Quantifying stratigraphic uncertainties by stochastic simulation techniques based on Markov random field

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Outline

- Introduction
- Methodology
- Example
- Conclusion

Soil heterogeneity

Soil heterogeneity can be attributed to two main sources:

1. Inherent spatial variation of soil properties : due to the difference of geological deposition history and human activities.

2. Stratigraphic or lithological uncertainty : uncertainty of interfaces between different soil layers or lithological units due to limited subsurface investigation data.

Research process

- In this research will focuses on stochastic modeling techniques for quantifying uncertainties of geological structure due to limited site exploration data.

Modeling used in this research

- Two stochastic modeling techniques are developed to generate to quantify stratigraphic uncertainty in the post processing stage.

1. ICM modeling
2. MCMC modeling

- Three types of site investigation data could be used as input in this model :

1. Ground surface soil types
2. Boundaries of different soil layers at each borehole log location
3. Strata orientation information

Neighborhood system

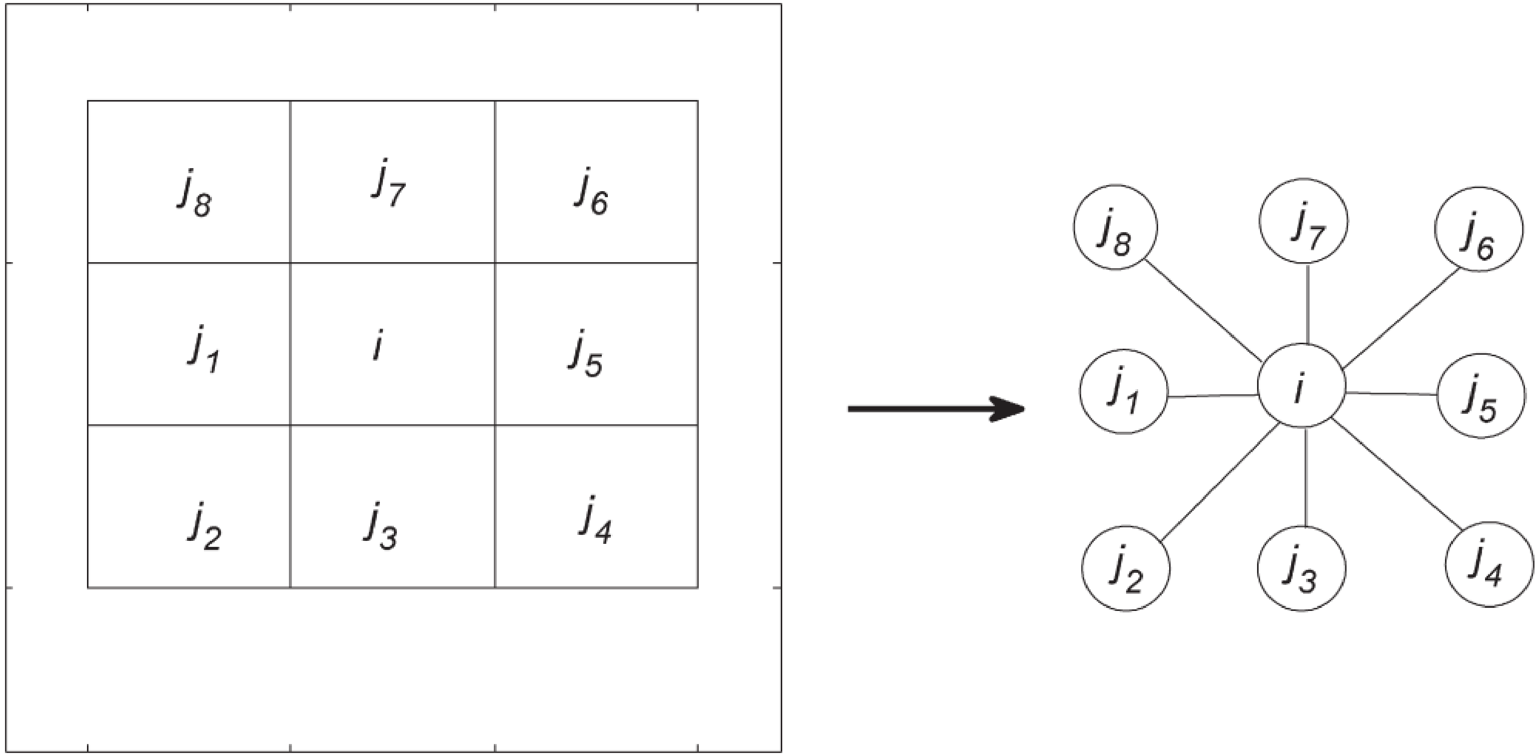


Fig. 1. Local neighborhood system.

1. Discretizing the geological body of interest into small square elements.

1. Element i has 8 neighbors: $j_1 \ldots j_8$ but not including itself.

2. Boundary element has fewer neighbors.

Spatial correlation of neighboring elements

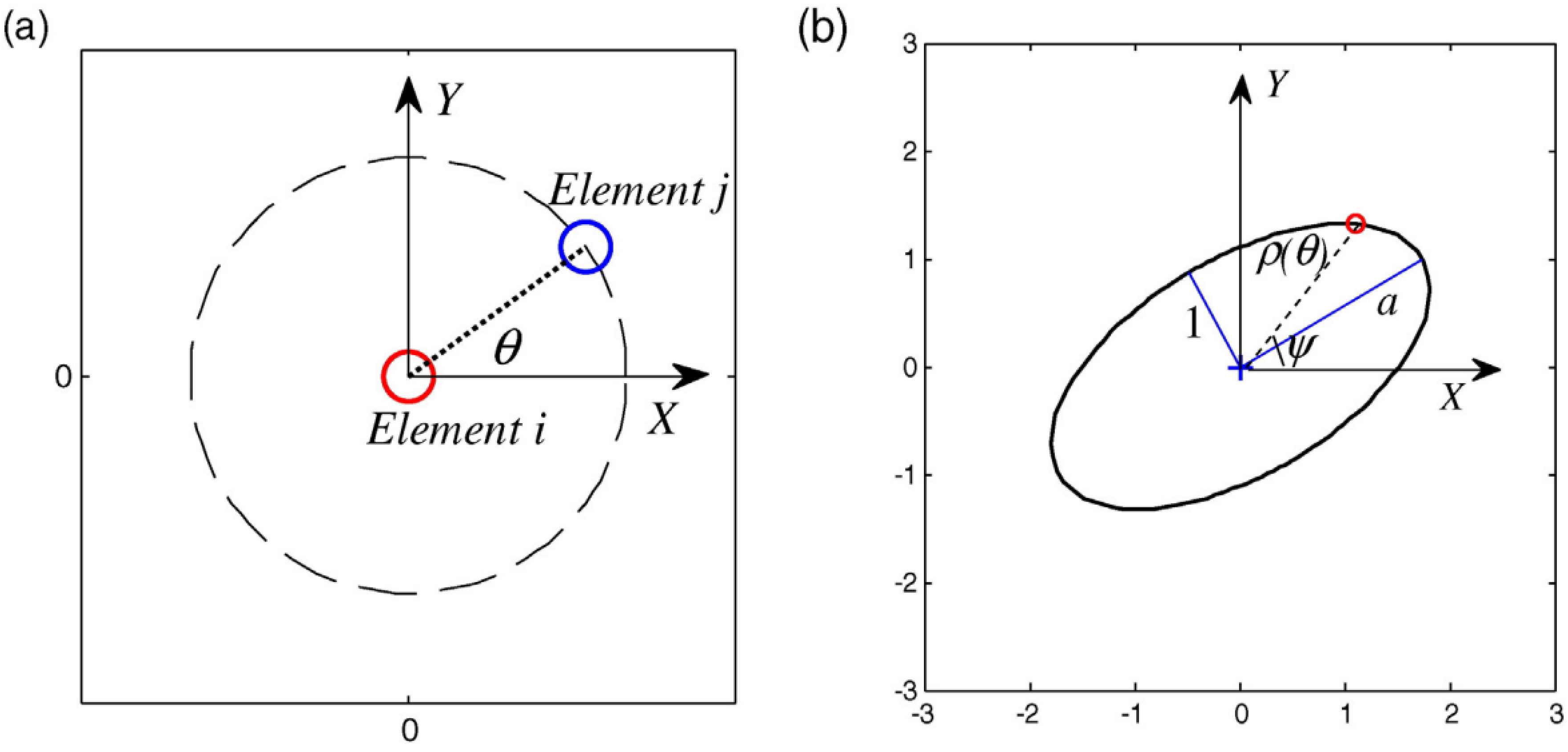


Fig. 2. Standard Geometric condition and corresponding spatial correlation model.

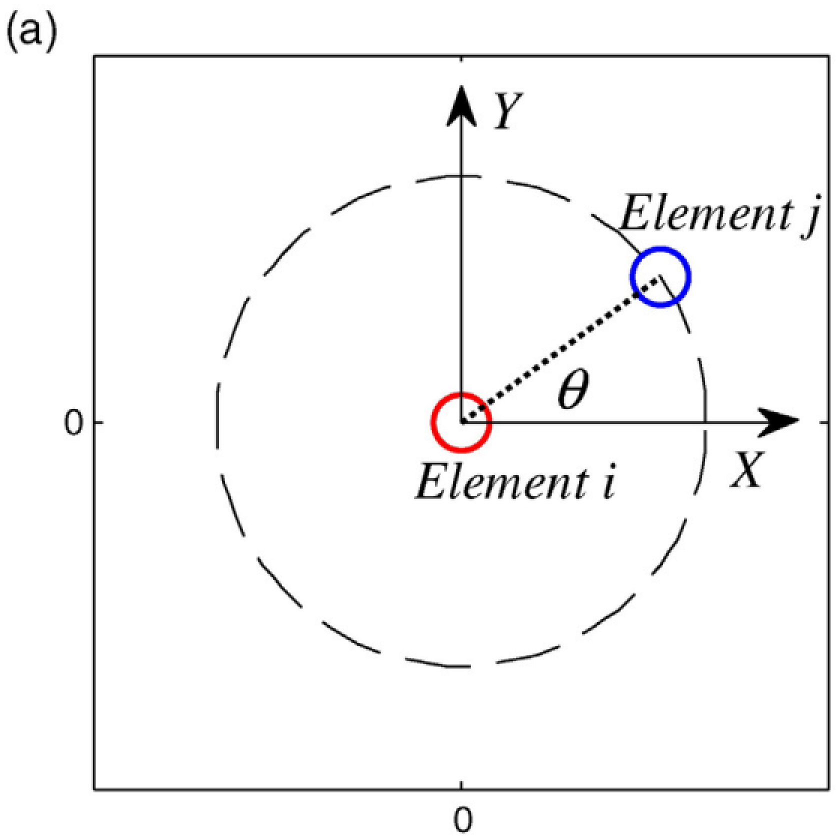
Two components :

1. normal correlation
2. Tangential correlation

regarding to plane orientation of geological formation.

Example : sedimentary plane, foliation, cleavage plane …

Spatial correlation of neighboring elements

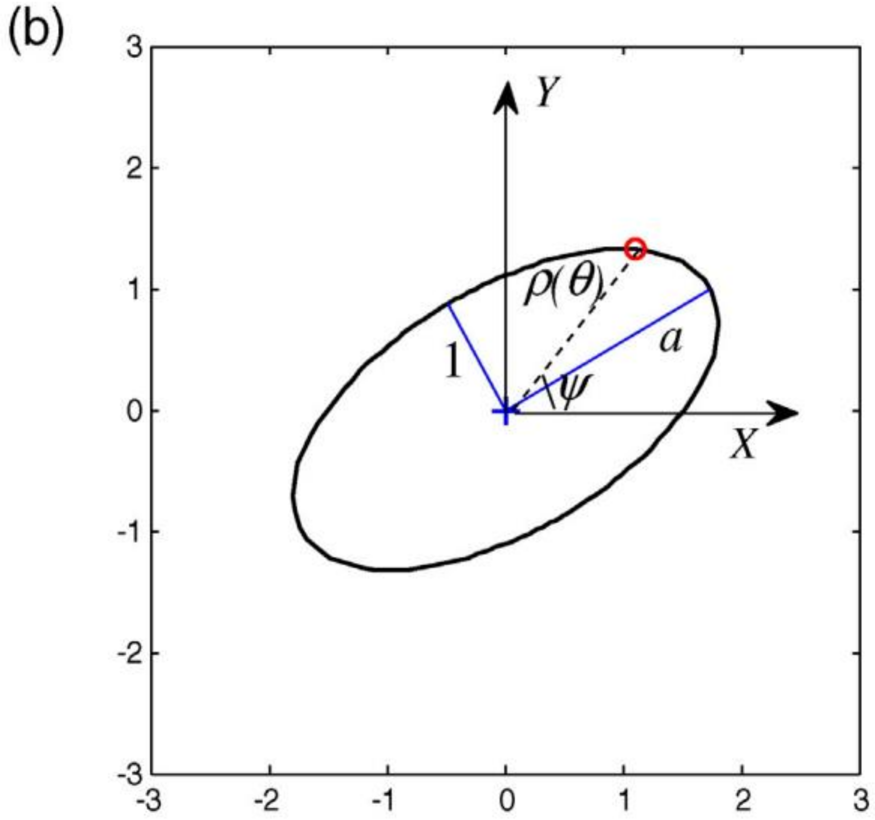


A standard geometric condition of element i and one of its neighbor elements j.

θ is the intersection angle between the line of the centroid of element i to that of element j and the X axis of Cartesian coordinate system.

Fig. 2. Standard Geometric condition and corresponding spatial correlation model.

Spatial correlation of neighboring elements

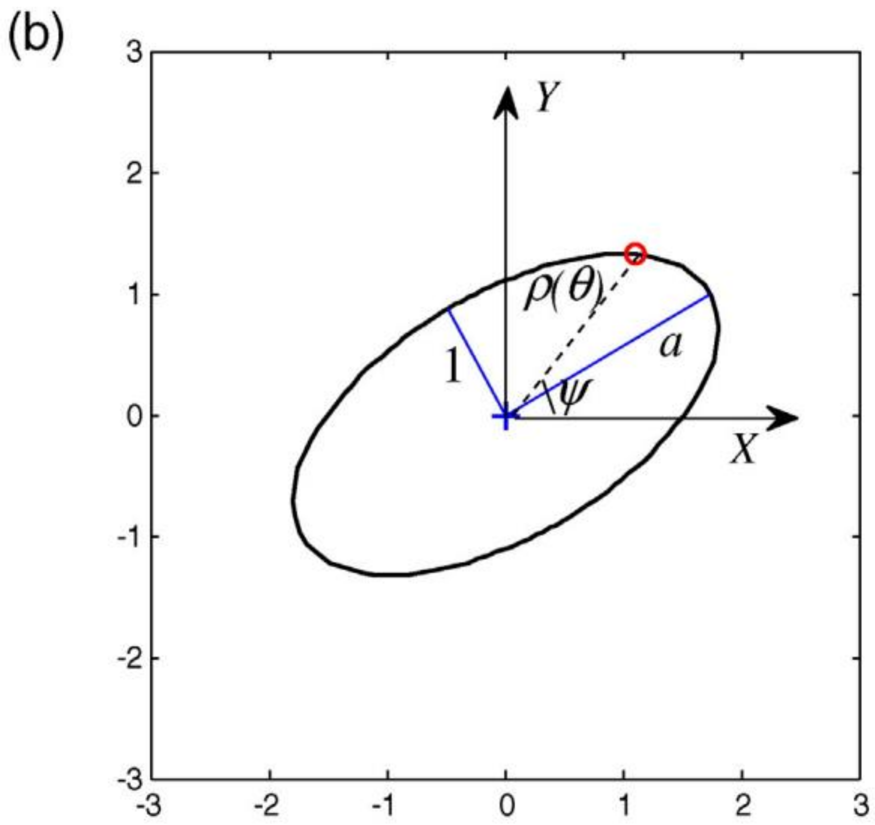


the radius length ρ(θ) of an ellipse centered at the centroid of element i with a rotation ψ with X axis of a Cartesian coordinate system represent the local spatial correlation between element i and element j.

The larger ρ(θ) means the stronger influence is from the neighbor element j on element i in terms of having the same geo-material type or lithology unit.

Fig. 2. Standard Geometric condition and corresponding spatial correlation model.

Spatial correlation of neighboring elements



The ellipse has a major axis “a” and a unit miner axis , indicating tangential correlation and normal correlation.

The rotation ψ is closely related to the orientation information of geological formations.

The parameter “a” indicates the ratio of strength of tangential correlation and normal correlation, which represents the degree of local anisotropy.

Fig. 2. Standard Geometric condition and corresponding spatial correlation model.

Simulation~ pre-process

Step1 : Discretized geological body by a suitable mesh scheme.

Step2 : Processing the geometric information of the meshed plot, including calculating orientation θ of element j to element i and constructing neighborhood systems.

Step3 : Assign known data to the corresponding elements.

Step4 : Values for parameters ψ and a in the $\rho(\theta)$-function.

Simulation

Step1 : Filling all the blank elements with lithological units using local transition sample to be the initial configuration.

