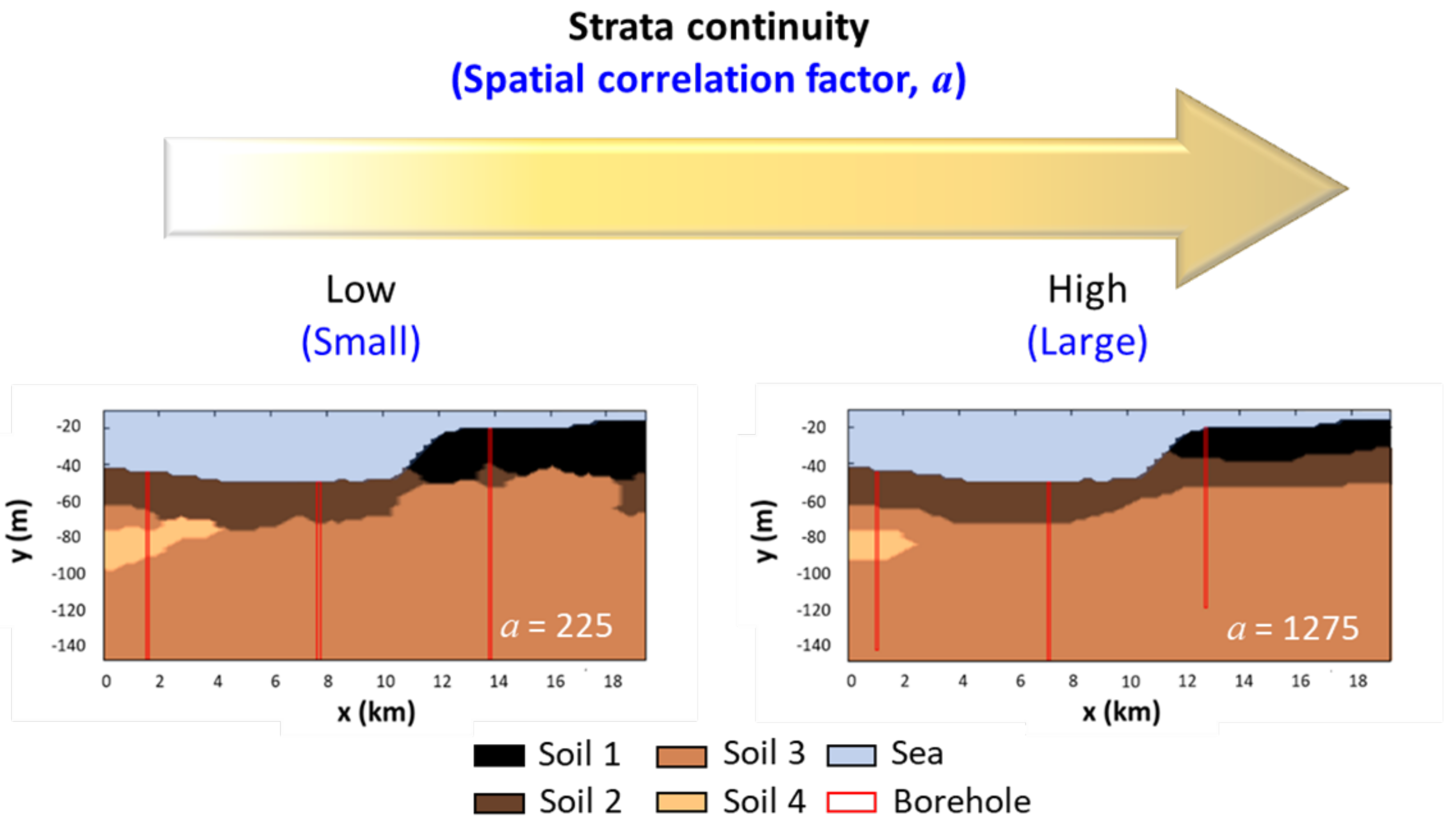
(*Chien et al., 2023b; Lu et al., 2023b*)

The Influence of Spatial Correlation Factor, a, in MRF Simulation

The definition of the spatial correlation factor, a



(after *Li et al., 2016*)

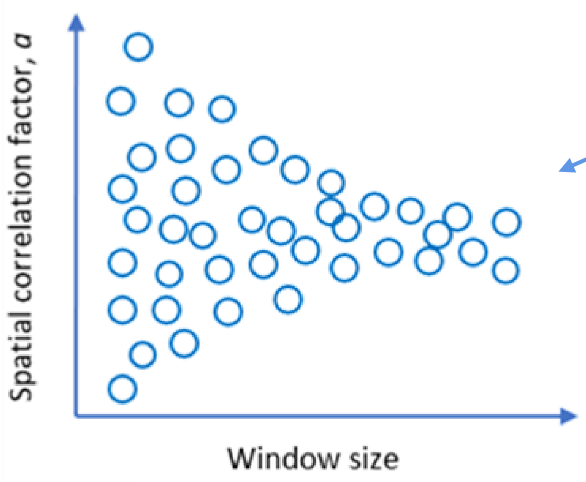


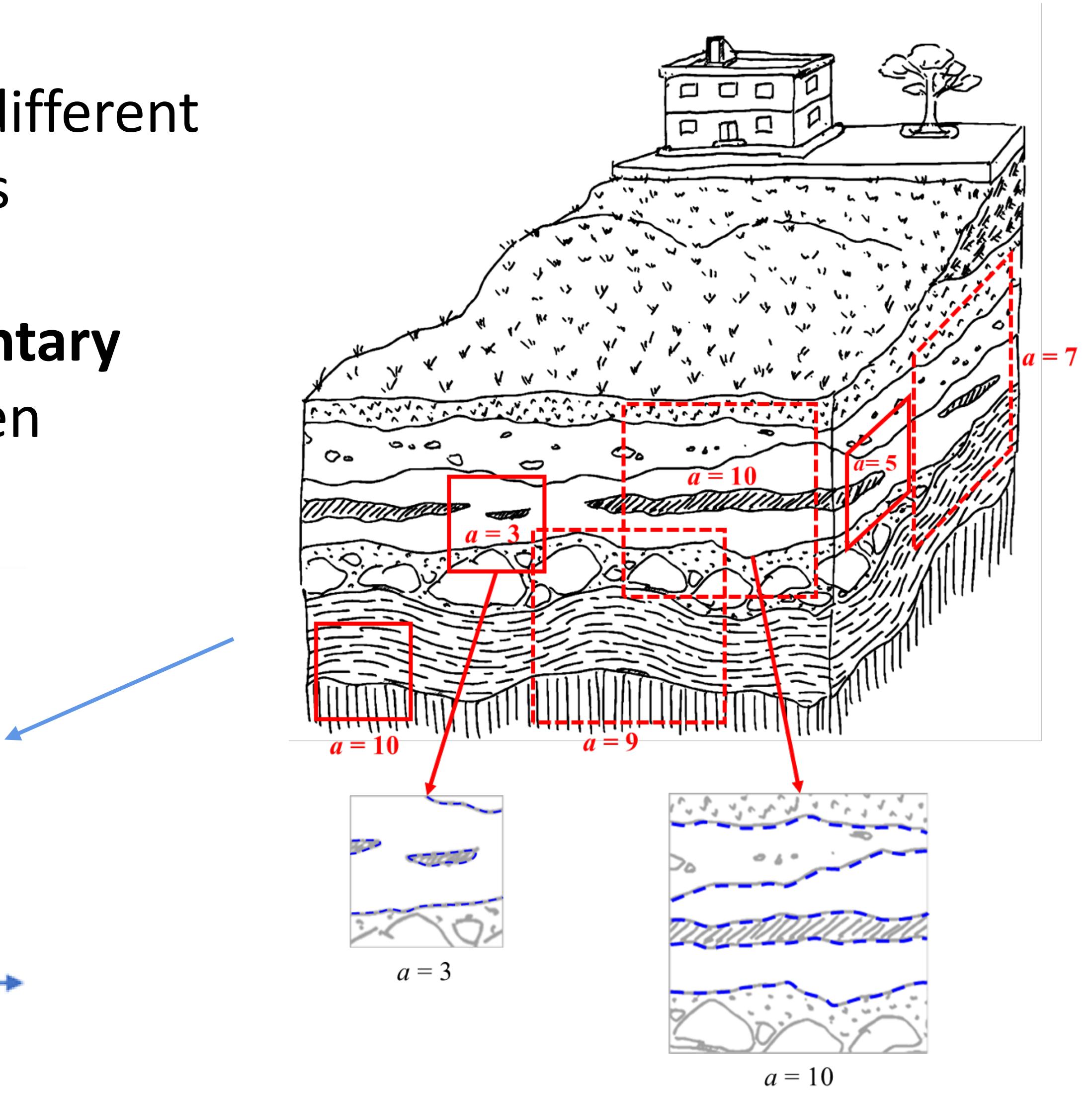
(*Hsu et al., 2022*)

Issue 1: Does the Spatial Correlation Factor Have Scale Effect?

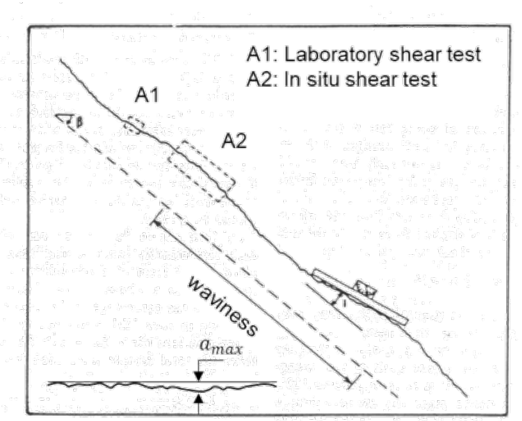
- In reality, a may be different with different scales of sampling geologic profiles

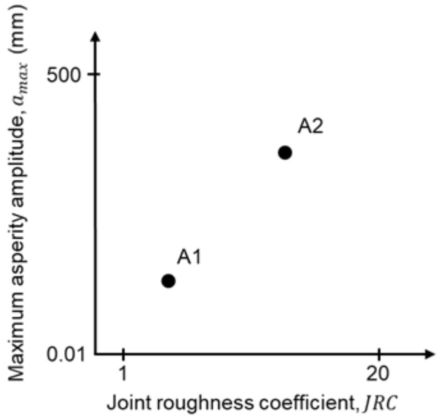
- What is the **representative elementary volume** of a sampling window when determining a?



