Analysis of multi-phase coupled seepage and stability in anisotropic slopes under rainfall condition

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Outline

- Introduction
- Methodology
- Results and Discussion
- Conclusions

Introduction

Heavy rainfall is a key factor that results in landslides.

→ The rainfall seepage into an unsaturated soil slope touches off complicated responses among the soil particle, liquid and air, these three phases of the soil mass and possibly slope instability.

The main stress on the Earth’s surface are naturally anisotropic.

→ The anisotropic behavior mainly occurs during the deposition process, causing the soil’s strength to vary depending on the direction of the stress.

Objective:

Examining a multi-field coupling model for studying the stability in unsaturated soil during rainfall infiltration.

1. This study discussed infiltration characteristics, air migration, deformation of the slopes and stability during rainfall infiltration.
2. The influences of slope anisotropy on rainfall infiltration considering solid - liquid - air three phase coupling are analyzed.

Methodology

The coupled model- Mass conservation equations

Assumptions for multi-field coupled analysis:

(1) Water is incompressible.

(2) Air diffusion in water and water vapor movement are ignored.

The mass conservation in a three-phase soil system

$$\frac{d}{dt}(\rho_\pi \eta^\pi) + \rho_\pi \eta^\pi \nabla \cdot v^\pi = 0$$

Ignoring the phase transition in unsaturated porous media:

$$\frac{(1-n)}{\rho_s}\frac{d\rho_s}{dt} - \frac{dn}{dt} + (1-n)\nabla \cdot v^s = 0 \qquad \text{solid}$$

$$\frac{d}{dt}(nS_w\rho_w) + \nabla \cdot (nS_w\rho_w v^{ws}) + nS_w\rho_w \nabla \cdot v^w = 0 \qquad \text{liquid}$$

$$\frac{d}{dt}(nS_a\rho_a) + \nabla \cdot (nS_a\rho_a v^{as}) + nS_a\rho_a \nabla \cdot v^a = 0 \qquad \text{air}$$

$\frac{d}{dt}(\cdot) = \frac{\partial}{\partial t}(\cdot) + v^\pi \cdot \nabla(\cdot)$ is the material derivative of the π phase

v^π is the velocity of the π phase

ρ_π is the inherent density of the π phase

v^{ws} is the relative velocity of the water phase with reference to the solid, $v^{ws} = v^w - v^s$

v^{as} is the relative velocity of the gas phase with reference to the solid, $v^{as} = v^a - v^s$

The detail equations of conservation of mass for water and air

$$\left[S_w\frac{\alpha-n}{K_s}\left(S_w + \frac{\partial S_w}{\partial p_c}p_c\right) + \frac{nS_w}{K_w} - n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t} + \left[S_w\frac{\alpha-n}{K_s}\left(1 - S_w - \frac{\partial S_w}{\partial p_c}p_c\right) + n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_a}{\partial t} + \alpha S_w\frac{\partial \varepsilon_v}{\partial t} + \nabla\tilde{v}^{ws} = 0$$

$$\left[(1-S_w)\frac{\alpha-n}{K_s}\left(1 - S_w - \frac{\partial S_w}{\partial p_c}p_c\right) + \frac{n(1-S_w)}{K_w} - n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_a}{\partial t} + \left[(1-S_w)\frac{\alpha-n}{K_s}\left(S_w + \frac{\partial S_w}{\partial p_c}p_c\right) + n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t} + \alpha(1-S_w)\frac{\partial \varepsilon_v}{\partial t} + \nabla\tilde{v}^{as} = 0$$

α *and* n is the empirical parameters

S_w is the saturation

ε_v is the bulk strain of soil mass

$\tilde{v}^{ws}$ is the Darcy velocity of water

$\tilde{v}^{as}$ is the Darcy velocity of air

p_w is the pore-water pressure

p_a is the pore-air pressure

α is the Biot coefficient

K_s is the bulk modulus of soil particles

p_c is the matric suction, $p_c = p_a - p_w$

K_w is the compression modulus of liquid

Equilibrium equation

The equation of the soil mass :

$$\nabla \cdot \sigma + \rho g = 0$$

Consider the pore-water and pore air, the total stress:

$$\sigma = \sigma' - \alpha(S_w p_w + S_a p_a)\delta$$

Consider transversely anisotropic, the constitutive relation:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E_x}{1-\mu_x\mu_y} & \frac{\mu_y E_y}{1-\mu_x\mu_y} & 0 \\ \frac{\mu_x E_x}{1-\mu_x\mu_y} & \frac{E_y}{1-\mu_x\mu_y} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