Step2 : sampling the geo-material type of elements which are the neighbors of those elements with known geo-material type, and then spread to the whole domain.

To avoid unsymmetrical information intensity caused by using a predefined scan order, two modeling techniques for generating an initial configuration are provide(ICM,MCMC) and will be discussed and compared in following parts.

two modeling approaches

1. Iterated Conditional Modes (ICM)

(a) Assume that geological body has the layered structure.

(b) The confidence on the layered system is based on prior knowledge.

Geo-material type with the maximum conditional probability is assigned to each element as a most likely “guess” of initial configuration.

With sufficient field investigation information, this modeling technique is considered as a better approach due to its ability using all available input data.

two modeling approaches

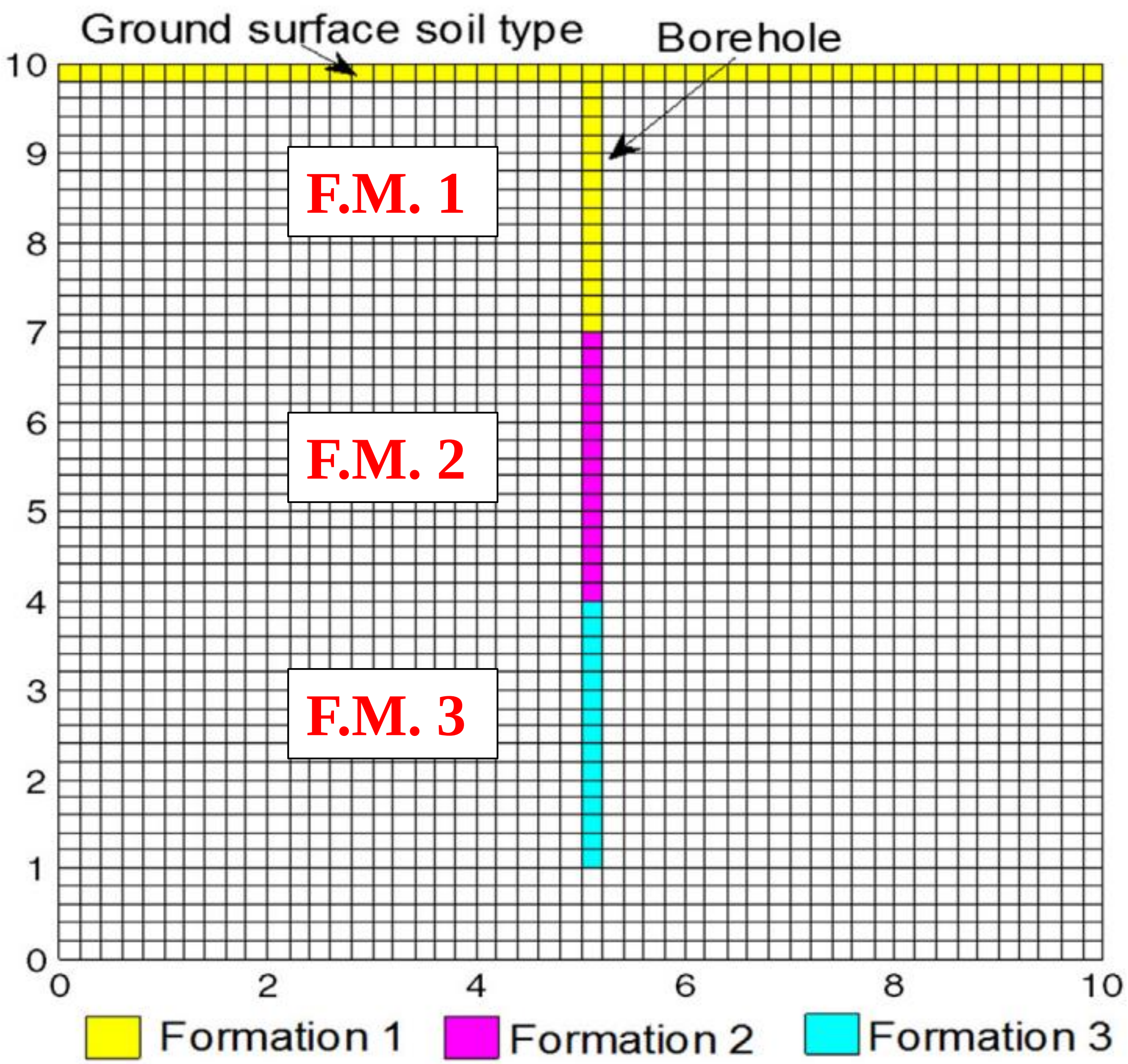
2. Markov chain Monte Carlo(MCMC)

If we concerns about the uncertainty, MCMC technique allows introducing more randomness into the initial simulation process, that means the simulated subsurface profile from MCMC will have greater degree of uncertainty.

Condition of model 1

- Physical domain is a 10 × 10 unit length area and is discretized into 2500 square elements.

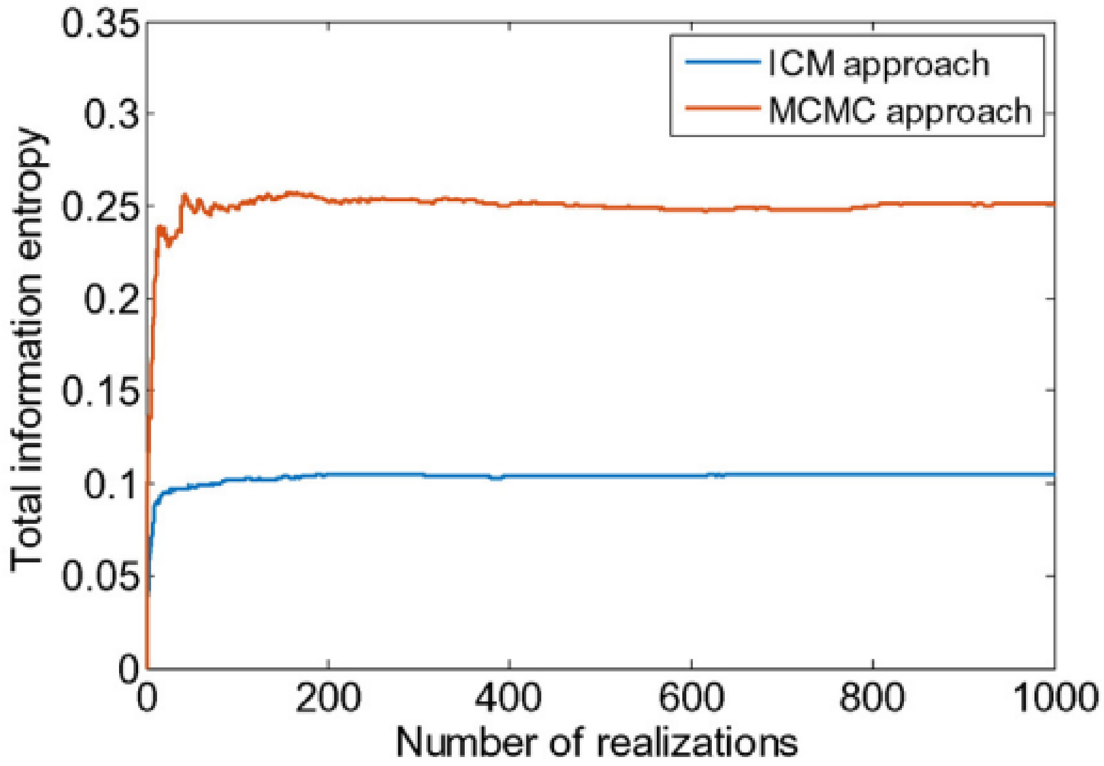
- Two parameters a=1.5 and ψ=0° are taken as constant for this whole area.

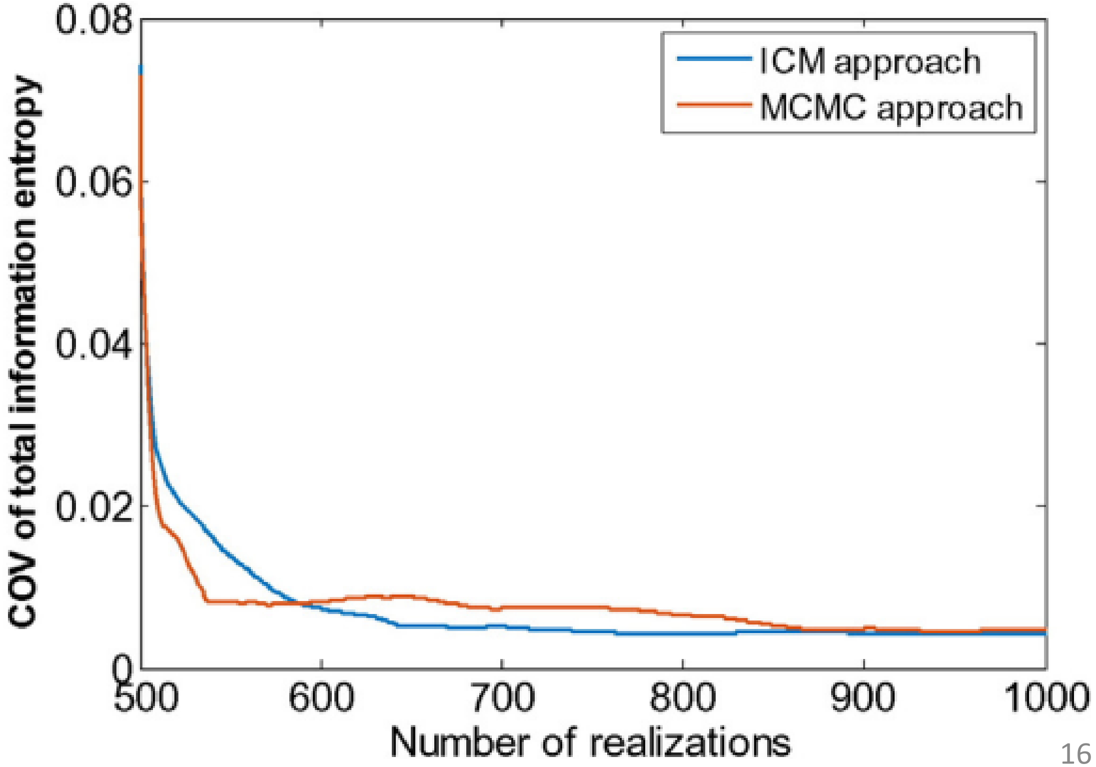


Total information entropy and COV along realization numbers

Once the COV of subsequent 500 newly added realizations is less than 0.5%, the total number of realizations is regarded large enough to represent all the possible stratigraphic profiles.

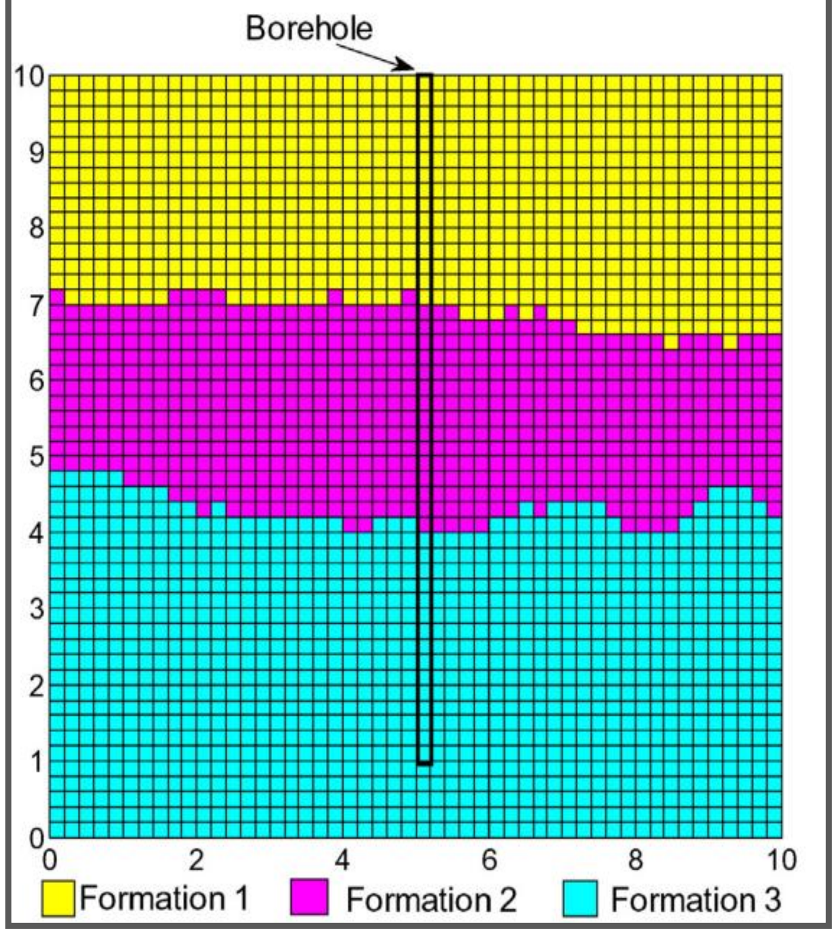
In this simulation, after 900 realizations, the convergence criterion has been met.