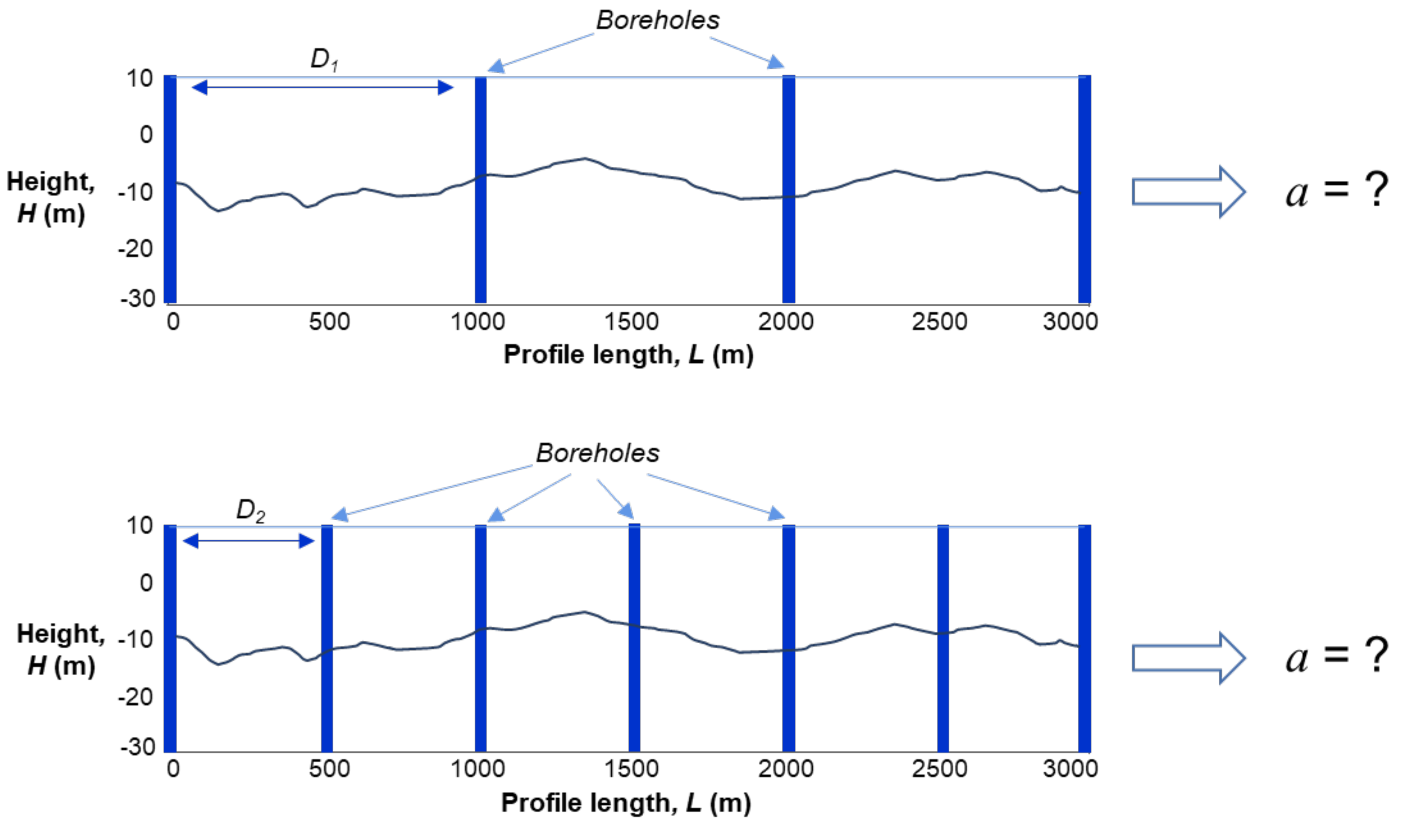


Issue 2: Does the Borehole Density Influence the a Determination in MRF Simulation?





Joint roughness coefficient (JRC) depends on the change in waviness



Estimation of spatial correlation factor (a) based on changes in the distance of borehole density

Objectives

1

Research on the scale effect of the Spatial correlation factor

- The **appropriate** $\boldsymbol{a}$ for the study site?
- What is the **representative elementary sizes (RES)** of a sampling window when determining the spatial correlation factor, a?
- The **uncertainty of** $\boldsymbol{a}$ under various sampling window sizes?

2

Research on the changing of borehole densities and its effect to the spatial correlation factor

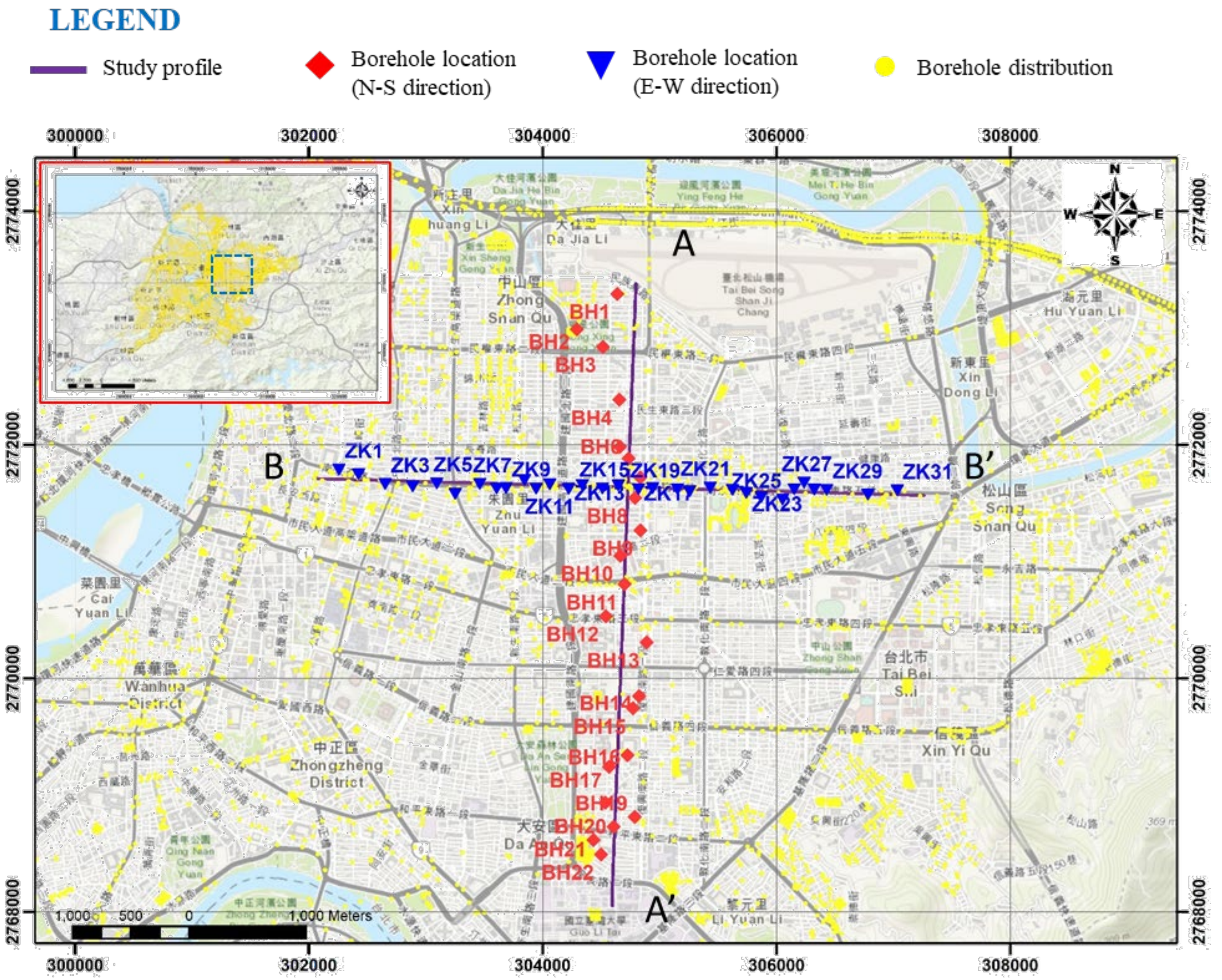
- The **relationship** between spatial correlation factor a and borehole densities?

Study Sites

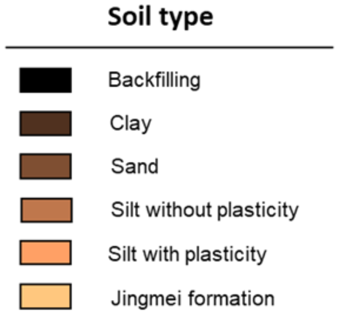
Introduction of the Study Sites

Two profiles in the Taipei basin were adopted as the study sites herein:

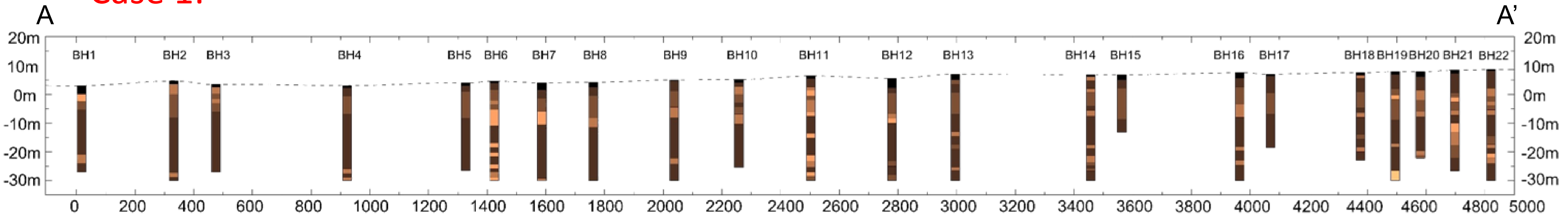
- Case 1: Section AA' (N-S) (5km) : 22 boreholes
- Case 2: Section BB' (E-W) (5km) : 31 boreholes



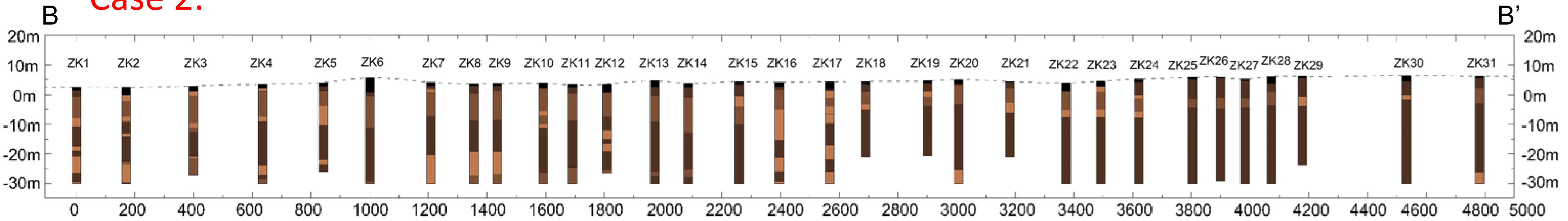
Soil Type Distributions on Borehole Locations for Two Cases



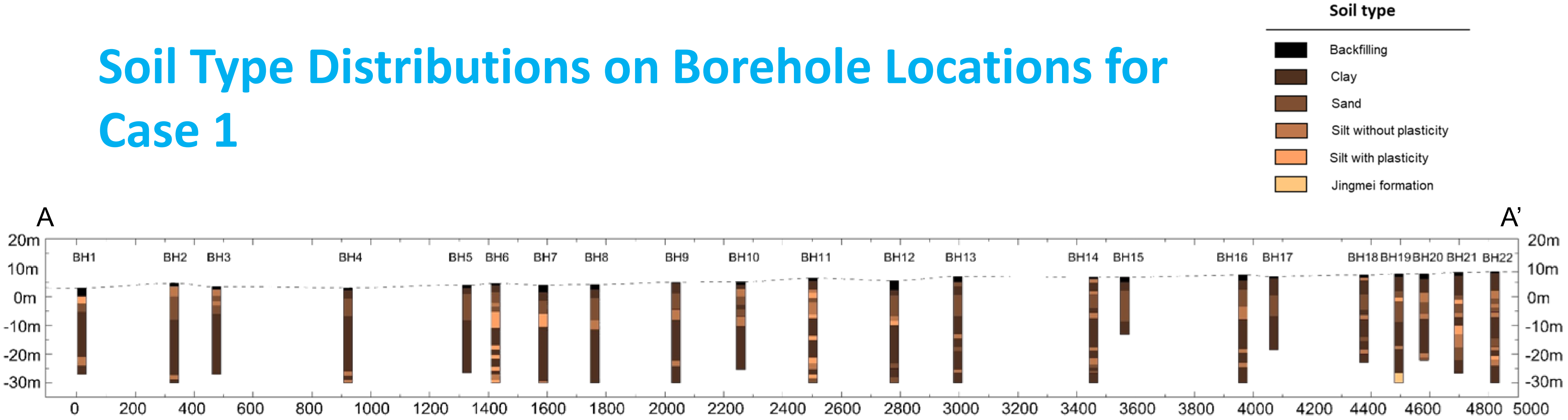
Case 1:

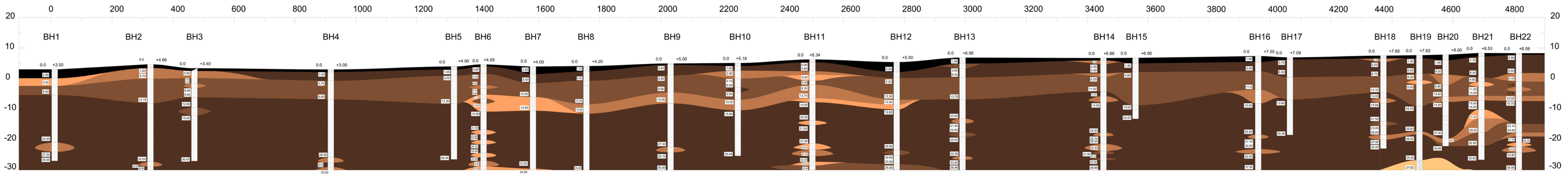


Case 2:

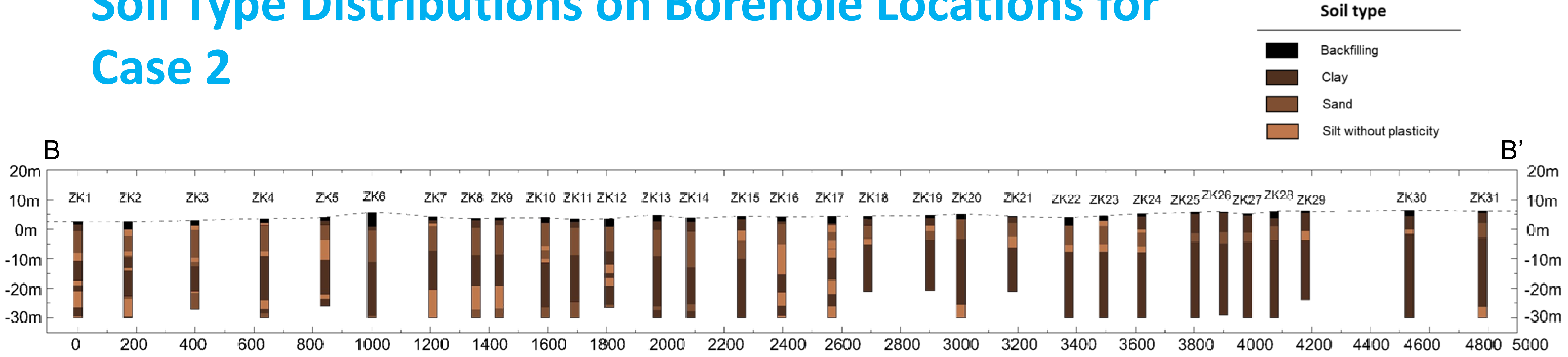


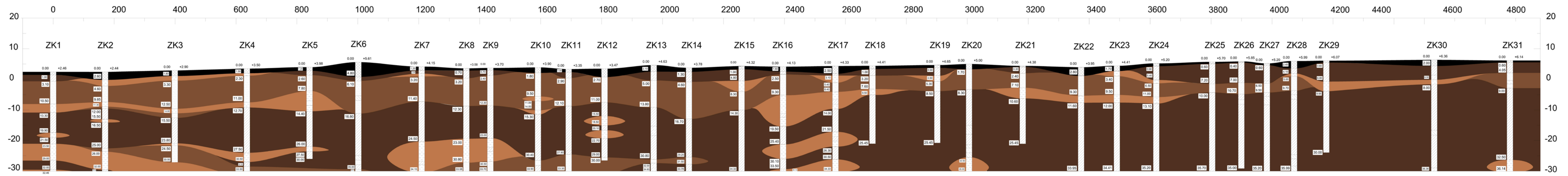
Soil Type Distributions on Borehole Locations for Case 1