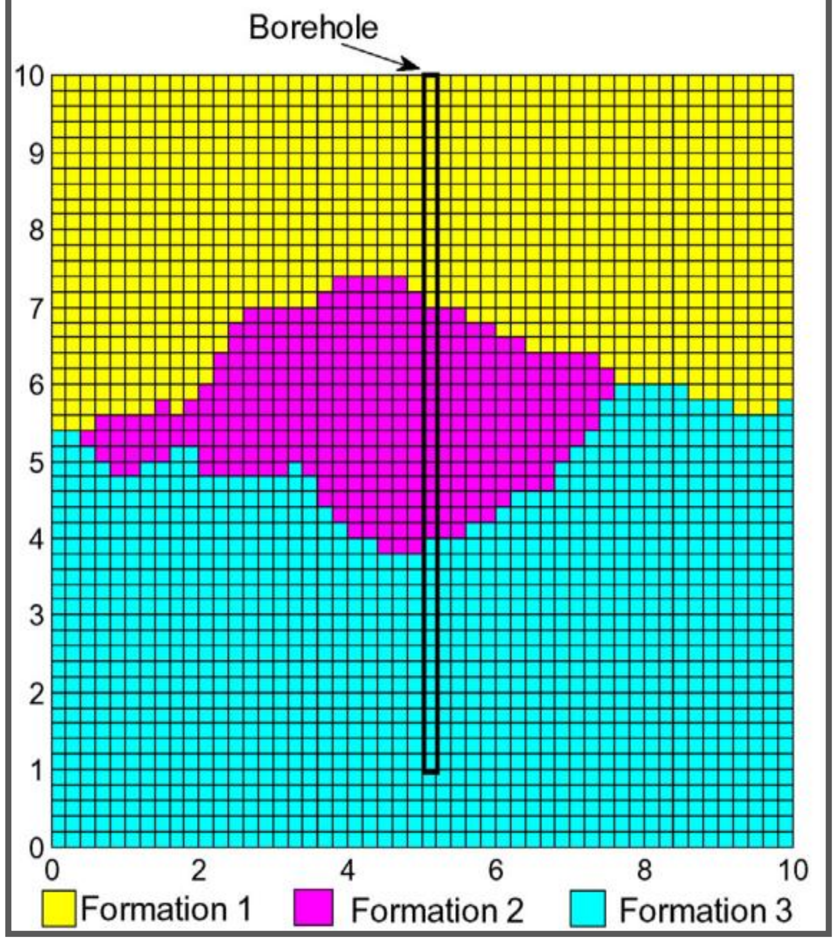


One of possible realization for modeling approach

ICM



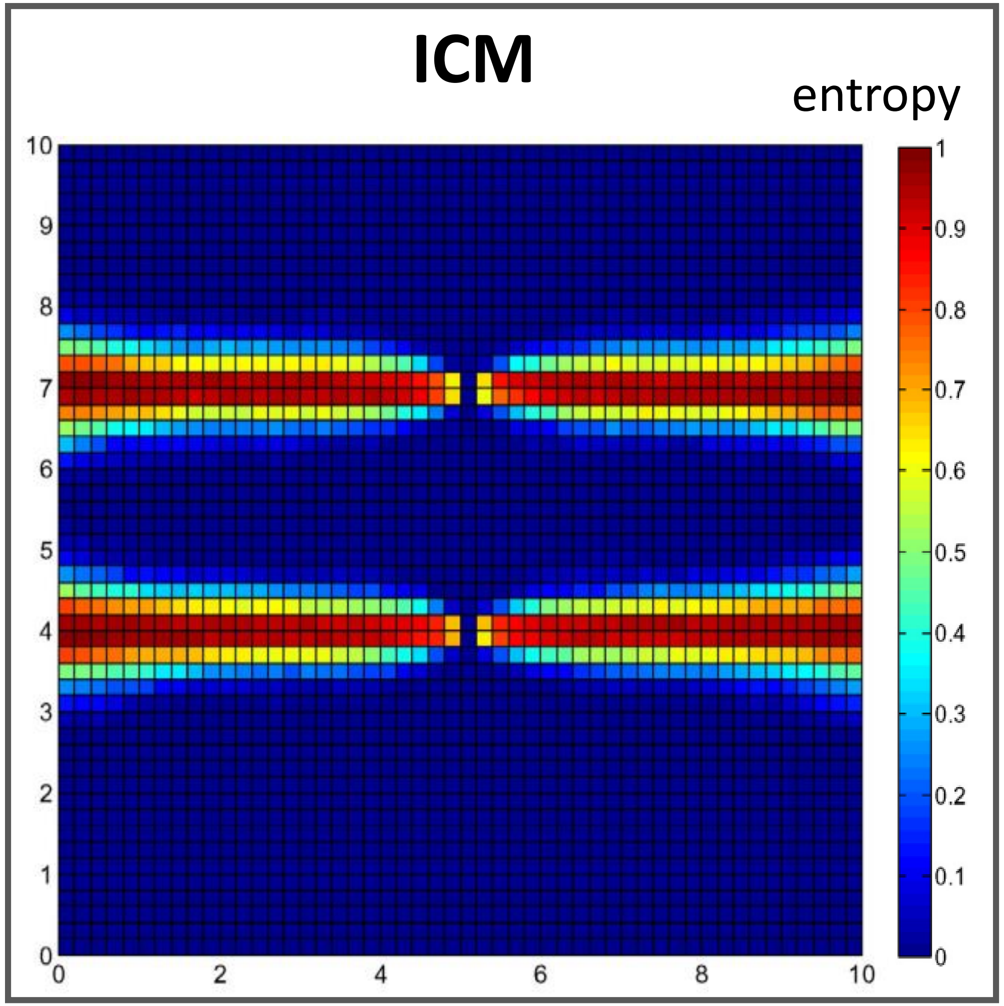
MCMC

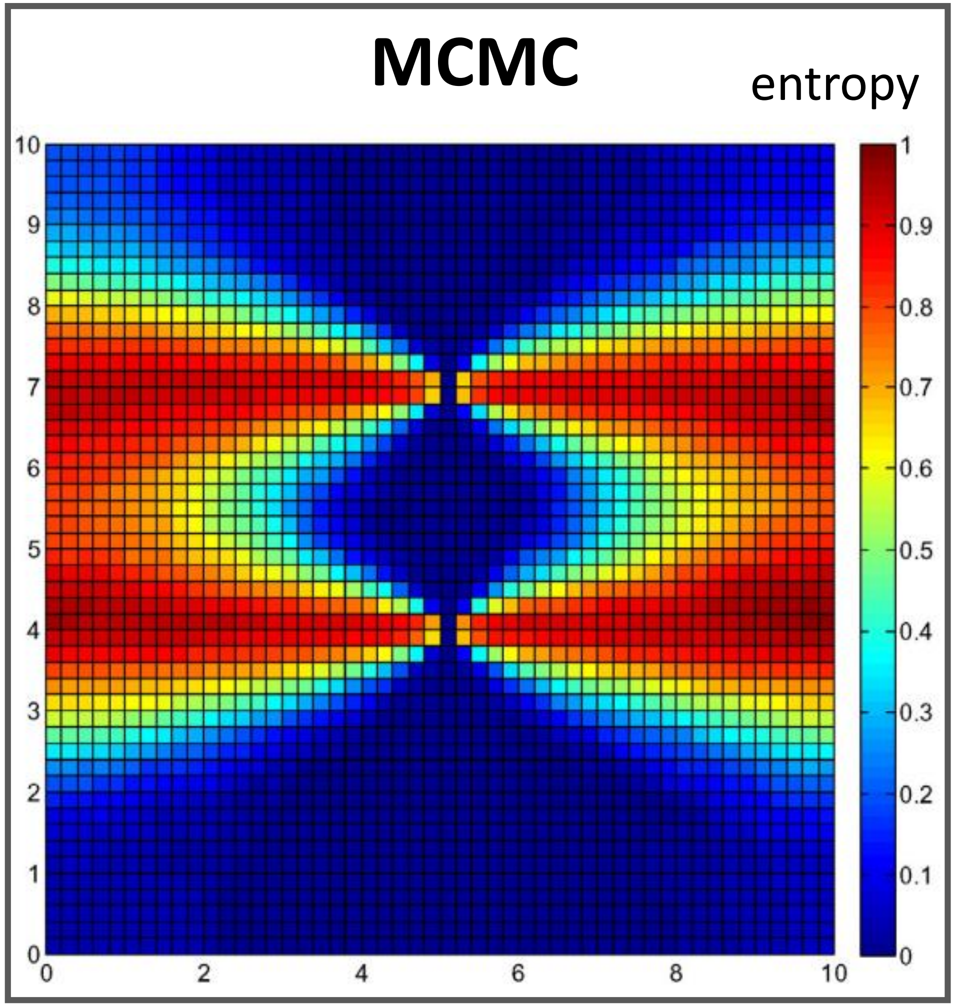


ICM simulation technique is applicable to a case where we are confident that the geo-strata are layered structure.

MCMC simulation technique is preferred for a situation where site exploration data is in sufficient and more randomness involved in subsurface structure.

Information entropy for modeling approach





MCMC allowed more uncertainties exist.
High entropy means that will have lower confidence and higher uncertainties in these area.

Sensitivity analysis in two modeling approaches

- The geological condition is simplified into two formations so that uncertain zone of possible boundary between two formations can be well quantified.
- The interested physical domain is a 10 $\times$ 10 unit length area.

1. different mesh density

The element lens/numbers : 0.05/400 , 0.025/1600 , 0.0125/6400

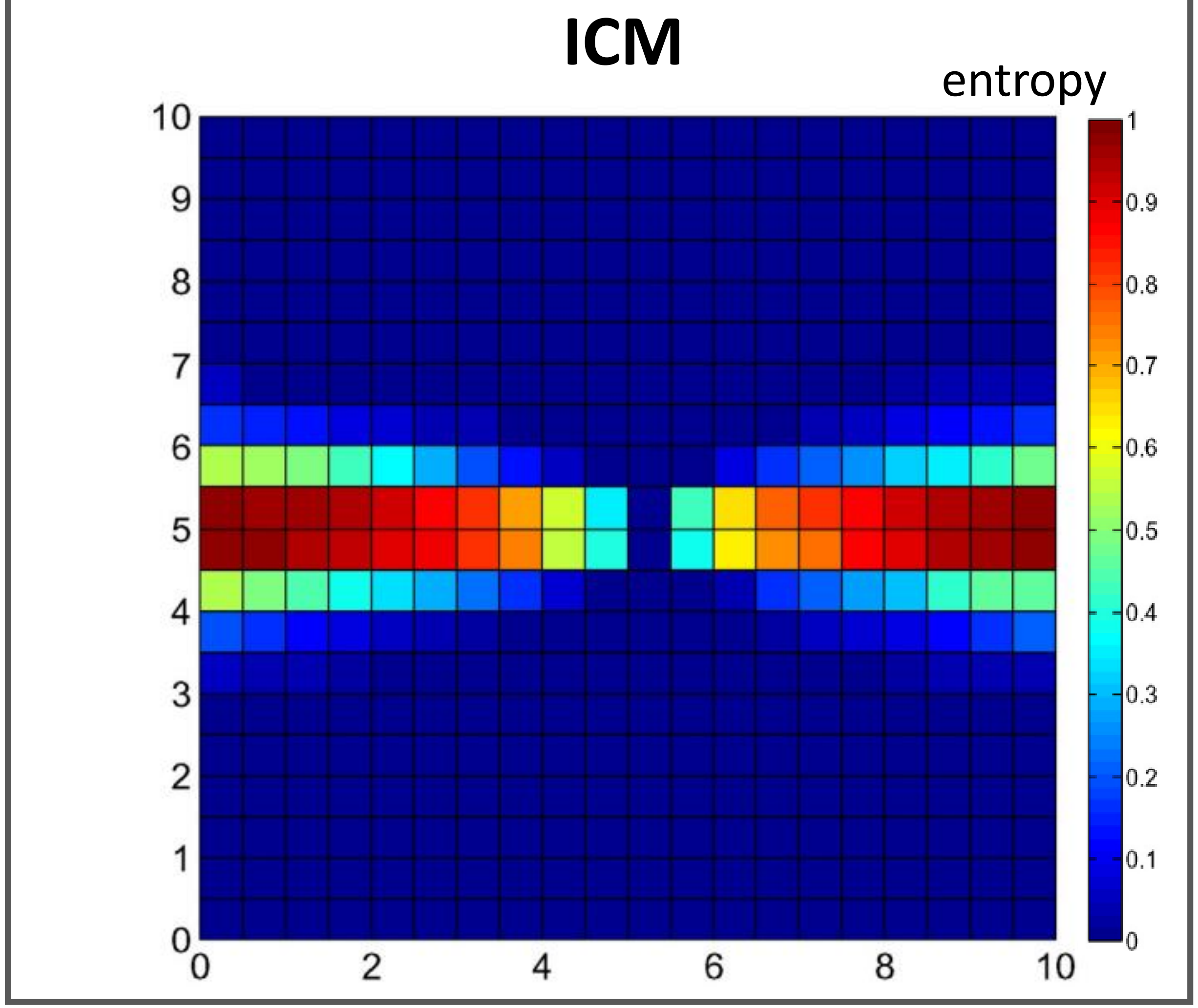
In this condition, a=3 and ψ=0 degree.

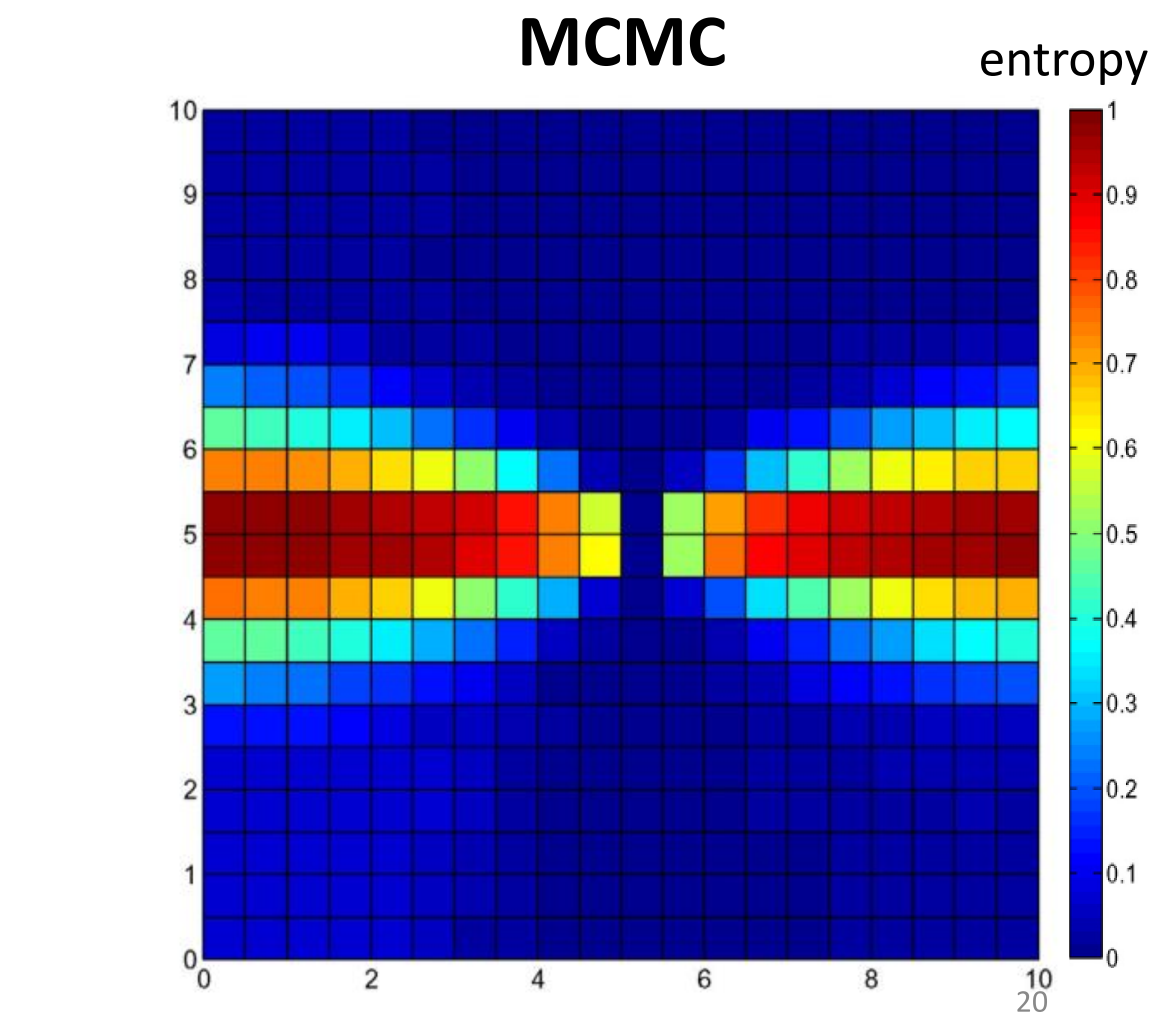
2. different parameter “a”

a=1.5 , a=3 , a=5

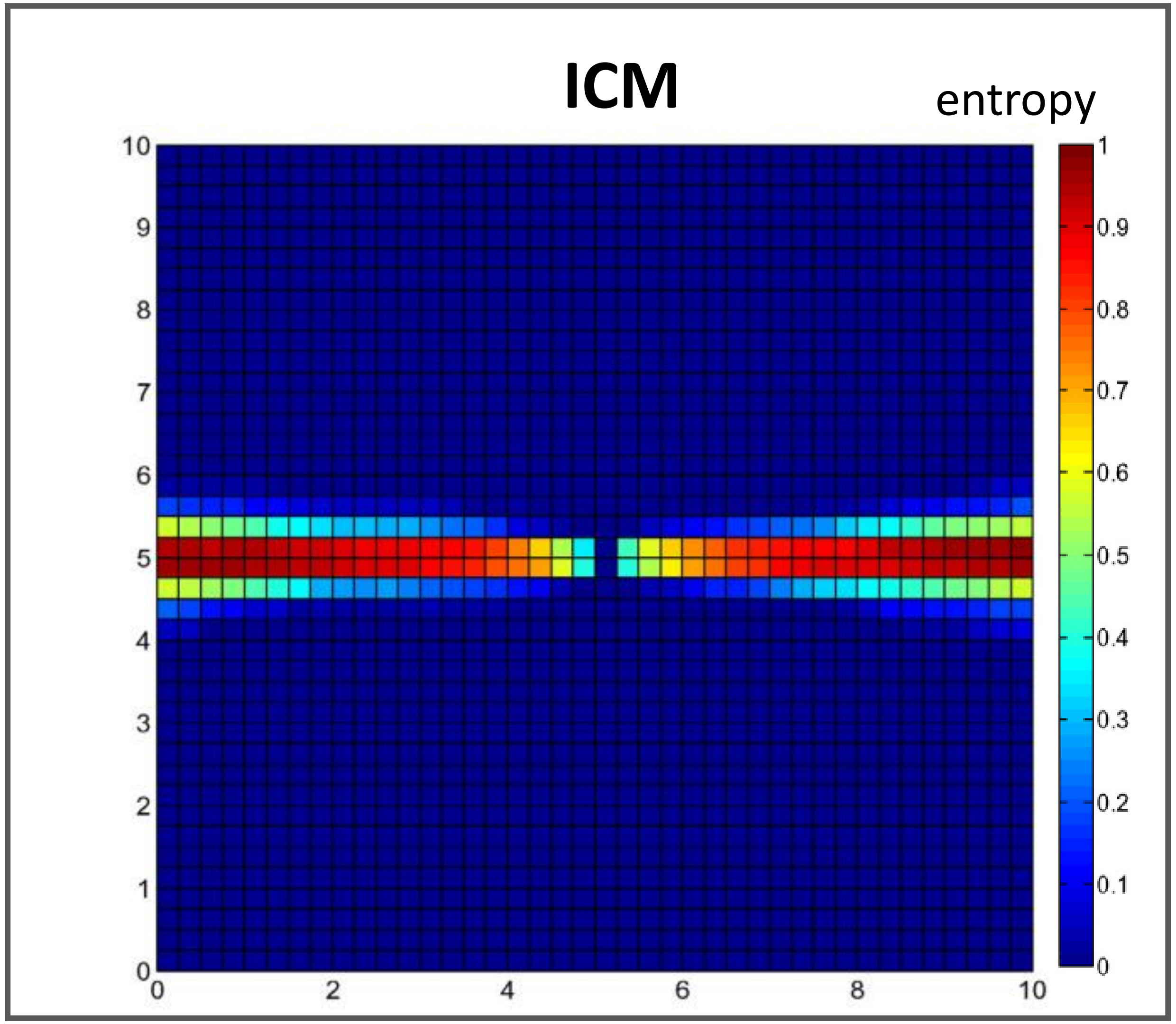
In this condition, ψ=0 degree and will have 1600 elements.