Soil Type Distributions on Borehole Locations for Case 2





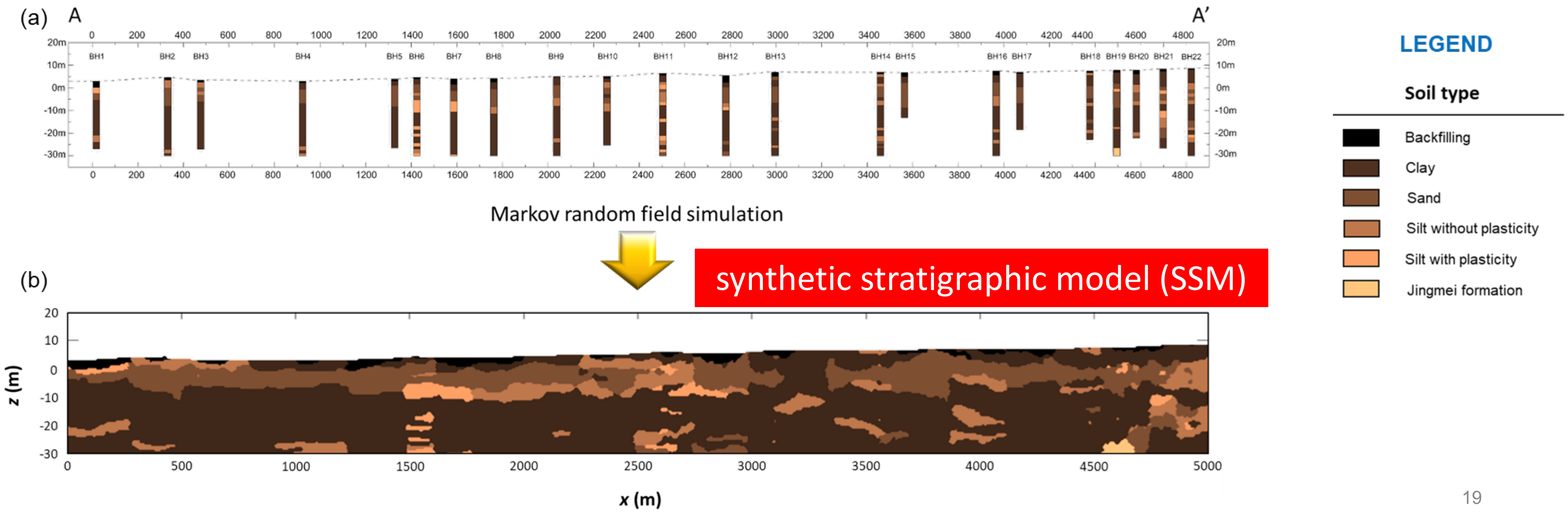
Methodology

Simulation Processes

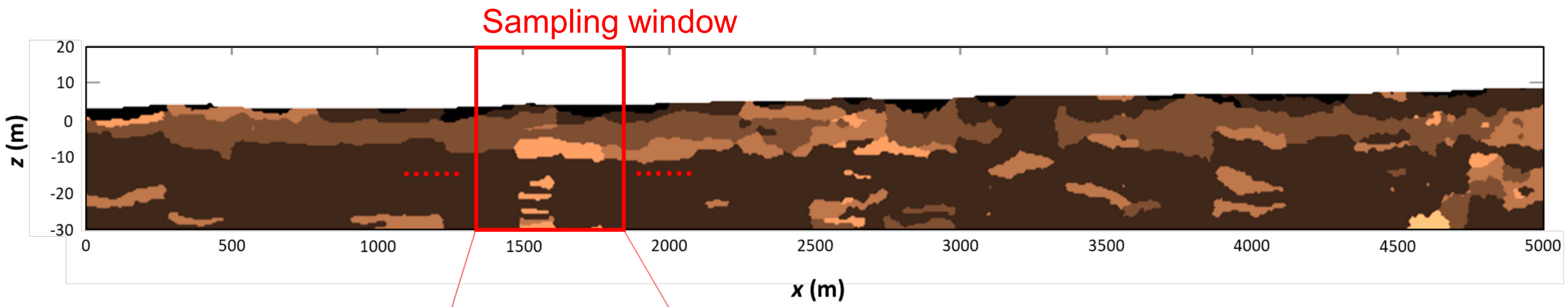
1. Step 1: Creating the synthetic stratigraphic model (SSM) based on borehole data
2. Step 2: Setting the sampling window on SSM
3. Step 3: Drilling virtual boreholes in a sampling window
4. Step 4: Simulating potential stratigraphic models using MRF for a sampling window for a given a
5. Step 5: Calculating the accuracy of MRF simulation
6. Step 6: Determining the most probable a for a sampling window using the *"maximum likelihood principle"*
7. Step 7: Calculating the mean, coefficient of variation, and confidence interval of a for various sampling window sizes

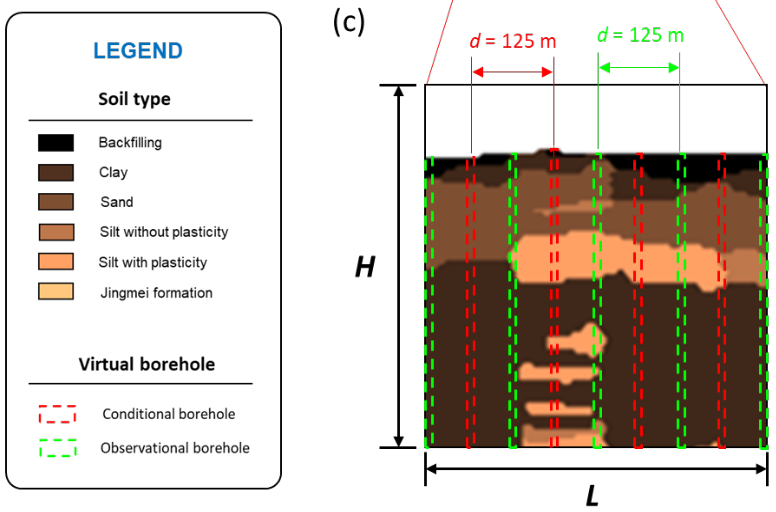
Step 1: Creating the synthetic stratigraphic model (SSM) based on borehole data

Based on **the drilling data and geological experience**, a potential stratigraphic model is generated by using MRF, called as **the synthetic stratigraphic model (SSM) and assumed as a "true" model**.



Step 2 & 3: Setting the sampling window on SSM & Drilling virtual boreholes in a sampling window



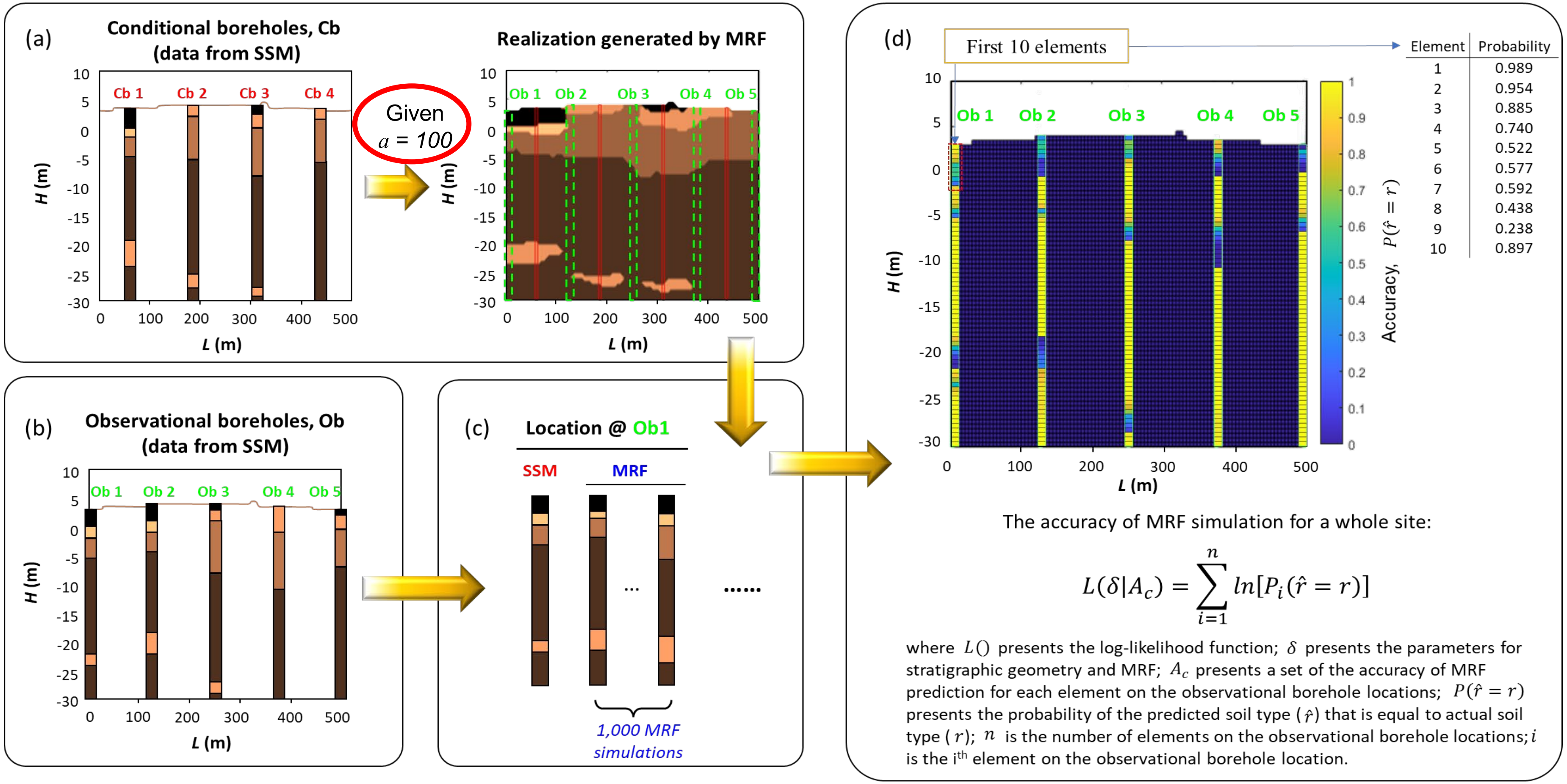


(d) **Table** Numbers of conditional and observational boreholes in various sampling window sizes

Sampling window size, H (m) × L (m)	Number of conditional boreholes	Number of observation boreholes
40 × 250	2	3
40 × 500	4	5
40 × 1000	8	9
40 × 2000	16	17
40 × 3000	24	25
40 × 4000	32	33

Two kinds of “virtual” boreholes were drilled in the sampling windows

Step 4 & 5: Simulating potential stratigraphic models using MRF for a sampling window for a given a & Calculating the accuracy of MRF



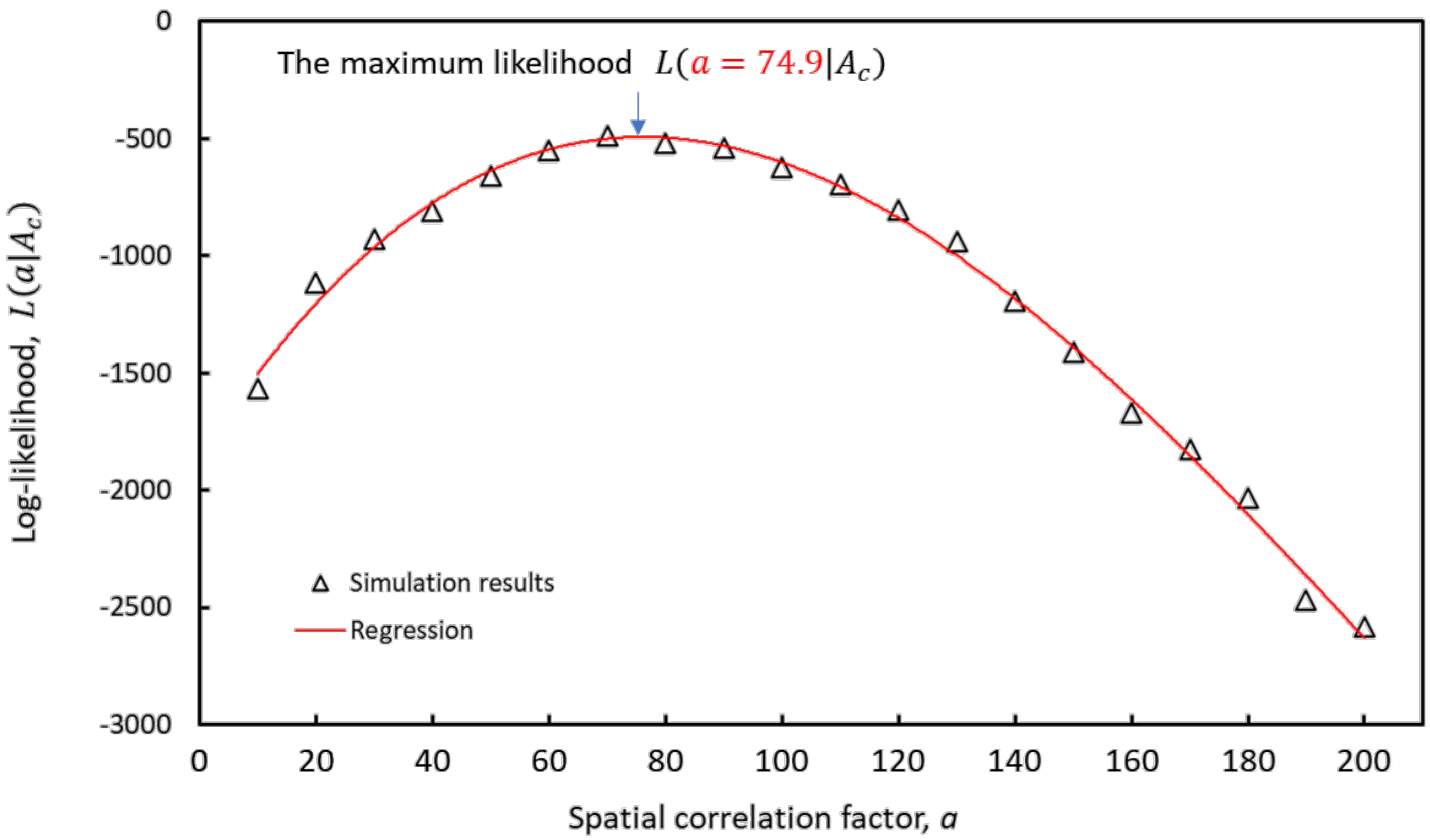
Element	Probability
1	0.989
2	0.954
3	0.885
4	0.740
5	0.522
6	0.577
7	0.592
8	0.438
9	0.238
10	0.897