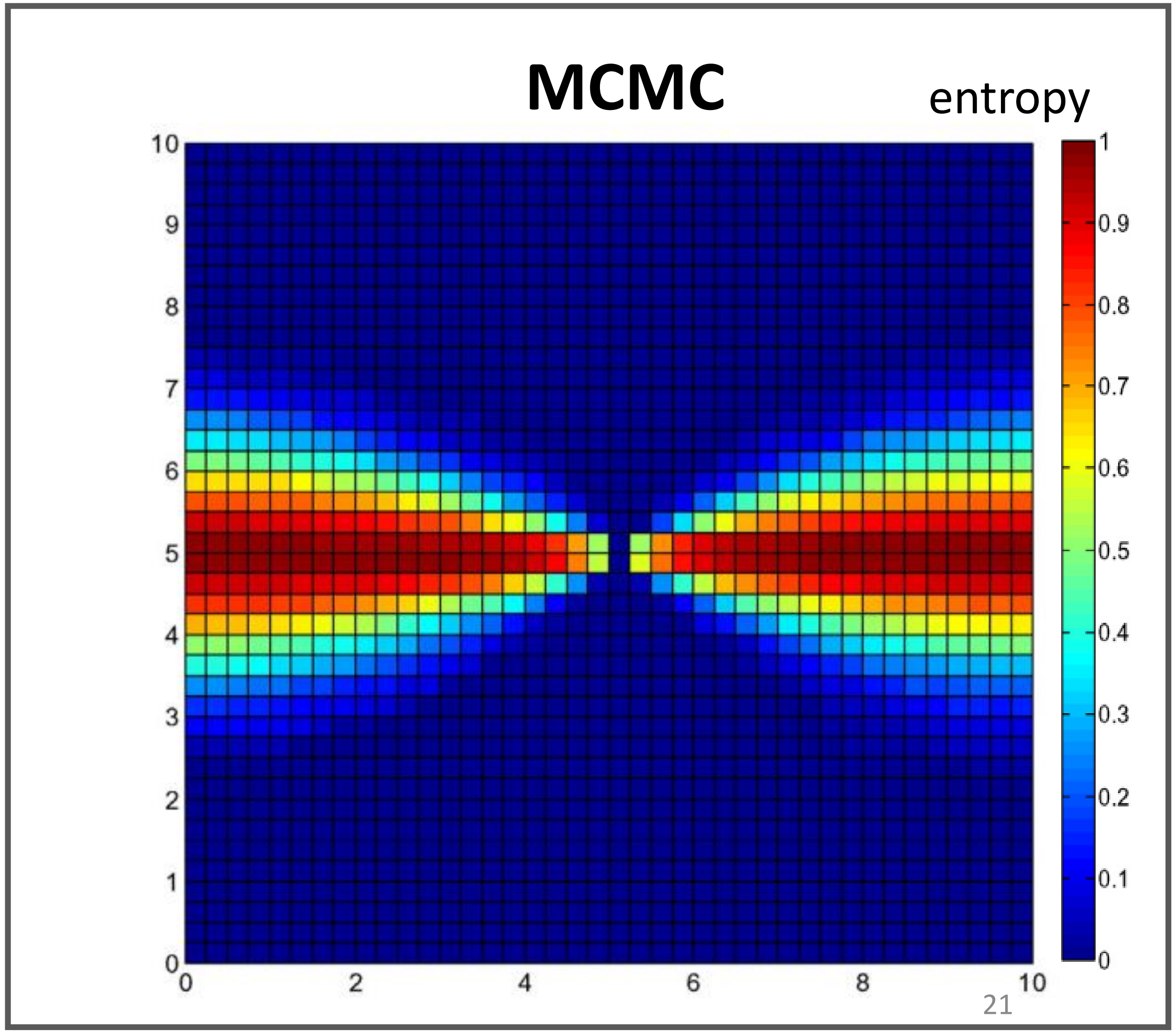
Sensitivity analysis ~ different mesh density (400)



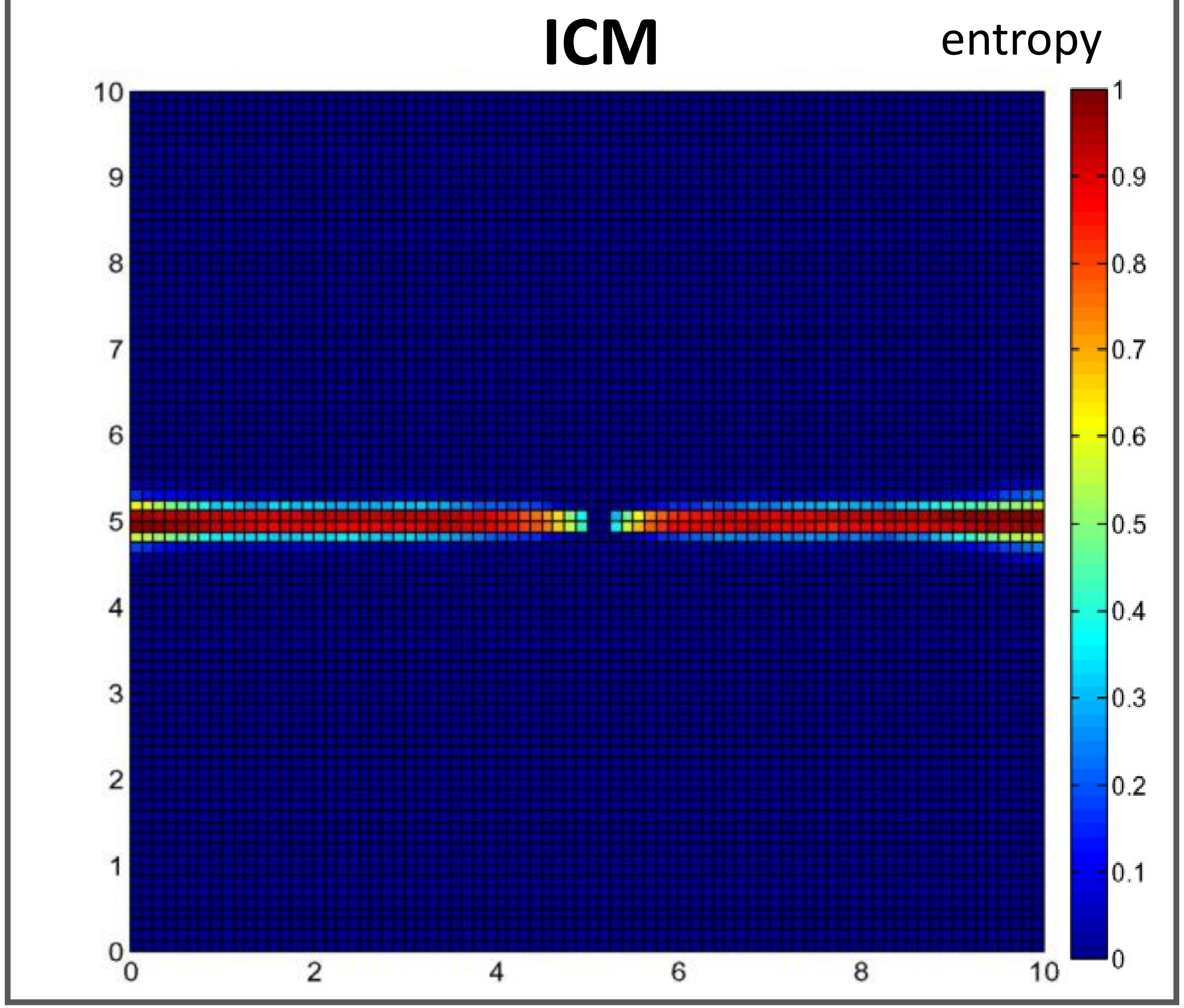


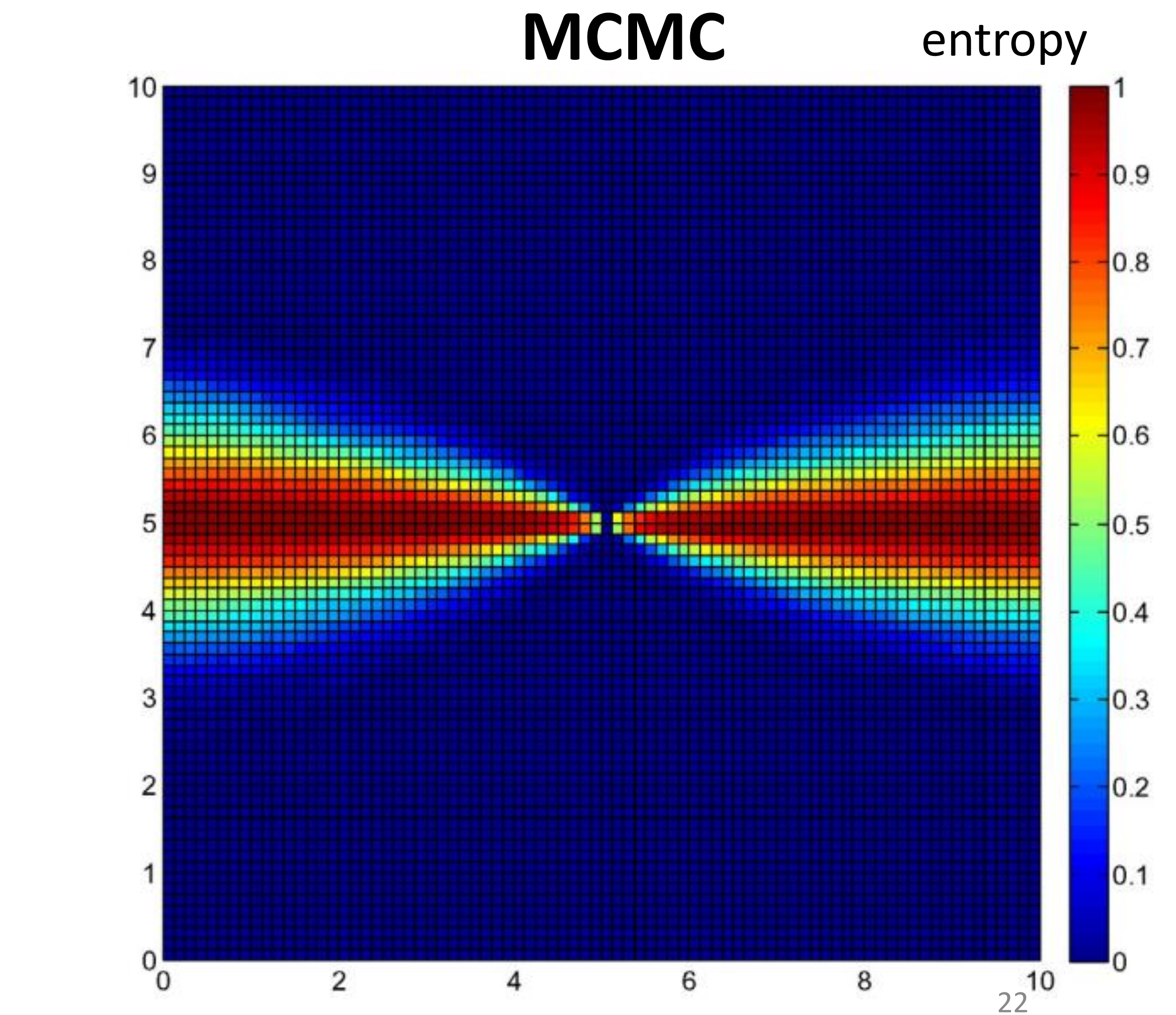
Sensitivity analysis ~ different mesh density (1600)





Sensitivity analysis ~ different mesh density(6400)

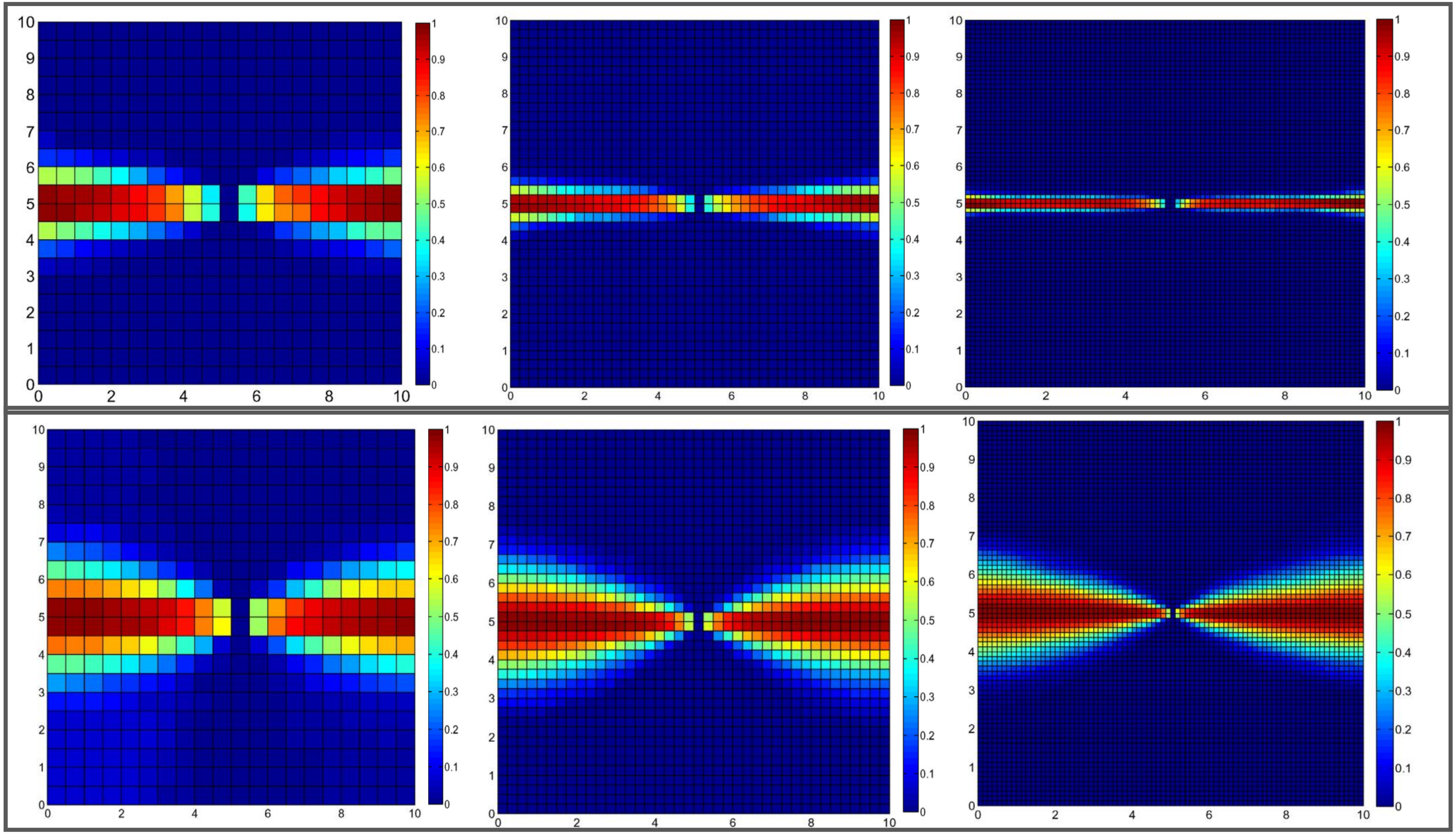




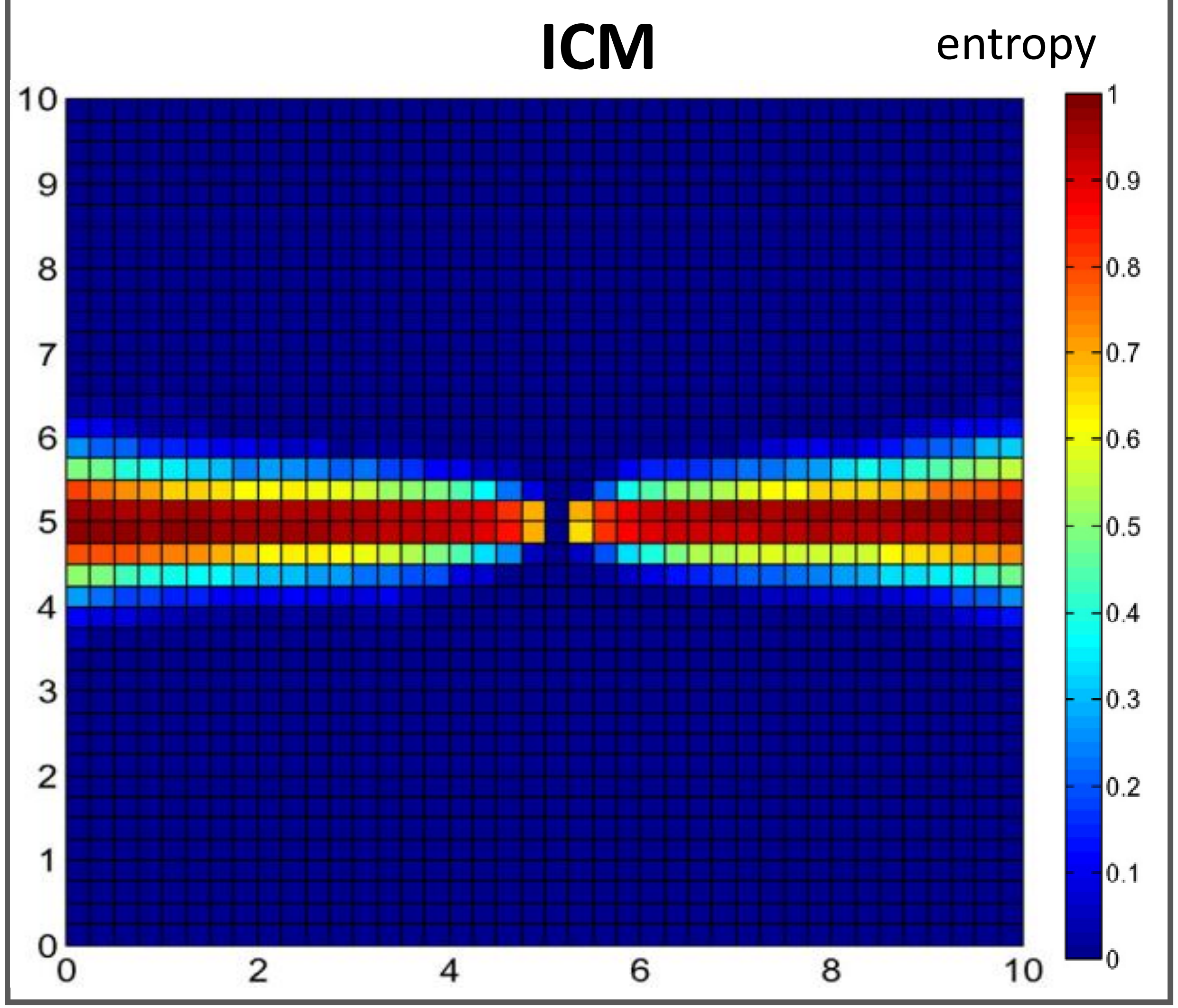
Sensitivity analysis discussion in different Mesh density

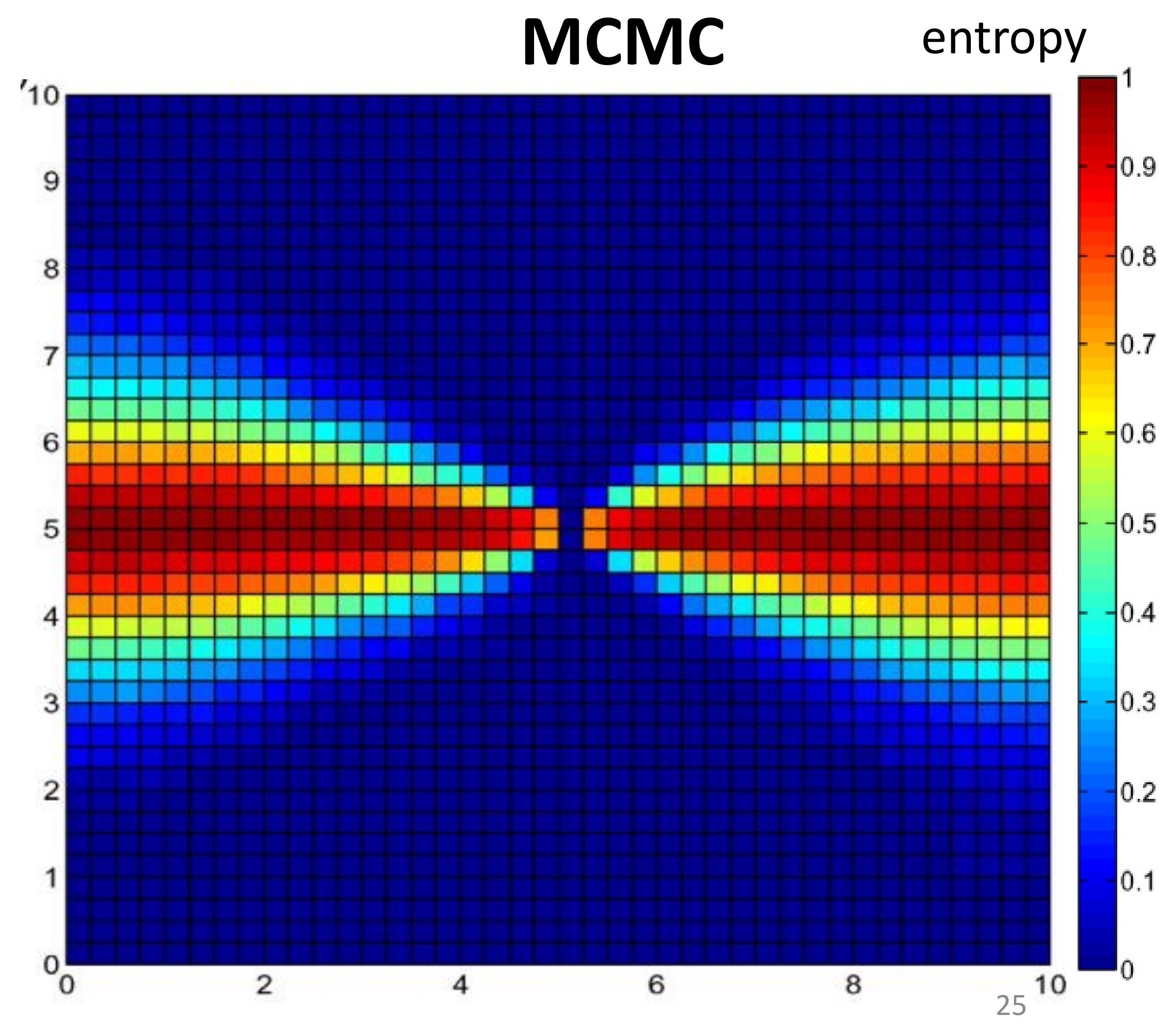
In ICM model, the uncertain area of the lithological unit boundary depends on the mesh density significantly. The larger the element size will have the wider the “uncertain band”.

In MCMC modal, “divergent zone” is shown not very sensitive to the mesh density.

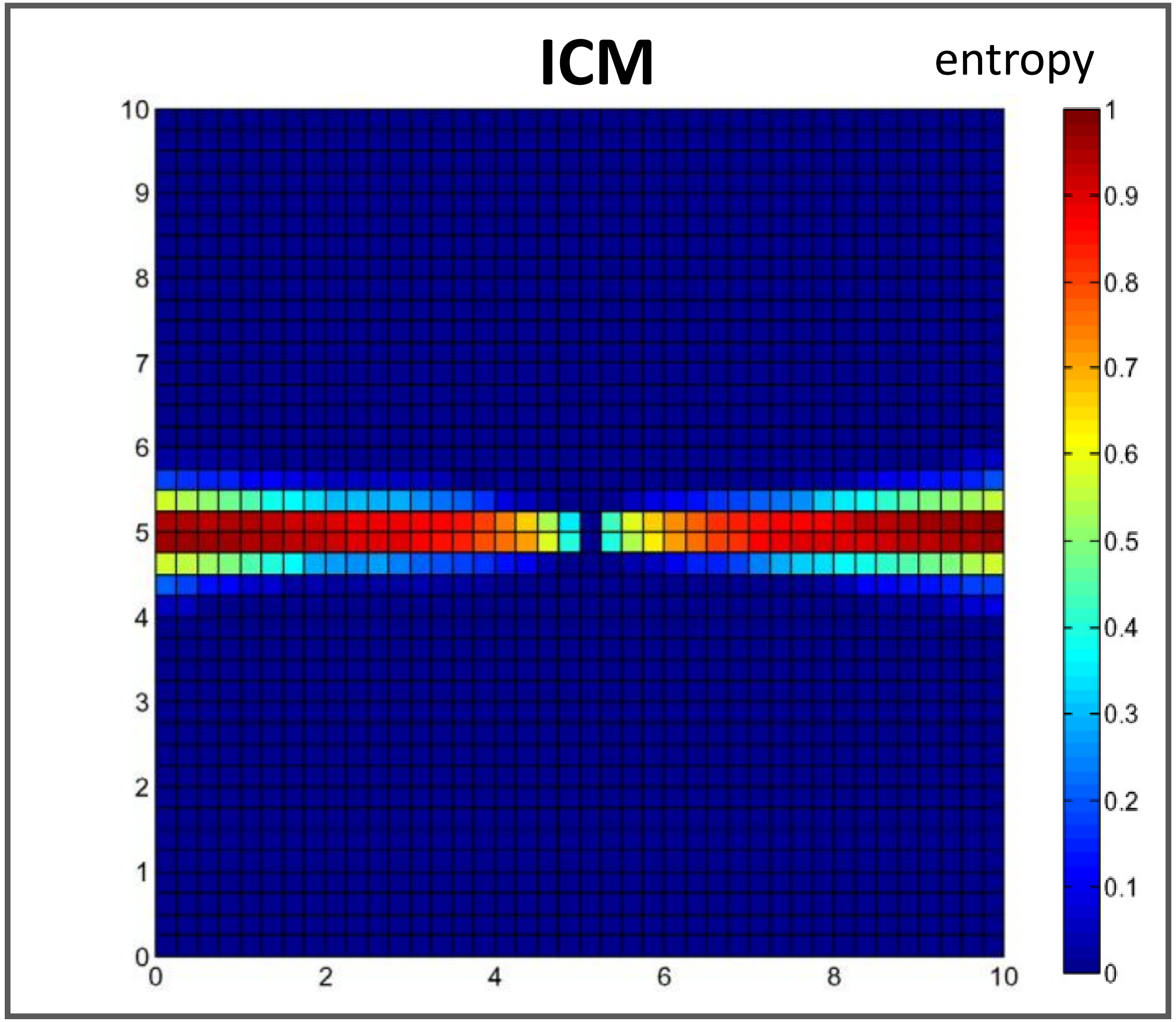

ICM
MCMC

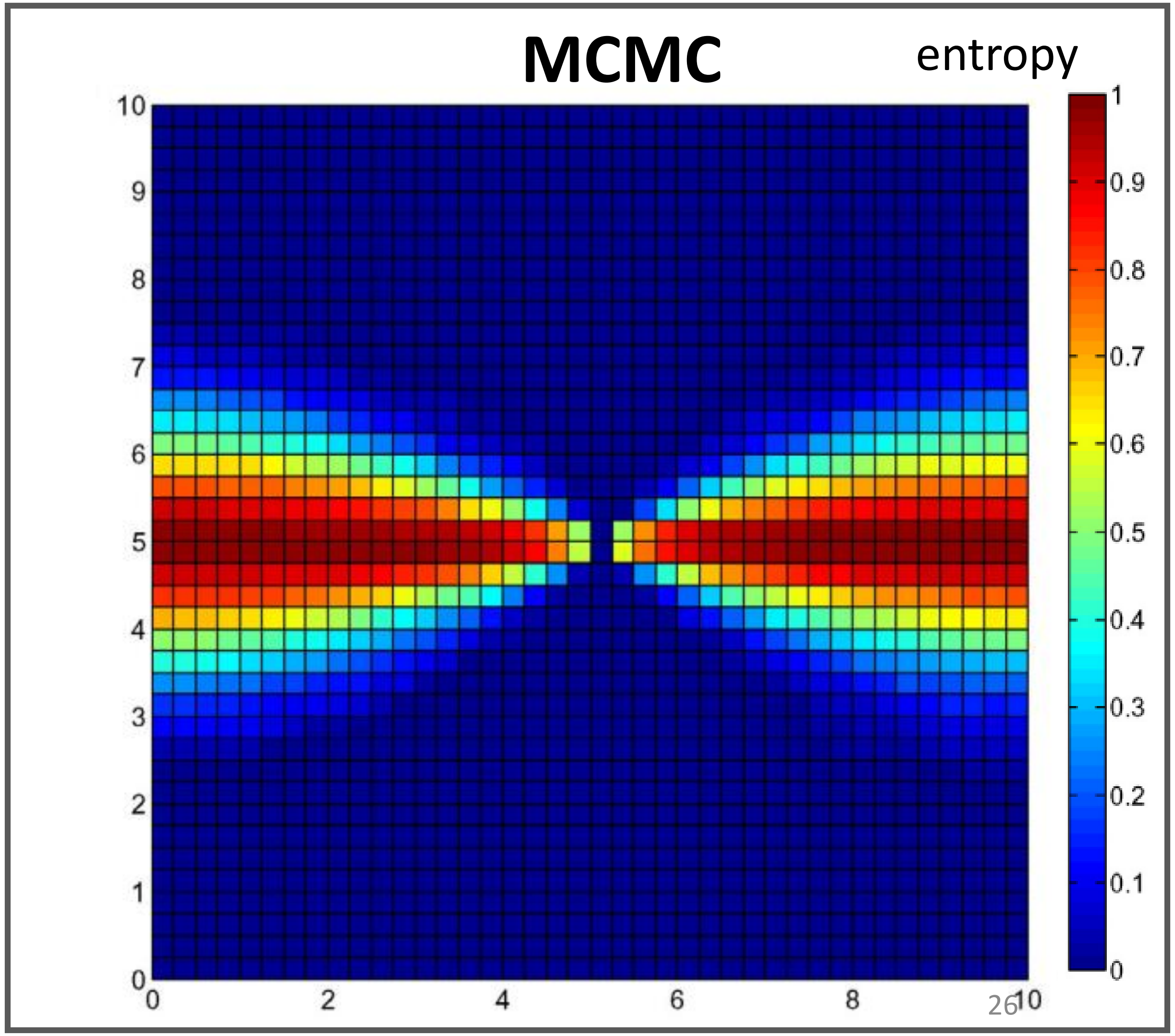
Sensitivity analysis ~ different a value(a=1.5)