The accuracy of MRF simulation for a whole site:

$$L(\delta|A_c) = \sum_{i=1}^{n} ln[P_i(\hat{r} = r)]$$

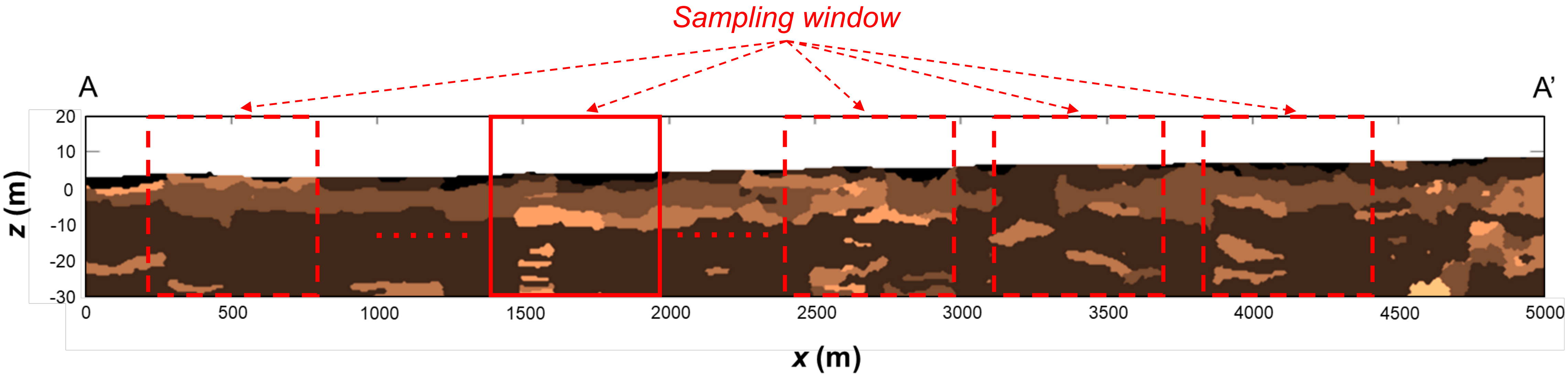
where $L()$ presents the log-likelihood function; δ presents the parameters for stratigraphic geometry and MRF; A_c presents a set of the accuracy of MRF prediction for each element on the observational borehole locations; $P(\hat{r} = r)$ presents the probability of the predicted soil type ($\hat{r}$) that is equal to actual soil type (r); n is the number of elements on the observational borehole locations; i is the ith element on the observational borehole location.

Step 6: Determining the most probable a for a sampling window using the "maximum likelihood principle" *(Qi et al., 2016)*



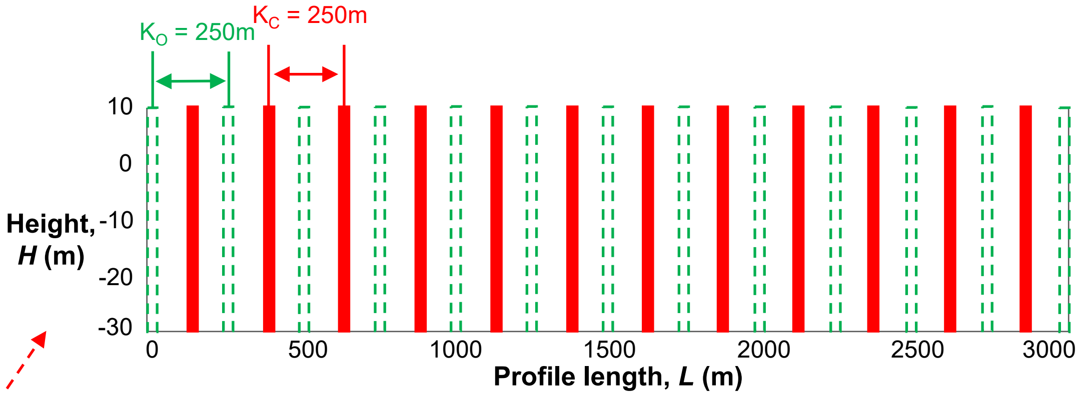
Step 7: Calculating the mean, coefficient of variation, and confidence interval of a for various sampling window sizes

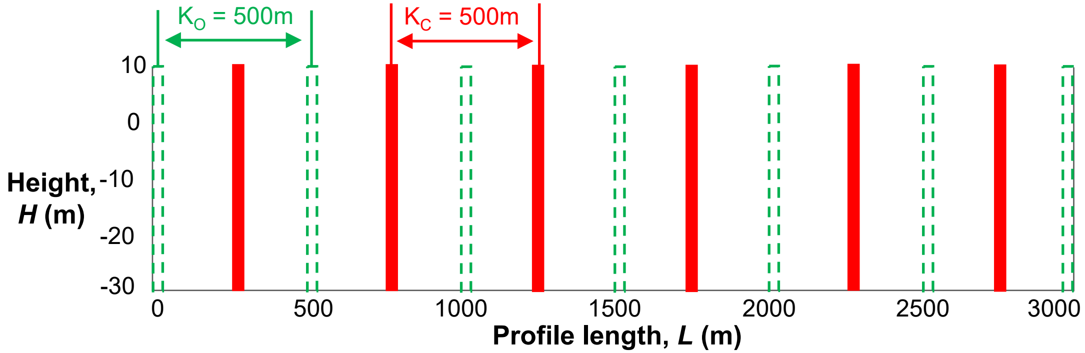
- Repeat Steps 2-6 30 times for different sampling locations under given sampling window sizes to obtain a series of a with the same sampling window size and borehole density
- **Calculate the mean, coefficient of variation (COV), and 95% confidence interval of** a for various sampling window sizes and borehole density.



Parametric Study of This Study

Length of profile, *H* (m) × *L* (m)	Distance between 2 boreholes, *K* (m)	Borehole density, *D* (number/km)	Number of conditional boreholes	Number of observational boreholes
40 x 1000	83.33	12	12	13
	100.00	10	10	11
	166.67	6	6	7
	250.00	4	4	5
	500.00	2	2	3
40 × 3000	83.33	12	36	37
	100.00	10	30	31
	166.67	6	18	19
	250.00	4	12	13
	500.00	2	6	7
40 x 5000	83.33	12	60	61
	100.00	10	50	51
	166.67	6	30	31
	250.00	4	20	21
	500.00	2	10	11





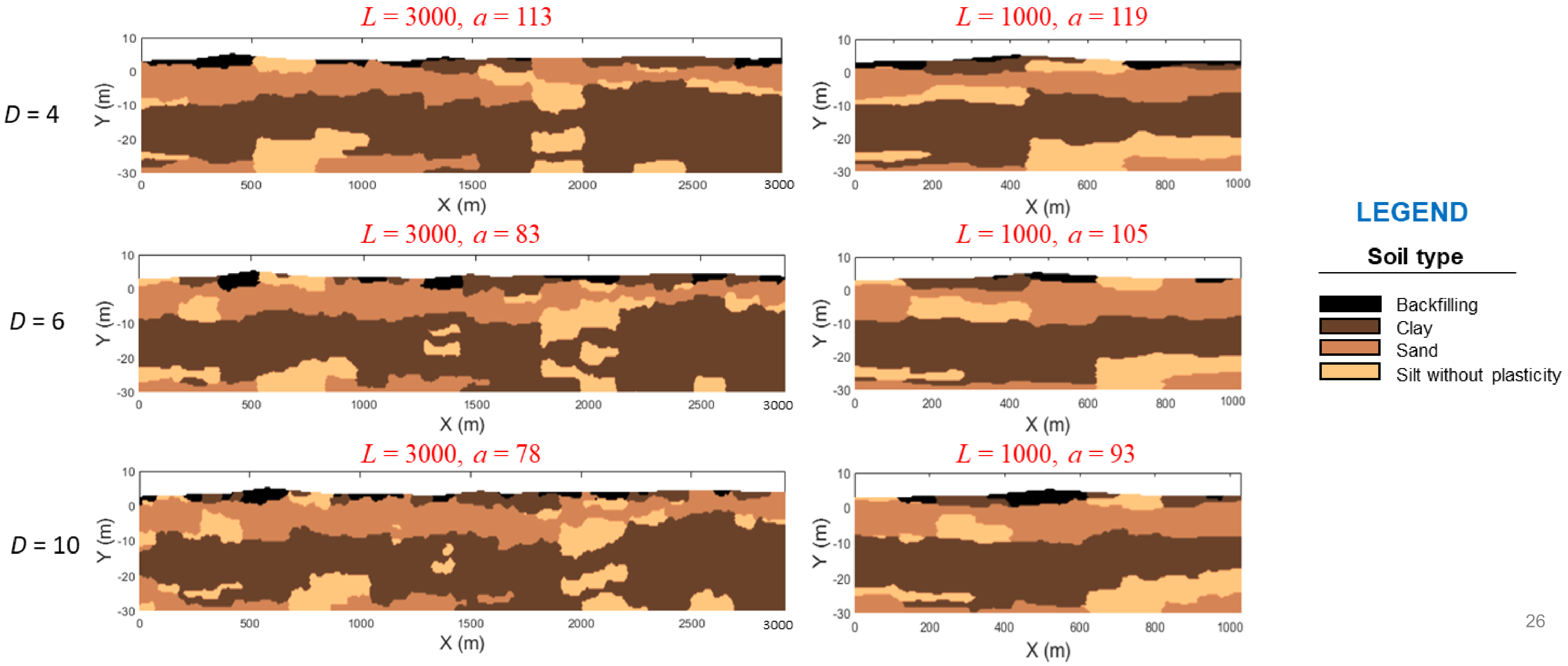
Where:

K_C: Distance of 2 conditional boreholes

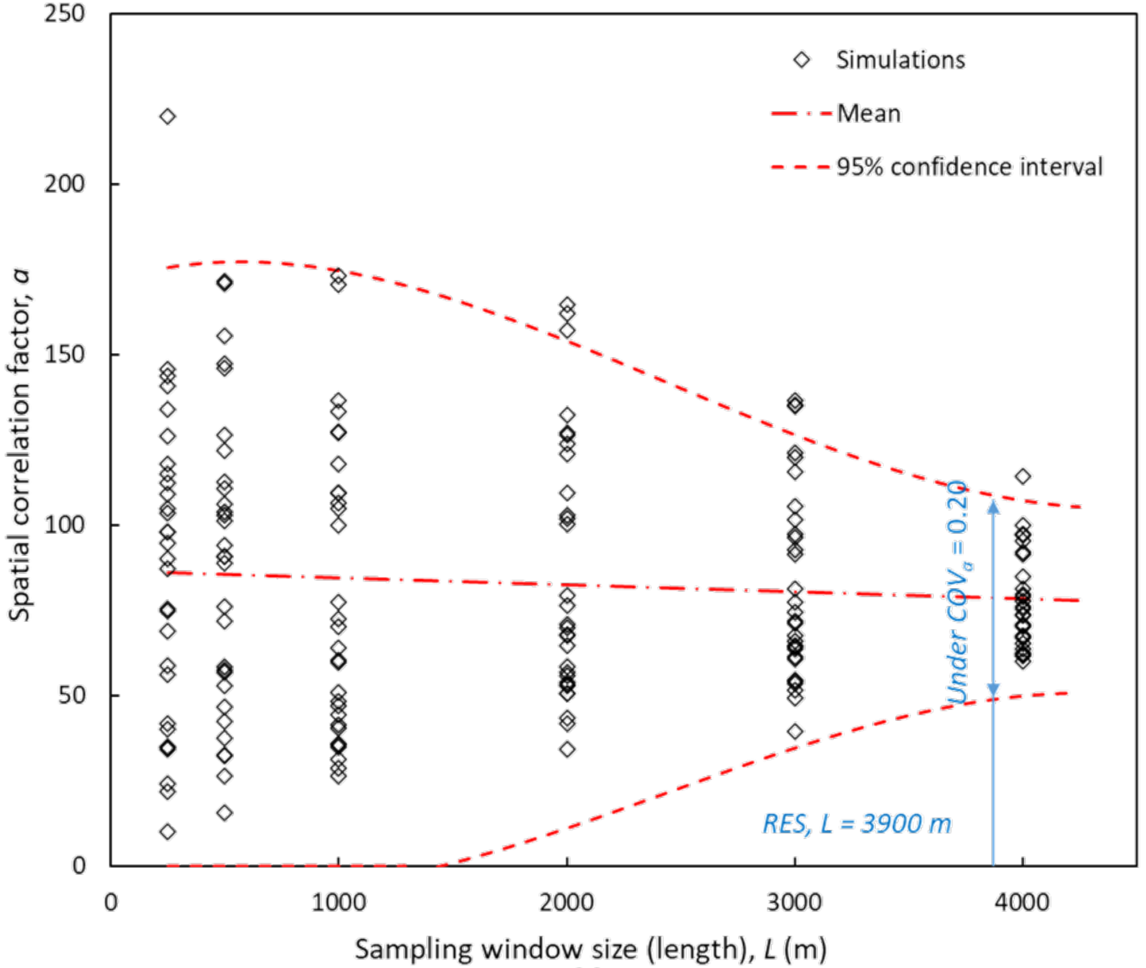
K_O: Distance of 2 observational boreholes

Simulation Results

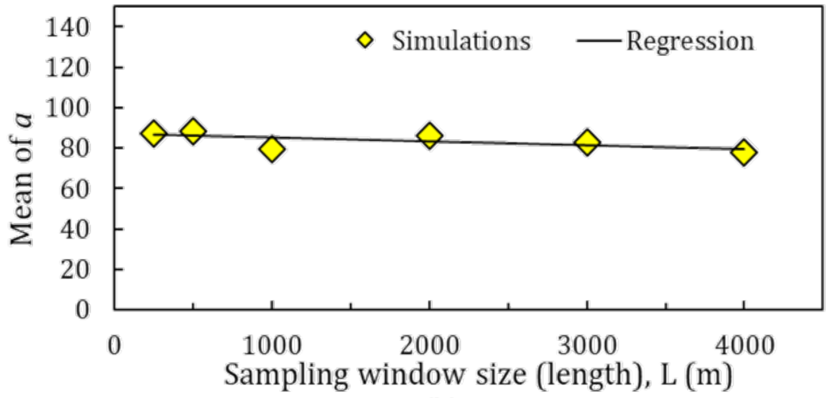
Geological Models Simulated by MRF for Various Sampling Window Sizes and Borehole Densities (Case 2)



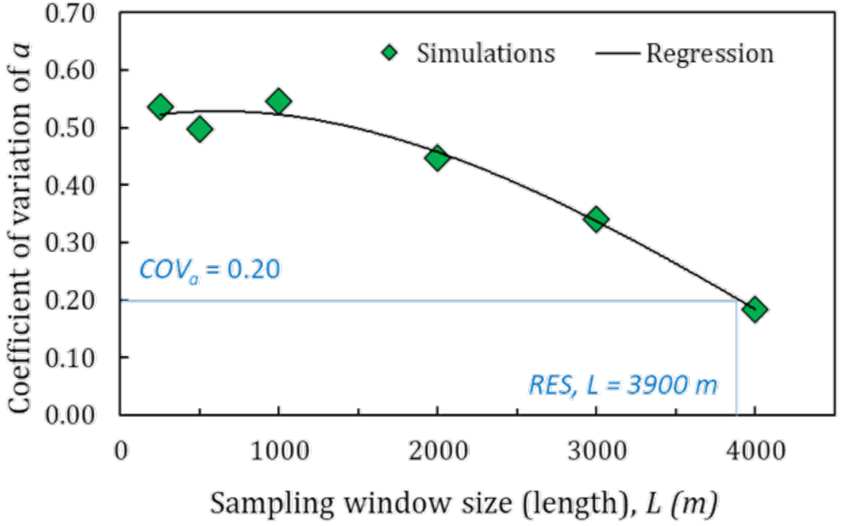
Scale Effect of a in Case 1



(a)



(b)



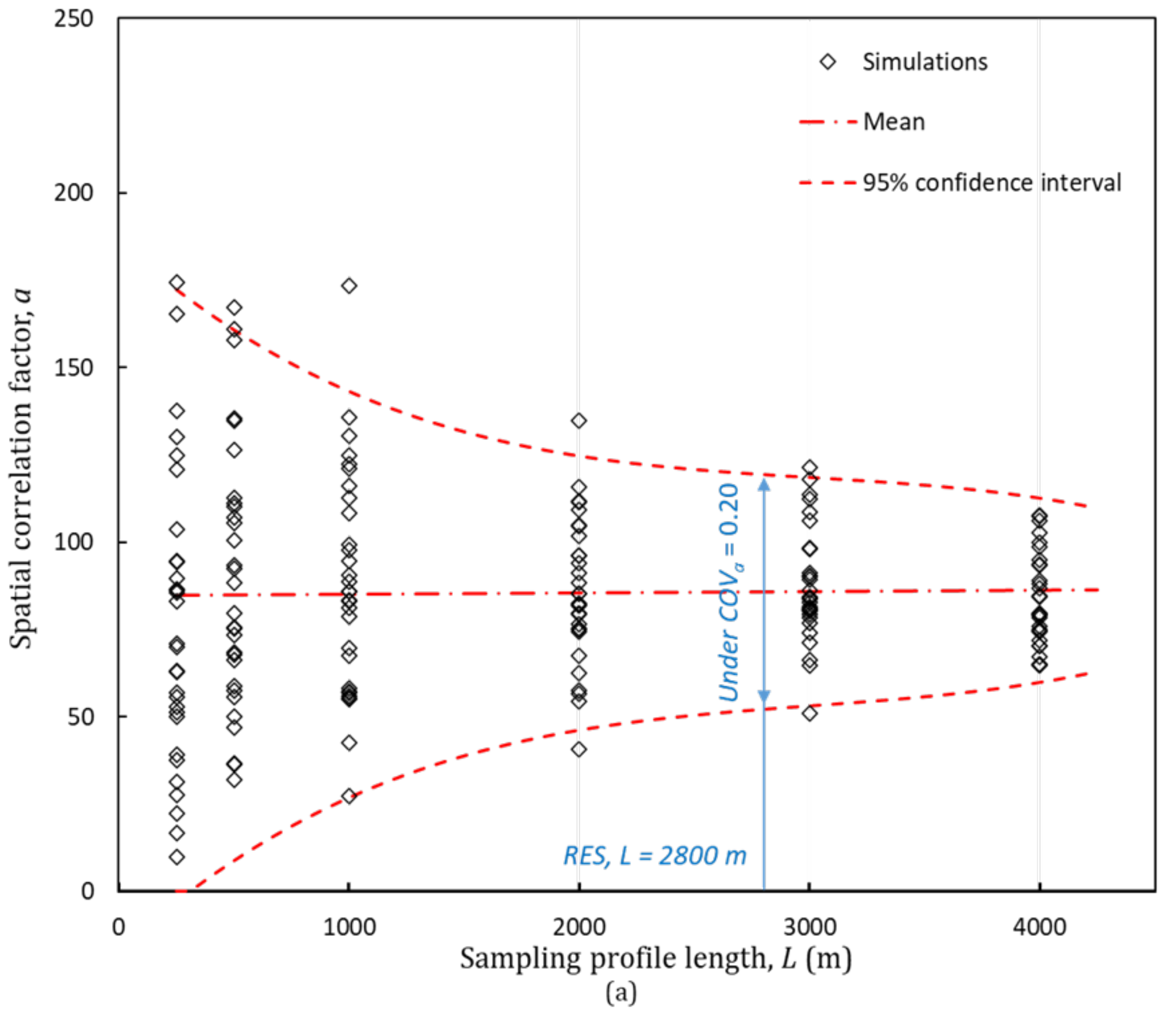
(c)

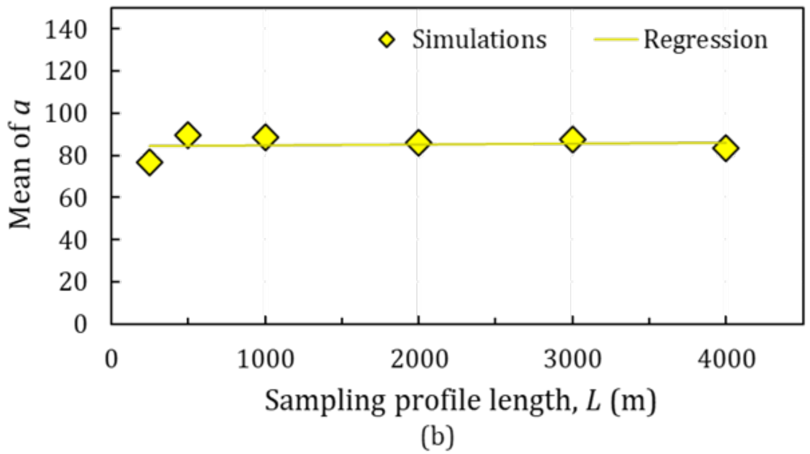
COV	Quality
< 0.10	Excellent
0.10-0.20	Good
0.20-0.30	Acceptable
> 0.30	Poor

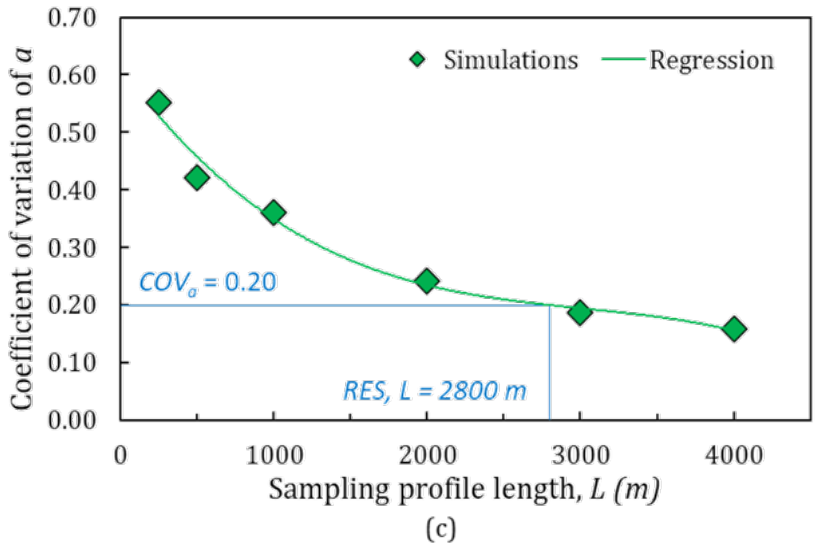
Source: Shimon Aronhime et al. (2013)

**RES: Representative elementary sizes*

Scale Effect of a in Case 2







COV	Quality
< 0.10	Excellent
0.10-0.20	Good
0.20-0.30	Acceptable
> 0.30	Poor

Source: Shimon Aronhime et al. (2013)

**RES: Representative elementary sizes*

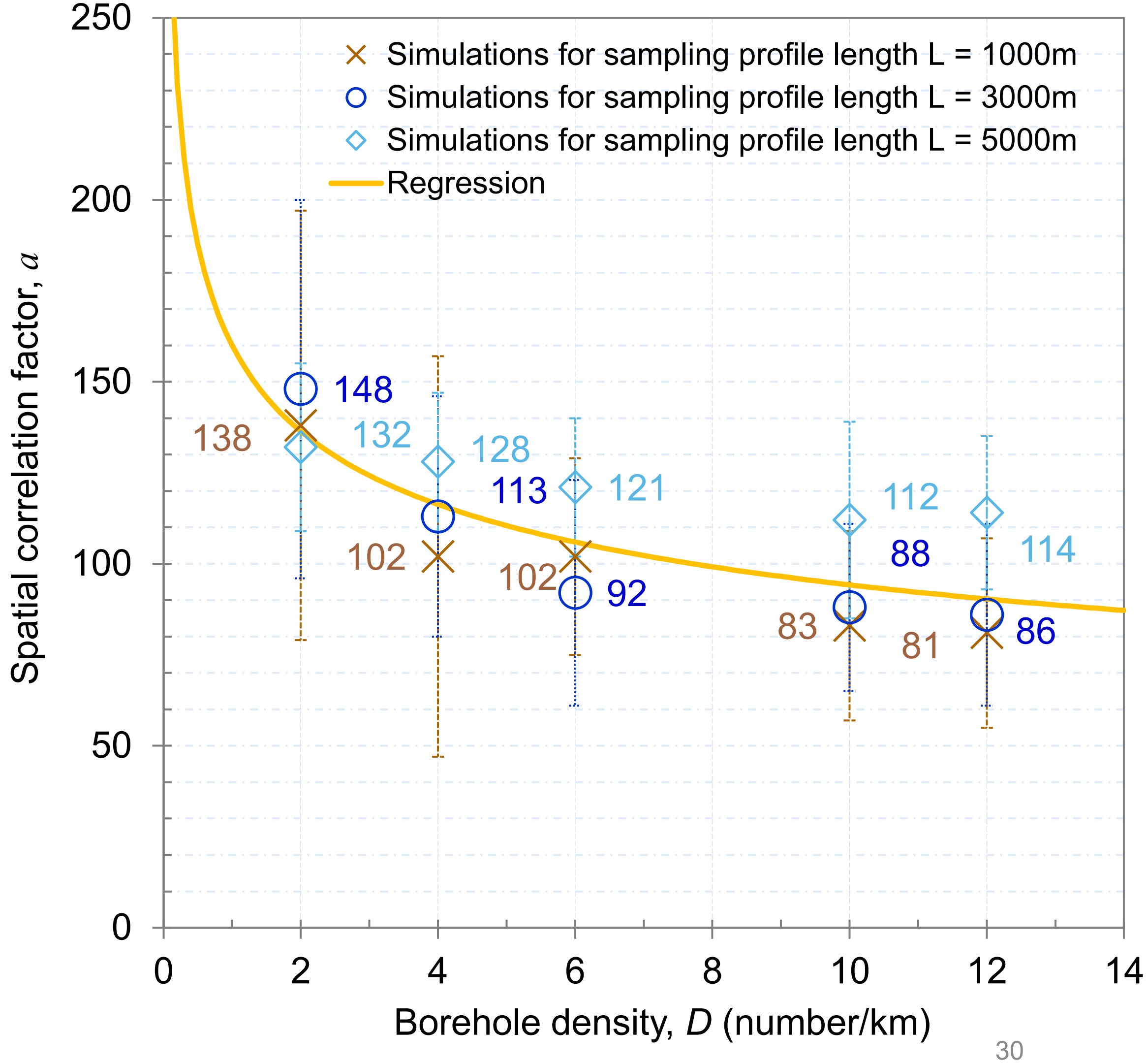
Comparison of Scale Effects of a in Case 1 and Case 2

	Case 1	Case 2
The range of mean of a under various sampling widow sizes	78.00 ~ 88.00	77.00 ~ 89.00
The range of COV of a under various sampling widow sizes	0.55 ~ 0.18	0.55 ~ 0.16
Representative elementary sizes of a sampling window (for acceptable COV of a = 0.2)	40 m × 3900 m	40 m × 2800 m

Spatial Correlation Factor *a* under Various Borehole Densities

Table Numbers of conditional and validation boreholes with various borehole densities

Length of profile, *H* (m) × *L* (m)	Distance of borehole density, *K* (m)	Borehole density, *D* (number/km)	Number of conditional boreholes	Number of validation boreholes	Spatial correlation factor, *a* *
40 x 1000	83.33	12	12	13	81
	100.00	10	10	11	83
	166.67	6	6	7	102
	250.00	4	4	5	102
	500.00	2	2	3	138
40 × 3000	83.33	12	36	37	86
	100.00	10	30	31	88
	166.67	6	18	19	92
	250.00	4	12	13	113
	500.00	2	6	7	148
40 x 5000	83.33	12	60	61	114
	100.00	10	50	51	112
	166.67	6	30	31	121
	250.00	4	20	21	128
	500.00	2	10	11	132



Spatial Correlation Factor *a* under Various Borehole Densities and Profile Length