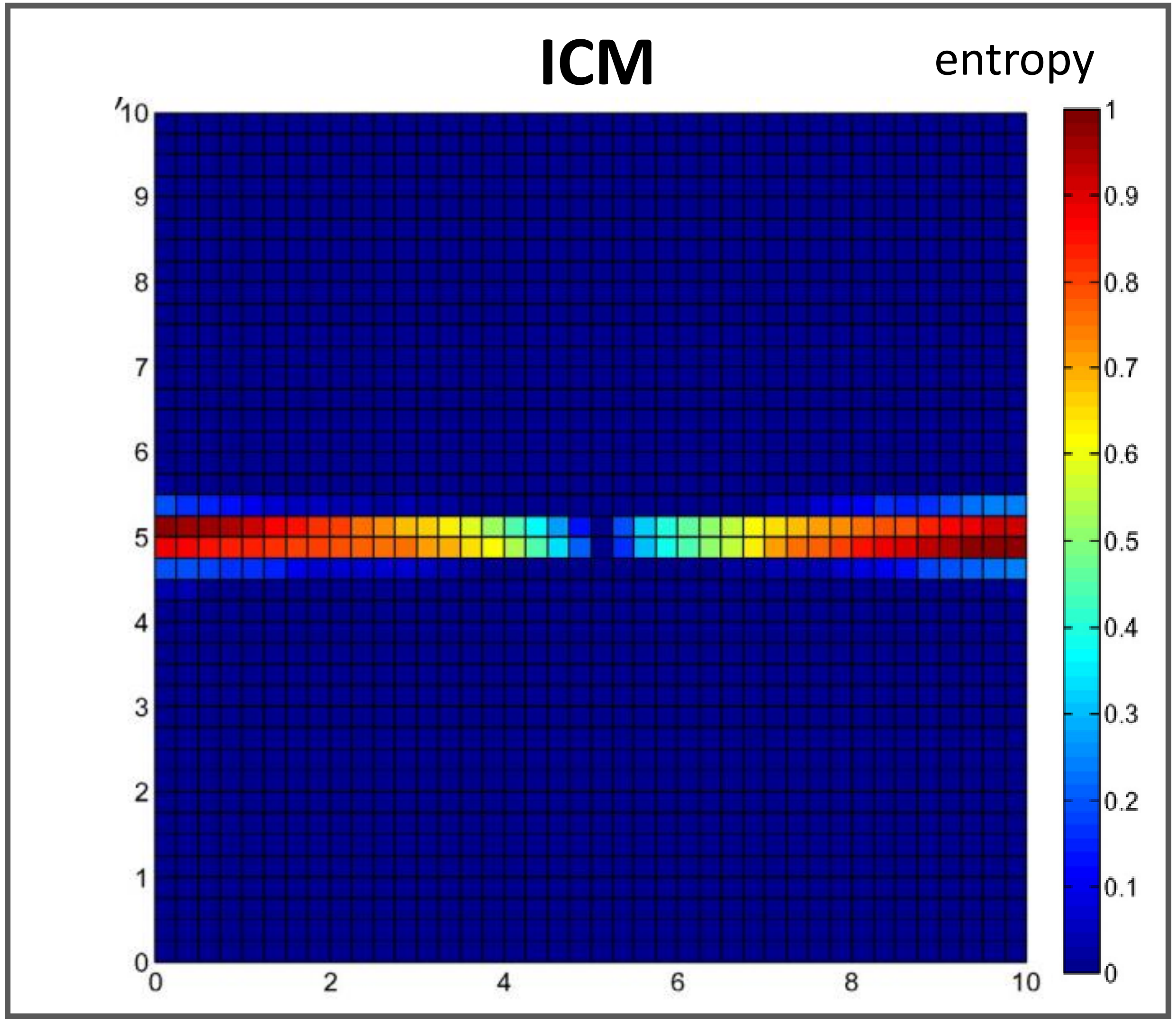


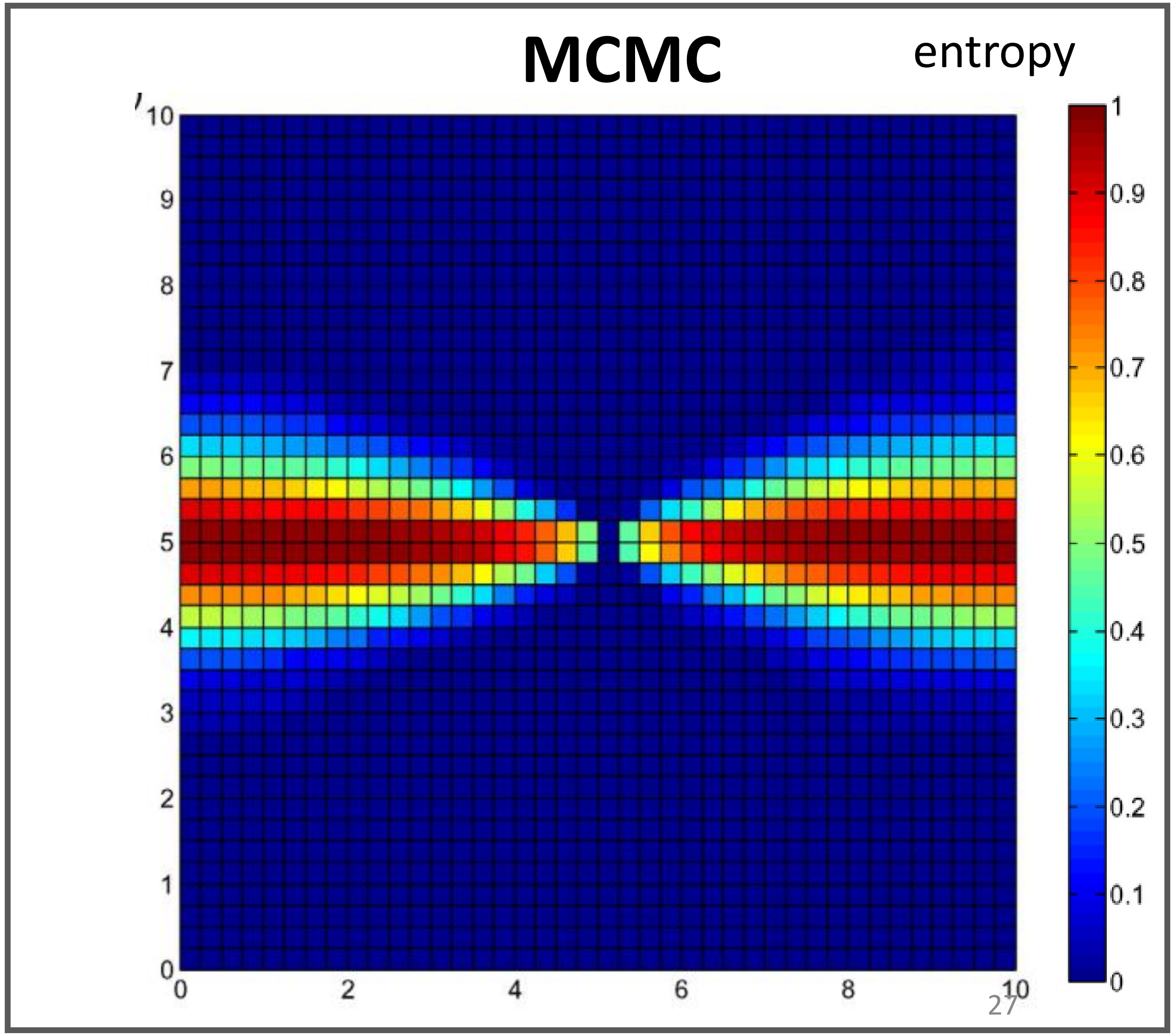
Sensitivity analysis ~ different a value(a=3)





Sensitivity analysis ~ different a value(a=5)





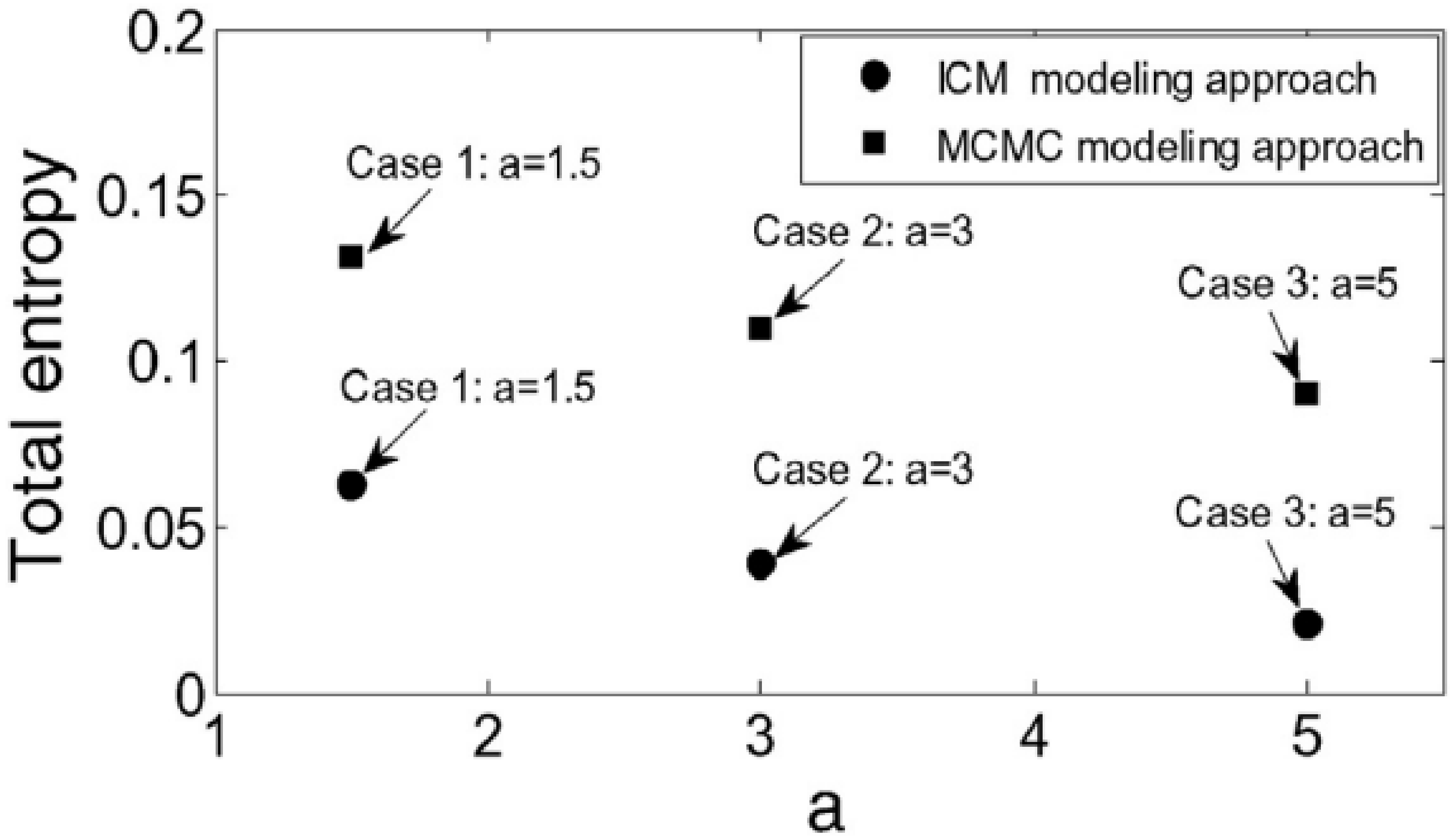
Sensitivity analysis discussion in different a value

In ICM model, “uncertain band” is very sensitive to parameter a. When larger value of a is used, the uncertain zone becomes narrower.

In MCMC modal, the “divergent zone” is inversely related to the value of parameter a. It becomes narrower with larger value of the model parameter “a”.

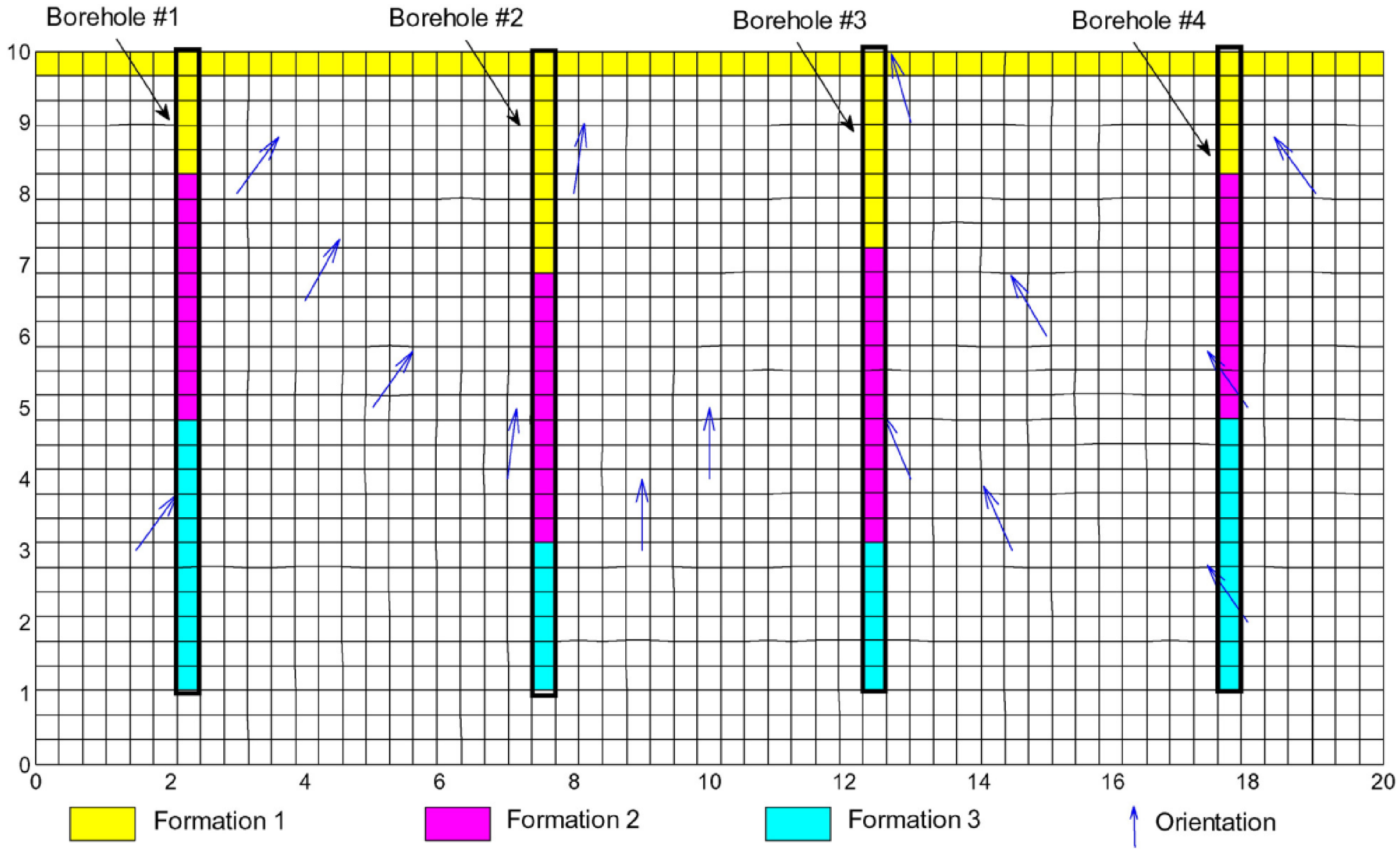
But value of parameter a should not be greater than 5 to avoid strong dominance of tangential correlation.

Total entropy for both modeling approaches



- Parameter “a” reflect the degree of anisotropy , higher total entropy due to more randomness involved.
- For both simulation approaches, the total entropy is decreased as parameter a is increased.
- But value of parameter a should not be greater than 5 to avoid strong dominance of tangential correlation.

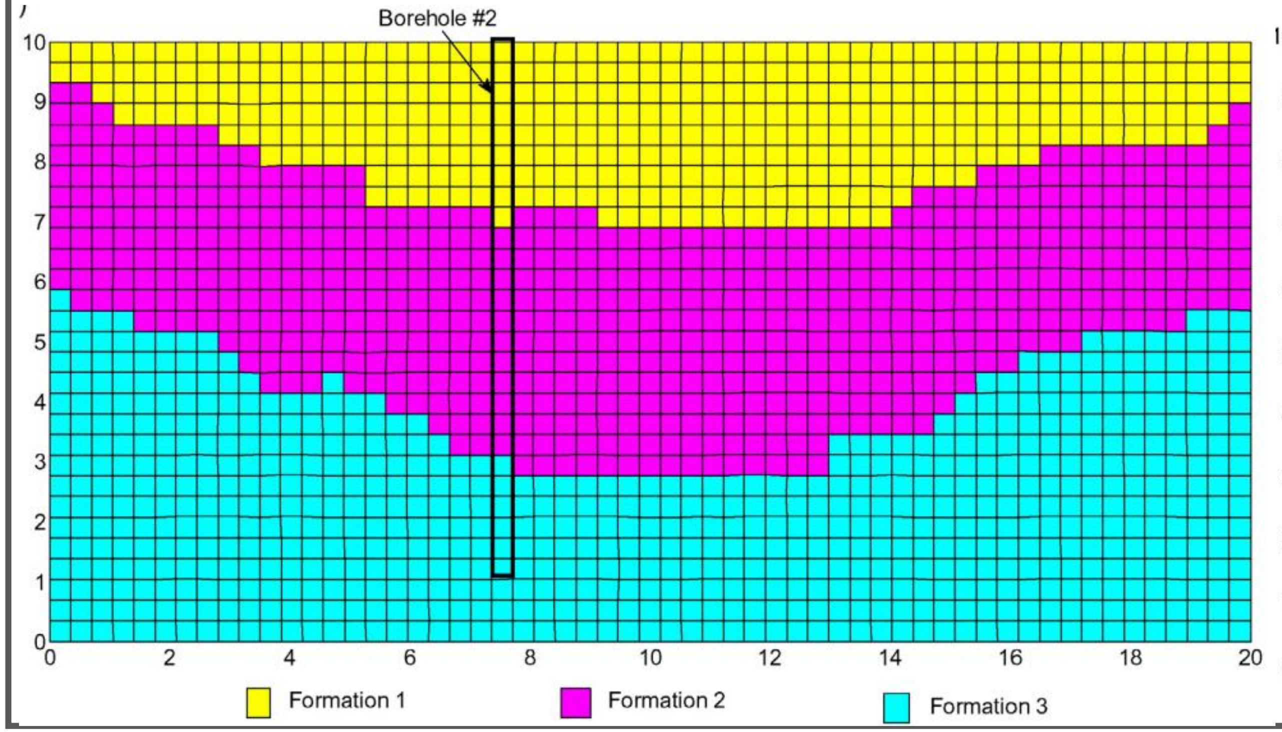
Example1: ICM modeling approach

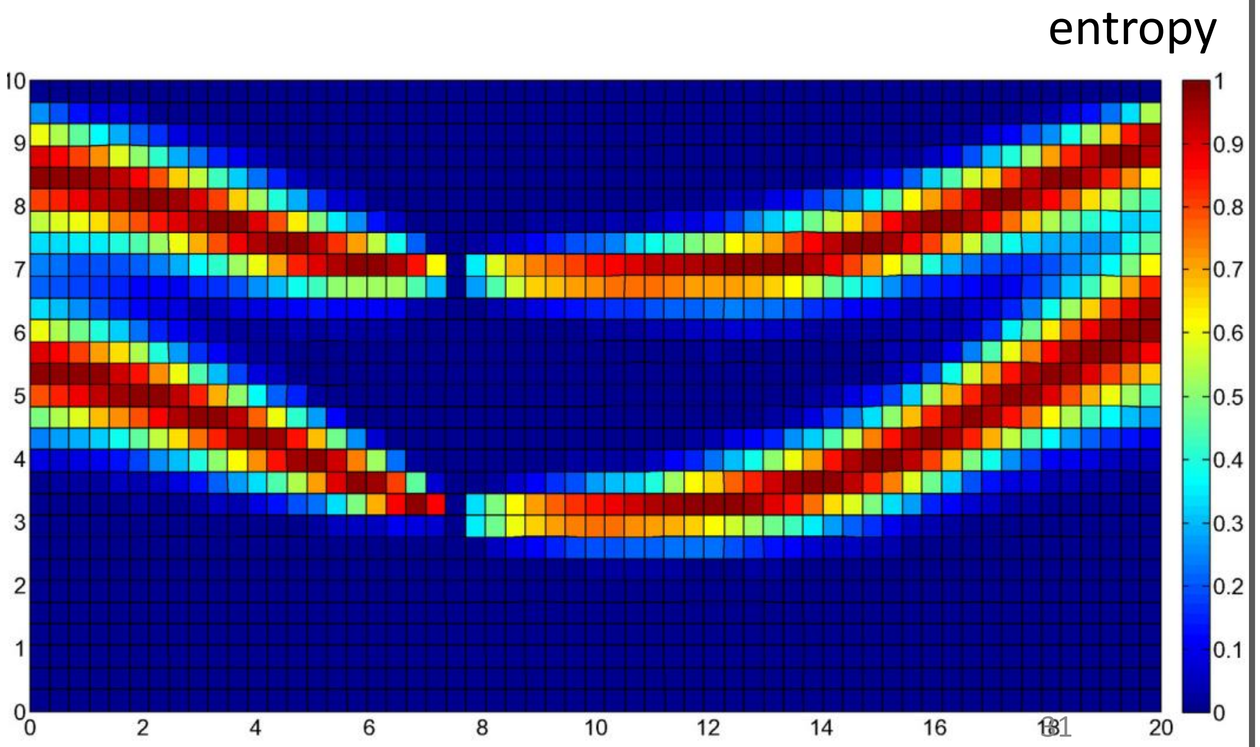


- assumed that the formation is layered-like, and then the ICM approach seems applicable.
- In this condition parameter ψ is not a constant. To obtain ψ at each element, we use ordinary kriging interpolation process to get the value ψ in each element.