L = 1000m

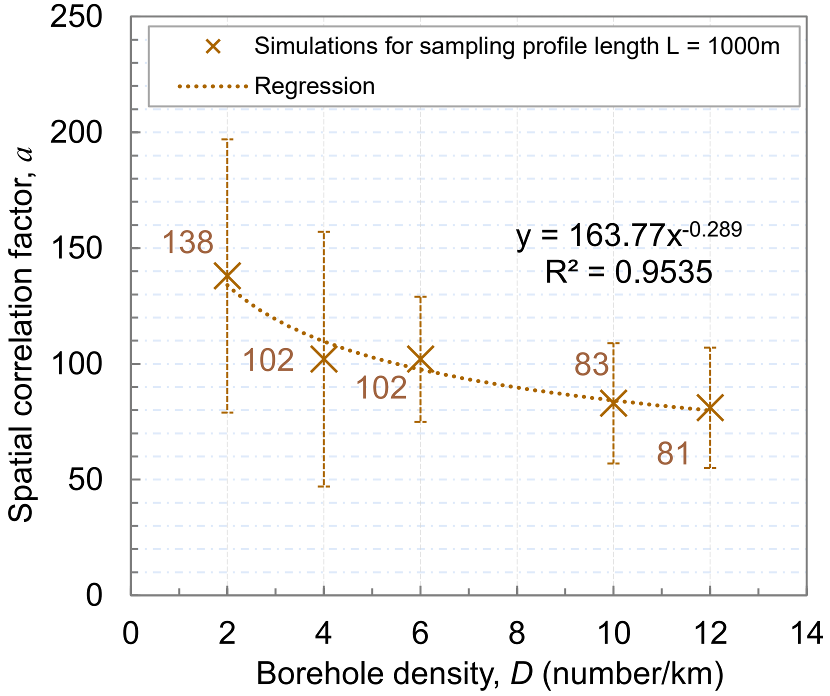
Criterion	Borehole density, *D* (borehole/km)				
	2	4	6	10	12
Mean	138	102	102	83	81
Min	30	10	50	30	30
Max	240	220	160	140	140
Std. Dev.	59	55	27	26	26
COV	0.43	0.54	0.26	0.31	0.32

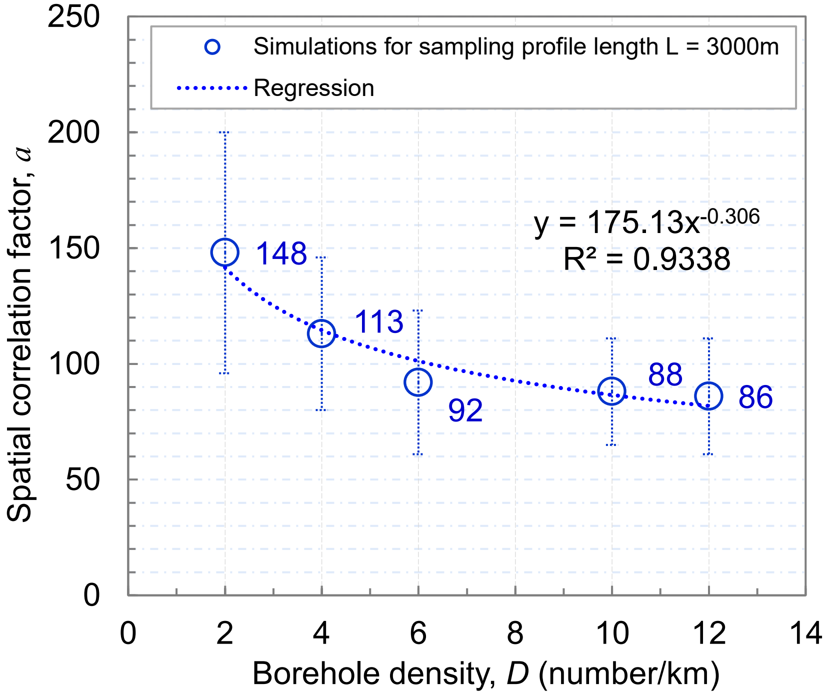
L = 3000m

Criterion	Borehole density, *D* (borehole/km)				
	2	4	6	10	12
Mean	148	113	92	88	86
Min	40	60	50	50	60
Max	240	200	160	140	150
Std. Dev.	52	33	31	23	25
COV	0.35	0.29	0.34	0.26	0.29

L = 5000m

Criterion	Borehole density, *D* (borehole/km)				
	2	4	6	10	12
Mean	132	128	121	112	114
Min	110	100	90	80	90
Max	170	160	150	160	150
Std. Dev.	23	19	19	27	21
COV	0.17	0.15	0.16	0.24	0.18





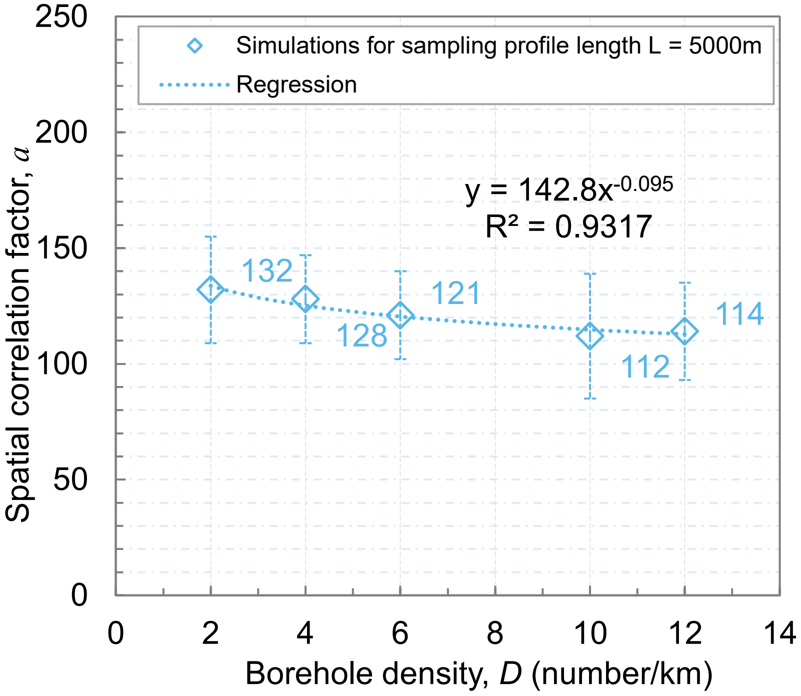
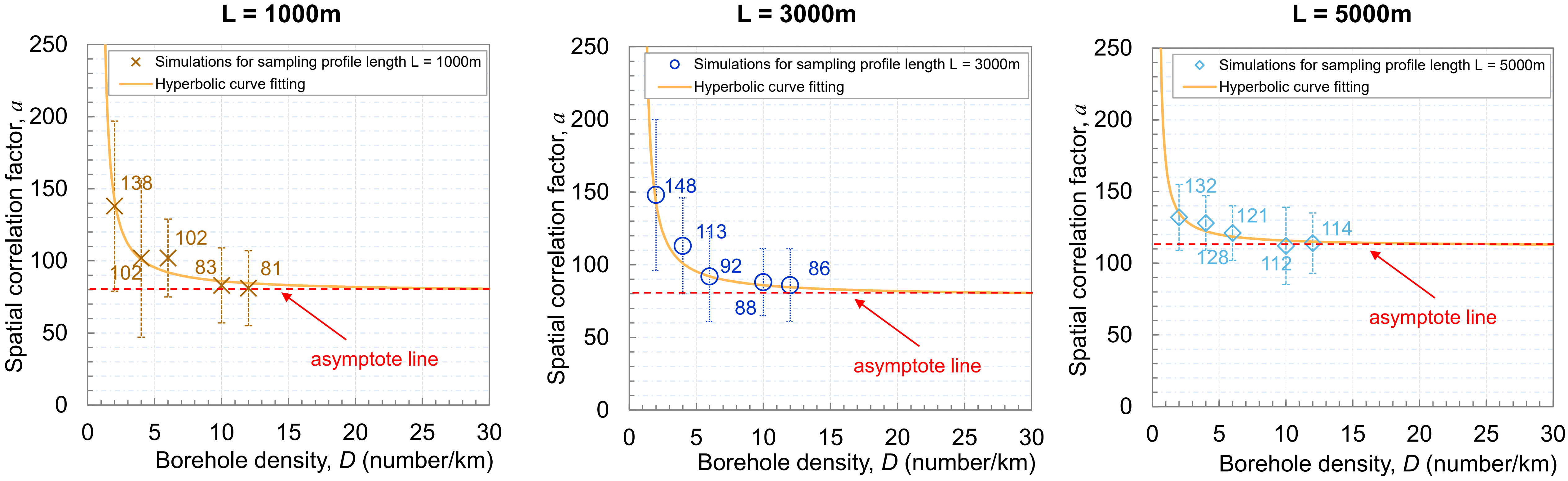


Fig. Spatial correlation factor *a* under various Borehole densities *D*

The relationship between *a* values and various borehole densities *D*. Find the sill value in the asymptote line using Hyperbolic curve fitting



The sill value in the asymptote line is equal to 81 when ***D* = 22.1**

The sill value in the asymptote line is equal to 83 when ***D* = 22.0**

The sill value in the asymptote line is equal to 113 when ***D* = 22.3**

Fig. Spatial correlation factor *a* under various Borehole densities *D*

- **Is the recommendation to choose a *D* = 22 boreholes/km for site investigation in this area?**

Concluding Remarks

Concluding Remarks

- The mean of a doesn't have a significant change for the different sampling window sizes. In both cases, the values range approximately from 77~89m. It means the distribution of a in this area may present isotropy.
- The coefficient of variation *COV* of a decreases with increasing sampling window size, which decreases from about 0.55 to 0.16 when the sampling window size increases from 40m $\times$ 250m to 40m $\times$ 4,000m.
- If the acceptable COV_a = 0.2:
  - In Case 1, the RES of the sampling window is equal to 40m $\times$ 3900m.
  - In Case 2, the RES of the sampling window is equal to 40m $\times$ 2800m.
- The mean of a increases with decreasing borehole density in MRF simulation.

Yangmingshan National Park

THANK YOU

To transfer the geological data to numerical models

It's an interesting process..

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Graduate Institute of Applied Geology

Seminar of Geotechnical Engineering | 大地工程專題討論

Scale effect on the Determination of the Spatial Correlation Factor Used in Markov Random Field

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