Example1: ICM modeling approach

Only used BH2 to build the model in the beginning.





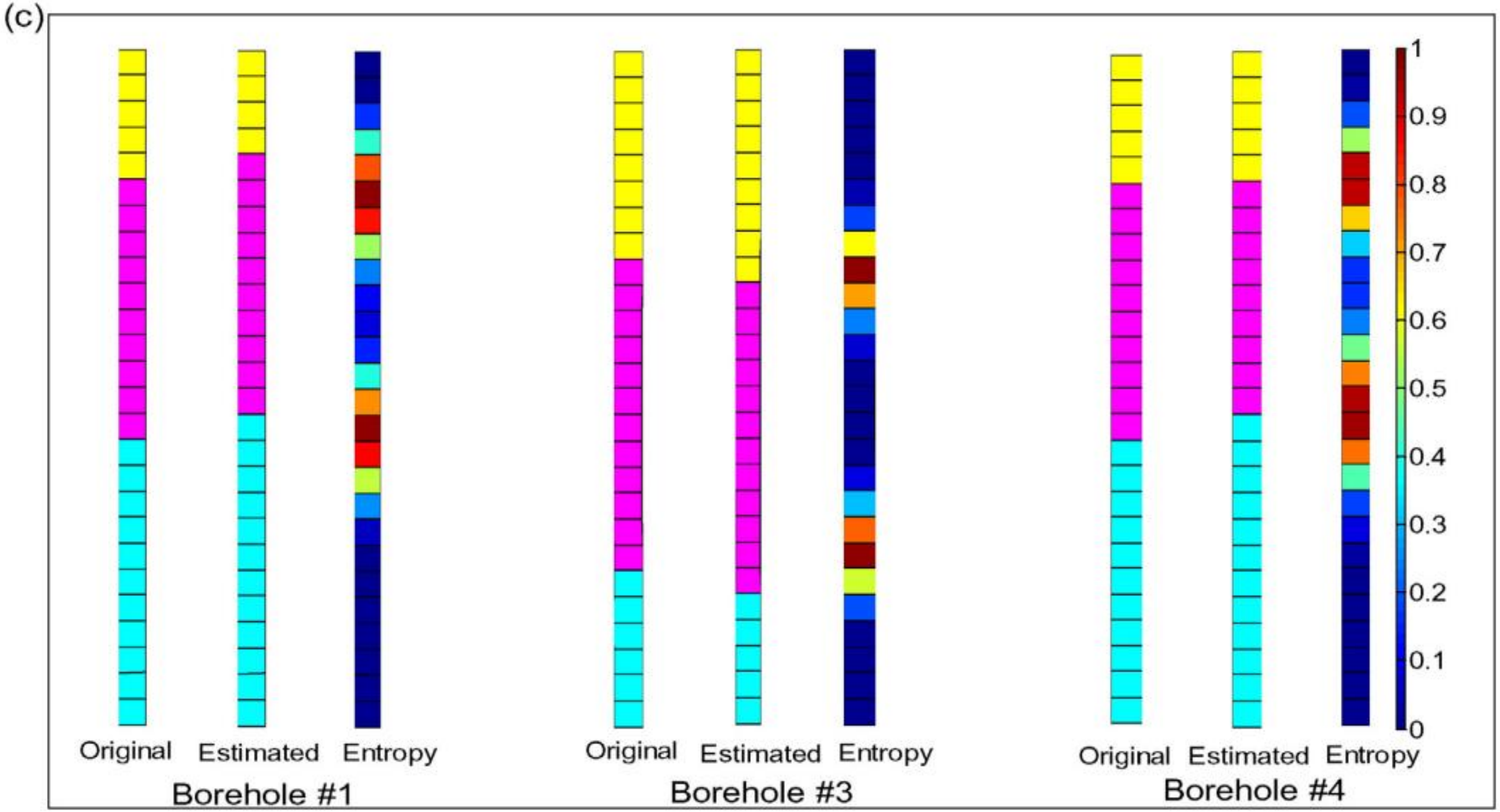
Example1: ICM modeling approach

Case 1: Borehole #2 + ground surface soil type + orientation vectors + Borehole #4 (additional data)

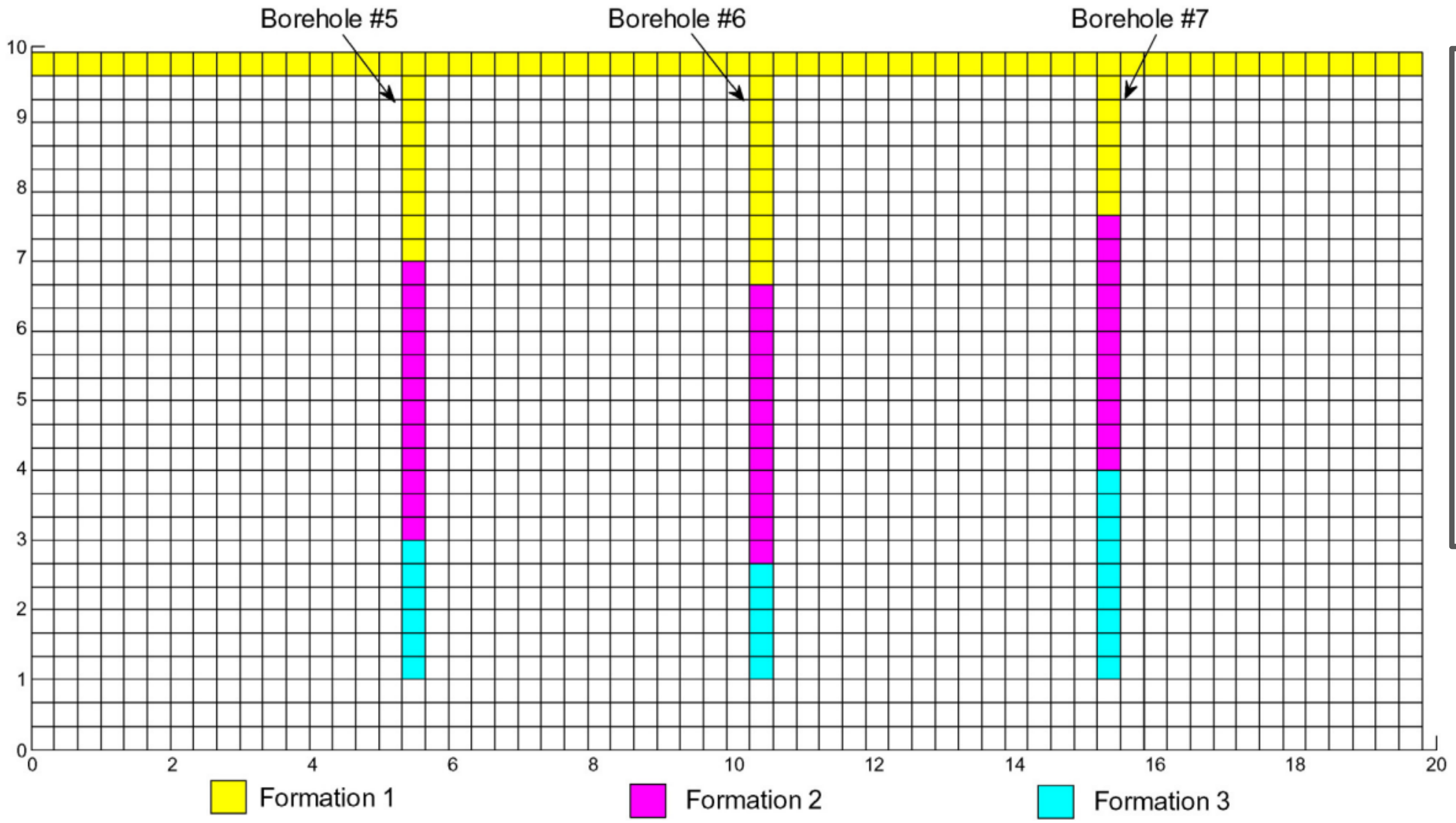
Case 2: Borehole #2 + ground surface soil type + orientation vectors + Borehole #4 and #1 (additional data)

Case 3: Borehole #2 + ground surface soil type + orientation vectors + Borehole #4 and #1and #3 (additional data)

Example1: ICM ~ correspond to the original formations



Example2 : MCMC modeling approach



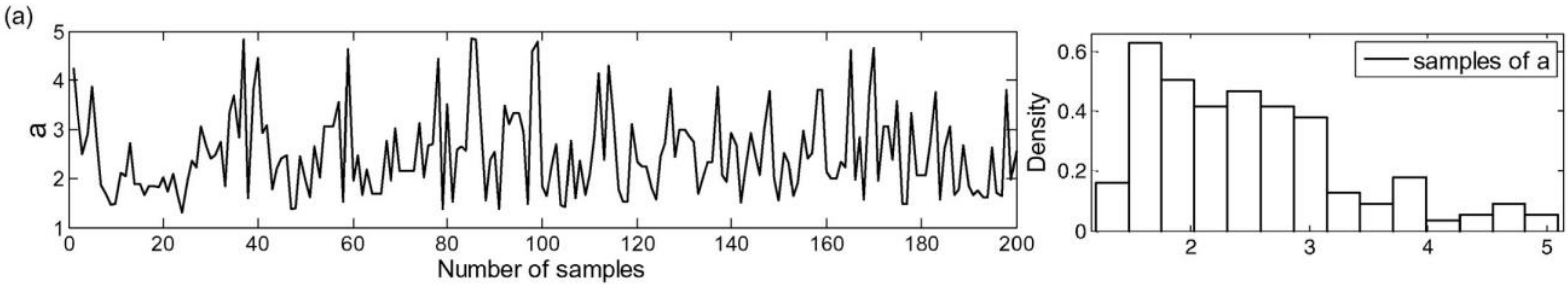
BH6 is used as the additional data to estimate parameter "a"

The orientation ψ=0° is set in this example.

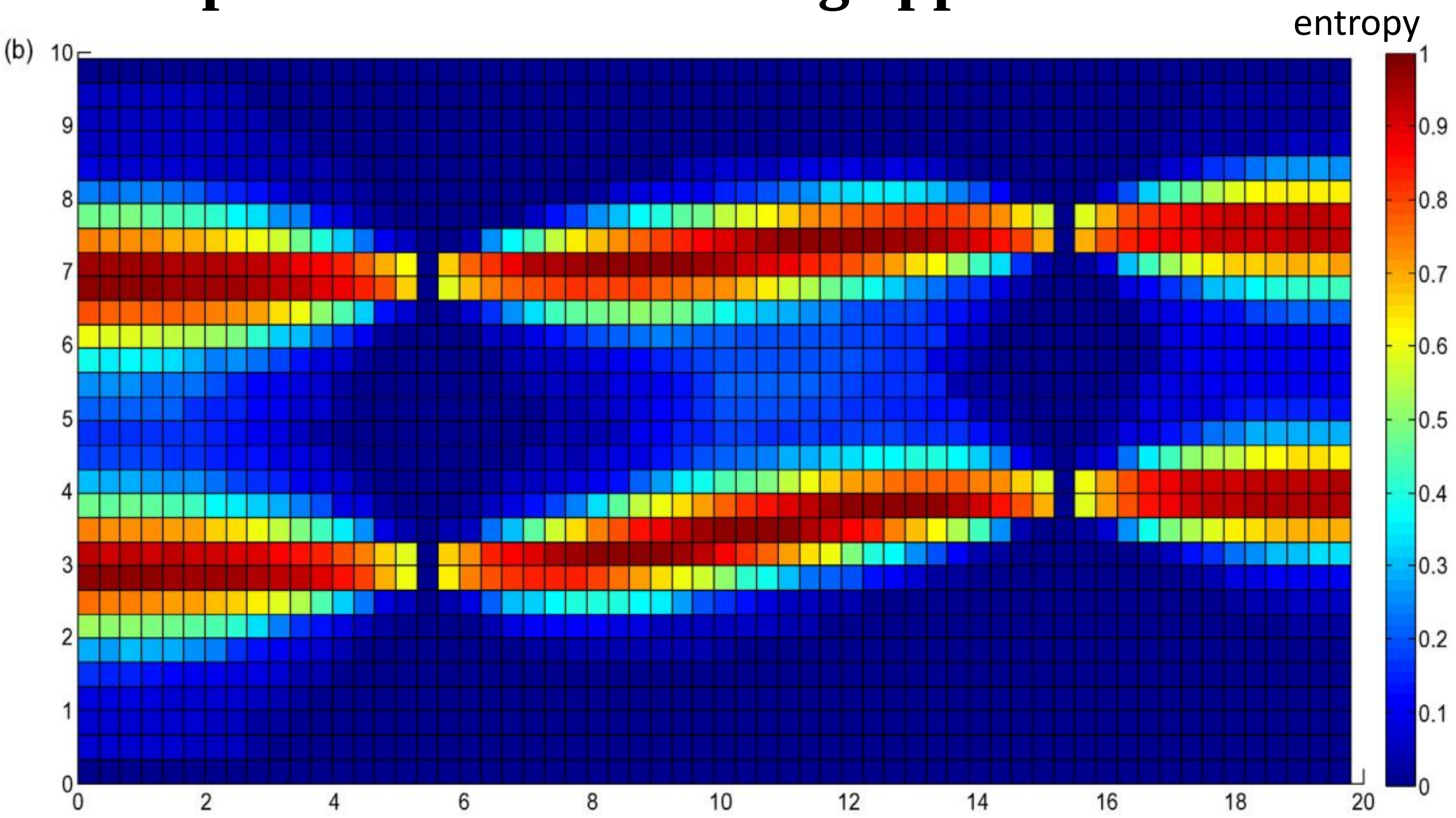
Example2 : MCMC modeling approach

In this case mean(a)= 2.4970 and variance=0.7141.

Cause more uncertainty involved in the MCMC simulation process, realizations can differ from each other.



Example2 : MCMC modeling approach



Conclusion

- Two geological modeling approaches involving the ICM and MCMC techniques were developed for modeling using three types of site investigation data.

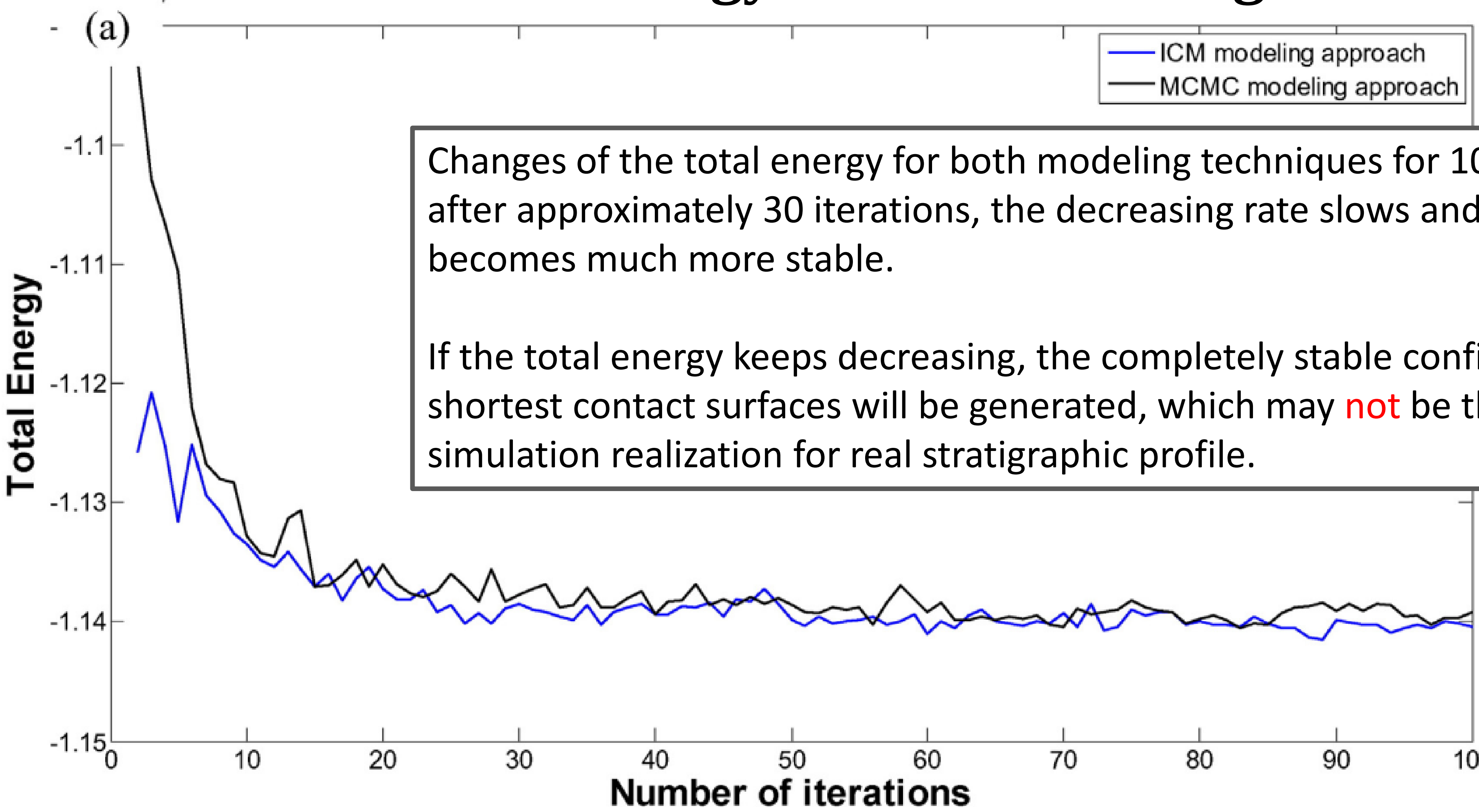
- ICM technique to generate initial configuration is applicable to the layered lithological structure. The “uncertain band” is generally located at the possible lithological unit boundary.

- MCMC technique can introduce more uncertainties into the initial configuration. The uncertain area is a “divergent zone” from the known formation boundary at borehole location.

- The estimation error (sample variance) decreases as more additional boreholes are incorporated as model evidence.

Thanks for your listening

The evolution of total energy for two modeling

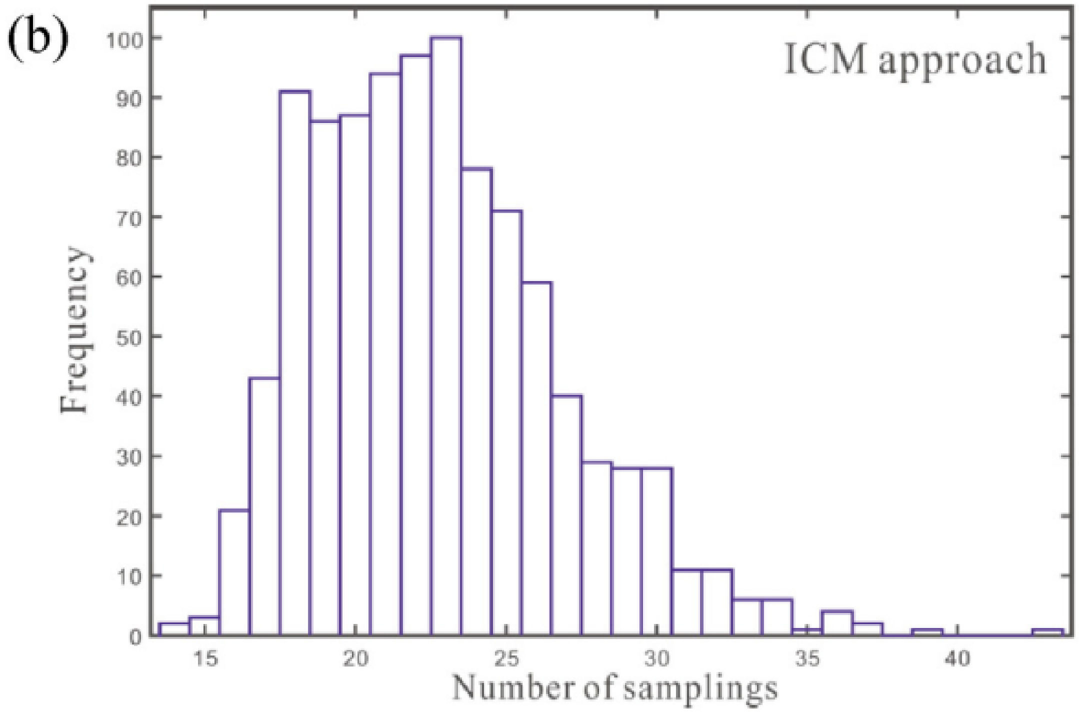


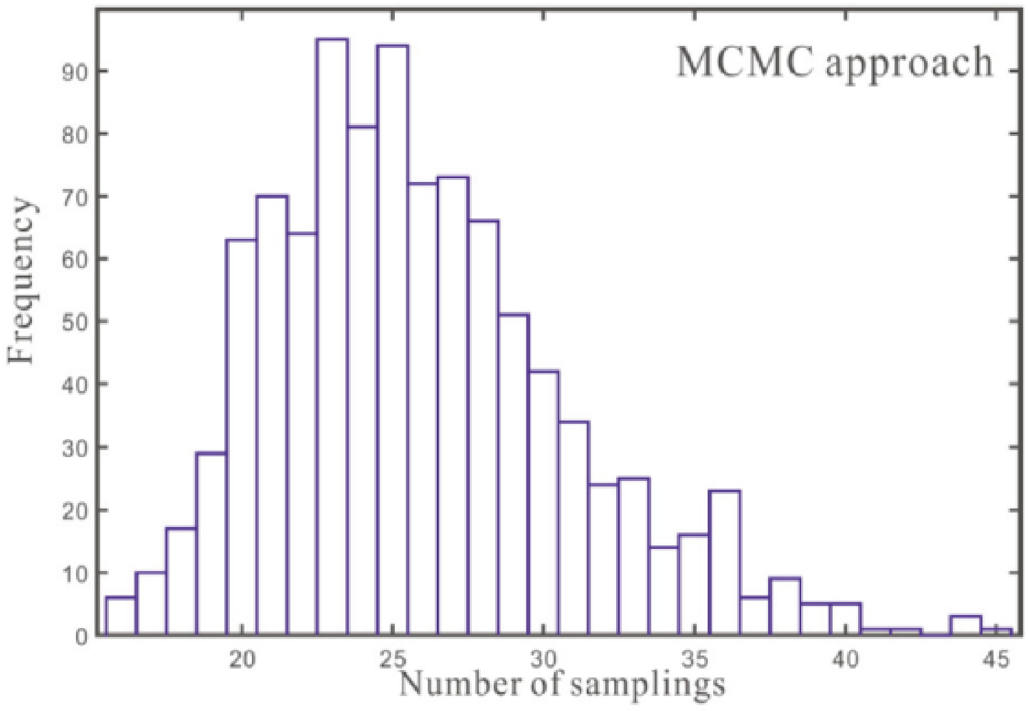
Changes of the total energy for both modeling techniques for 100 iterations, after approximately 30 iterations, the decreasing rate slows and the total energy becomes much more stable.

If the total energy keeps decreasing, the completely stable configuration with the shortest contact surfaces will be generated, which may not be the appropriate simulation realization for real stratigraphic profile.

Histogram of total model iterations for 1000 chains

If the COV of total energy values from subsequent 10 times of sampling is less than 0.1%, the decreasing rate of total energy is regarded as unobvious and the stochastic energy relaxation is regarded as completed to obtain a MAP estimate (one chain).





Appendix

Table 1
The locations and magnitude of known orientation vectors.

Number	X	Y	Orientation angle (°)	Number	X	Y	Orientation angle (°)
1	3.0	8.0	−37.5	9	13.0	4.0	25.0
2	1.5	3.0	−37.5	10	13.0	9.0	16.7
3	4.0	6.5	−30.0	11	15.0	6.0	30.0
4	7.0	4.0	−8.3	12	14.5	3.0	25.0
5	5.0	5.0	−37.5	13	19.0	8.0	37.5
6	8.0	8.0	−8.3	14	18.0	5.0	37.5
7	9.	3.0	0.0	15	18.0	2.0	37.5
8	10.0	4.0	0.0				

Note: "0°" indicates the strata orientation is parallel to X axis of Cartesian coordinate system; "+" indicates the strata orientation is counterclockwise from X axis; "-" indicates the strata orientation is clockwise from X axis.

Example1: ICM modeling approach ~ frequency diagram

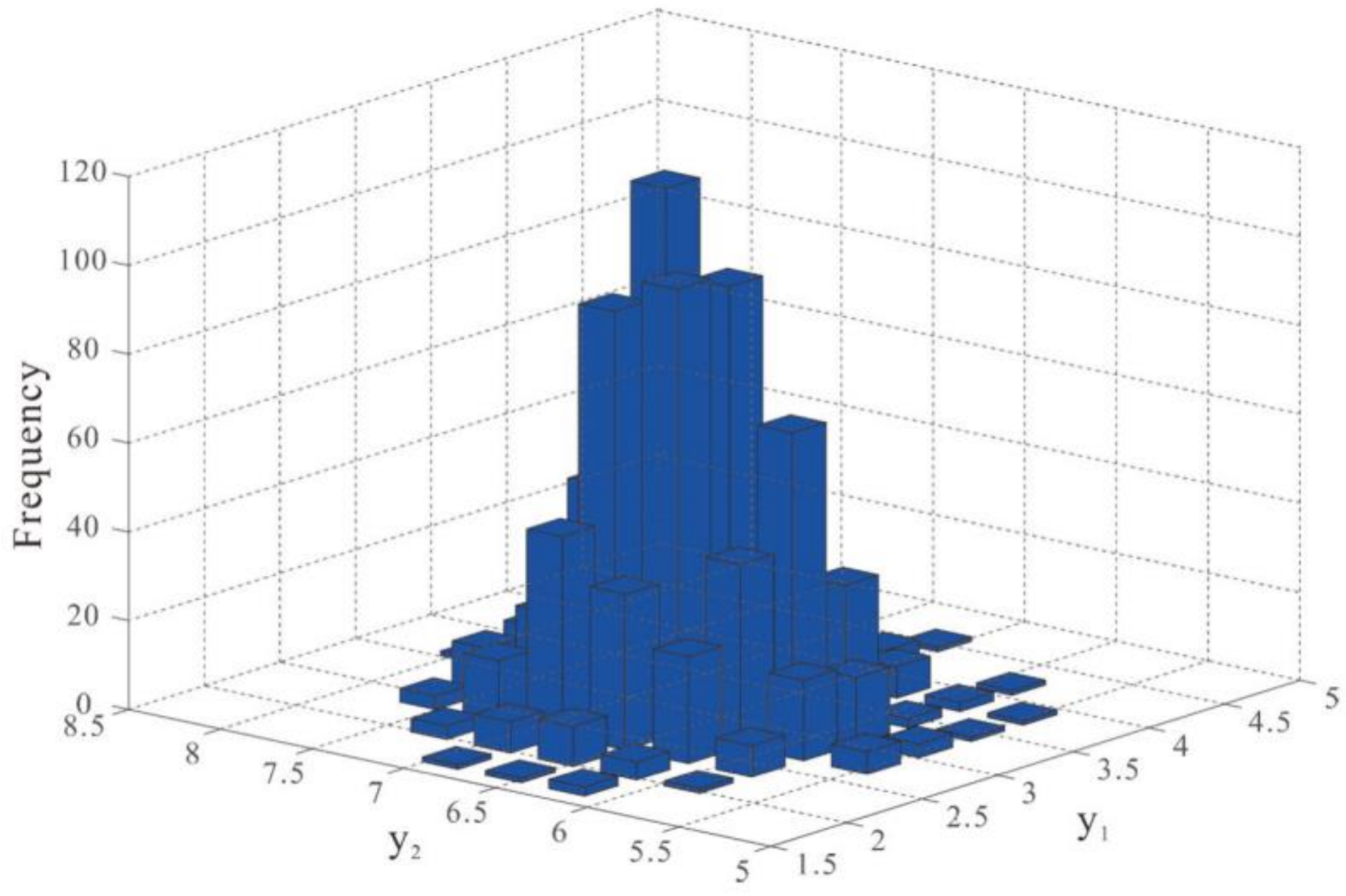


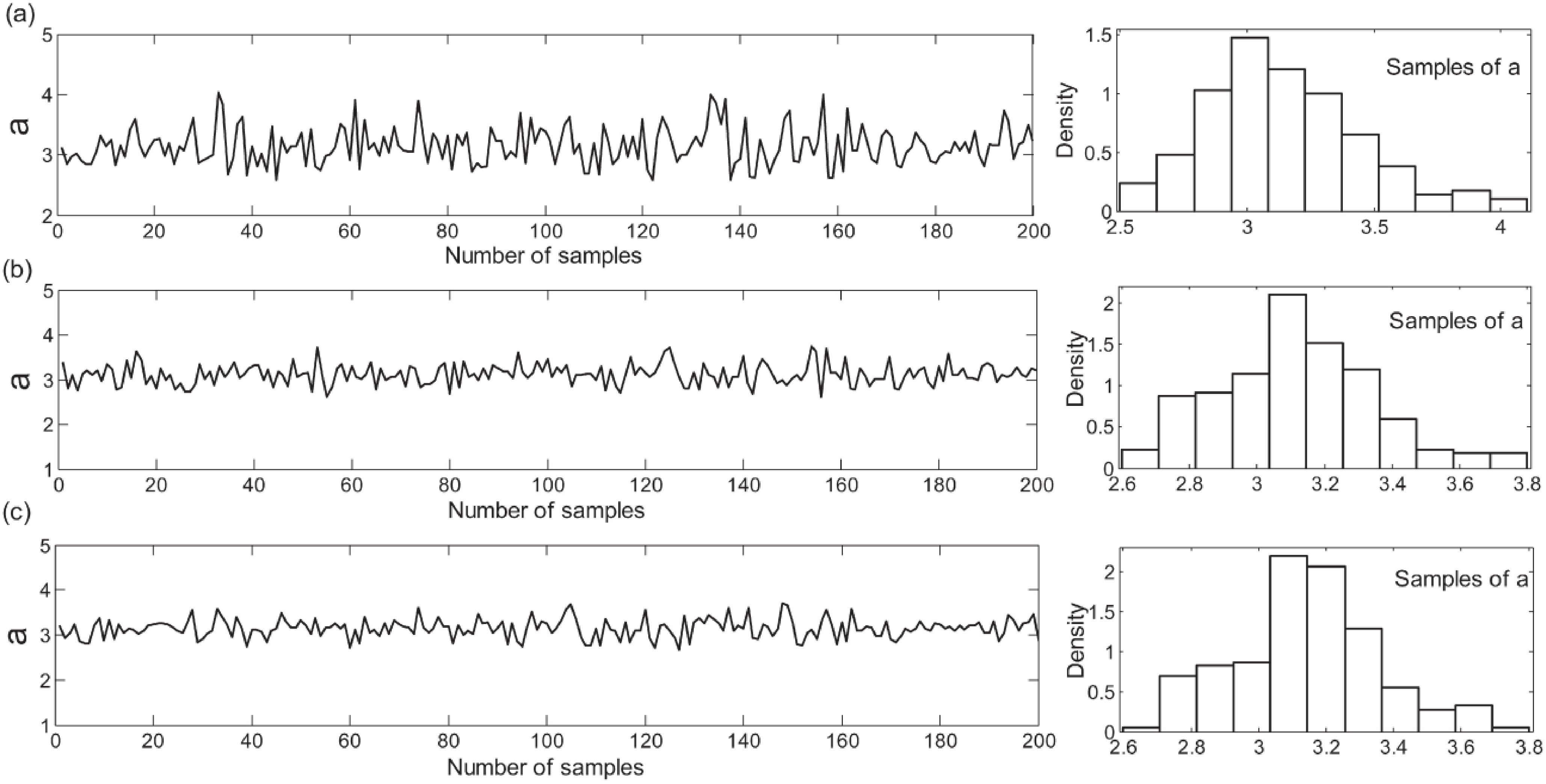
Fig. 11. Joint frequency diagram of the pair of depth (y_1,y_2).

In addition data:
y1 is the contact surface(depth) between formation 1 and formation 2.

y2 is the contact surface(depth) between formation 2 and formation 3.

different "a" will have different frequency diagram, in this case is a=3.0

Example1: ICM modeling approach



Mean(a):3.1481
Var(a):0.0976

Mean(a):3.1147
Var(a):0.0537

Mean(a):3.1459
Var(a):0.0455