The shear strength equation:

$$\tau = c' + (\sigma - p_a)\tan\varphi' + (p_a - p_w)\tan\varphi^b$$

The factor of safety, F_s, considering the coupled effects :

$$F_s = \frac{\sum_{i=1}^{N}\{[c'^{b_i} + (W_i - p_a b_i)\tan\varphi' + b_i(p_a - p_w)\tan\varphi_b]/[\cos\alpha_i(1 + \tan\alpha_i \tan\varphi'/F)]\}}{\sum_{I=1}^{N}(W_i \sin\alpha_i)}$$

σ is the total stress tensor
g is the acceleration of gravity
ρ is the total density of the three phases.
σ' is the effective stress tensor
δ is the Kronecker delta
S_w is the saturation
p_w is the pore-water pressure
p_a is the pore-air pressure
E_x and E_y is the x- and y-orientation elastic modulus of the soil mass
μ_x *and* μ_y is the x- and y-orientation Poisson's ratios of the soil
φ' is the effective angle of internal friction of unsaturated soils.
φ^b is the effective internal friction angle
α_i is the angle between the bottom plane and horizon
W_i is the weight of the soil
b_i is the width of the soil slice
N is the number of soil slices

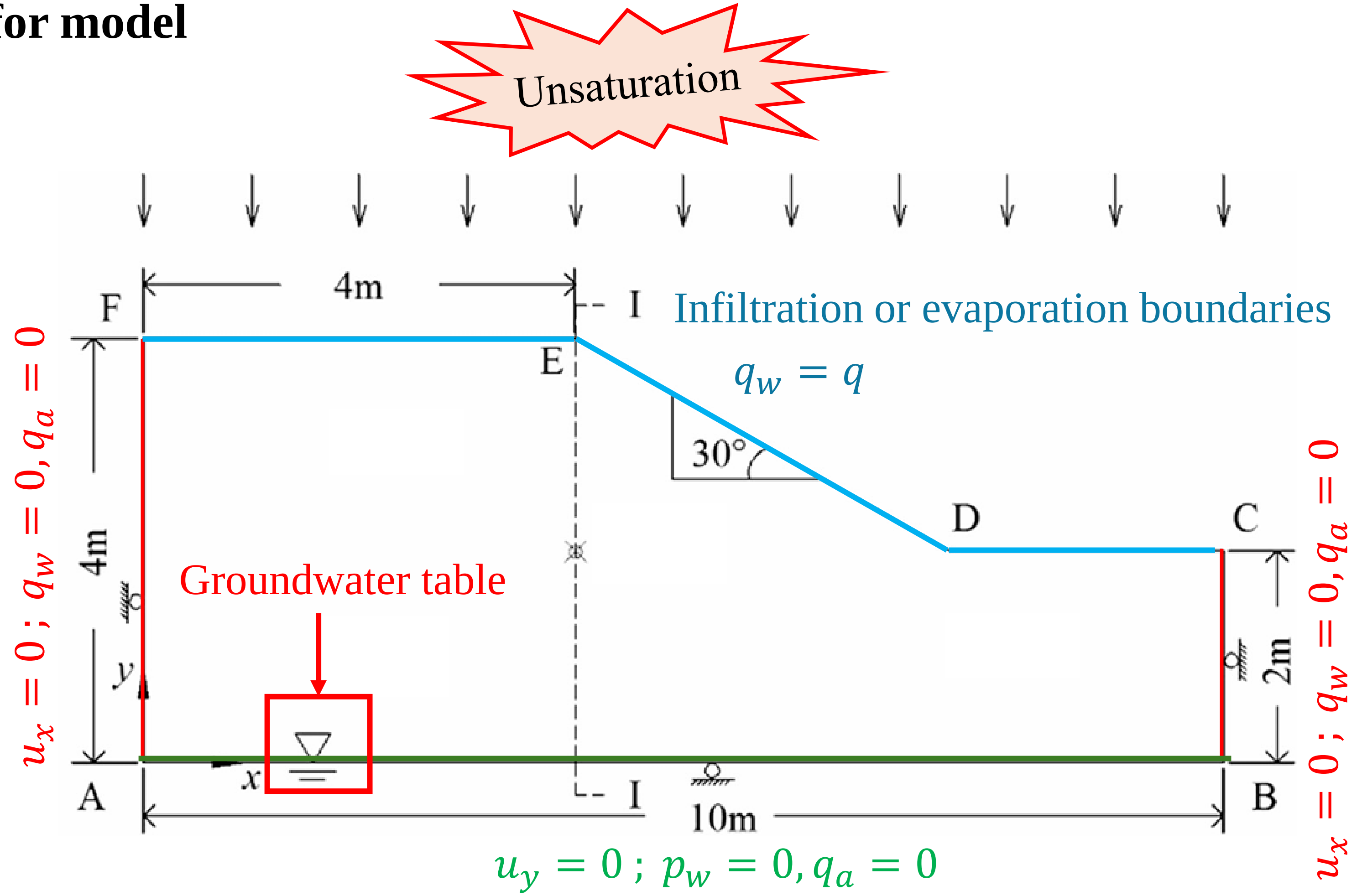
E_x and E_y, and μ_x and μ_y the equation:

$$\frac{\mu_y}{E_x} = \frac{\mu_x}{E_y}$$

Parameters, boundary and initial conditions for model

(1) Physical parameters

Parameter	Unit	Value
E	MPa	7.2
μ		0.3
K_s	MPa	1.0×10^6
ρ_s	kg/m^3	2850
n_0		0.42
k_0	m^2	4.5×10^{-13}
K_w	MPa	2.0×10^3
ρ_{w0}	kg/m^3	1000
μ_w	kg/m·s	1×10^{-3}
ρ_{a0}	kg/m^3	1.25
μ_a	kg/m·s	1.8×10^{-5}
p_{atm}	kPa	101.0



(2) The shear strength

C' (kPa)	φ' (°)	φ^b (°)
10	25	20

(3) Hydraulic parameters

P_b (kPa)	N	S_s	S_r	K_{s0} (m/s)	Q/k_{s0}
35	2	1	0.2	4.5×10^{-13}	0.9

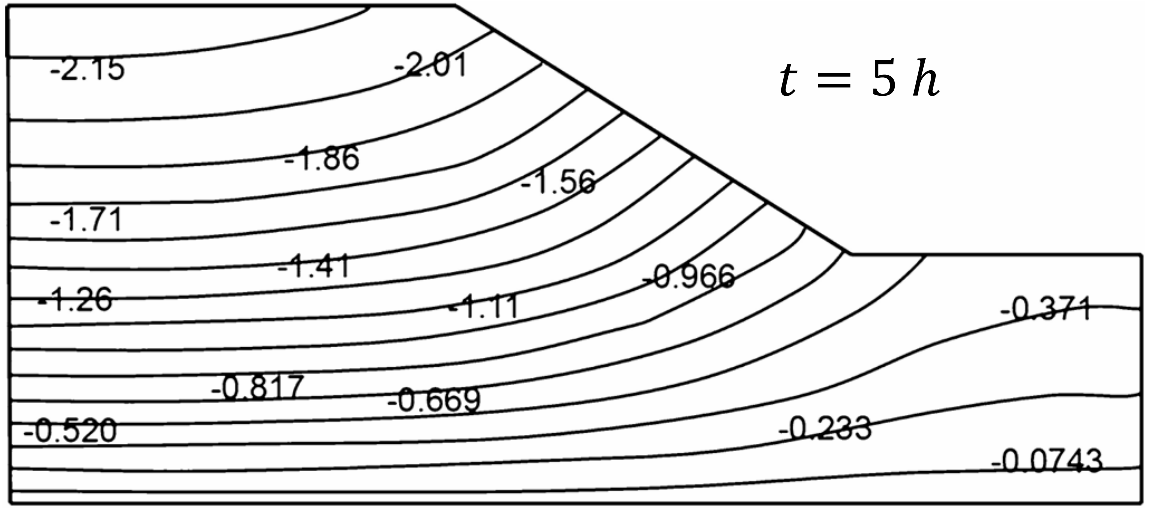
Duration: 48 h

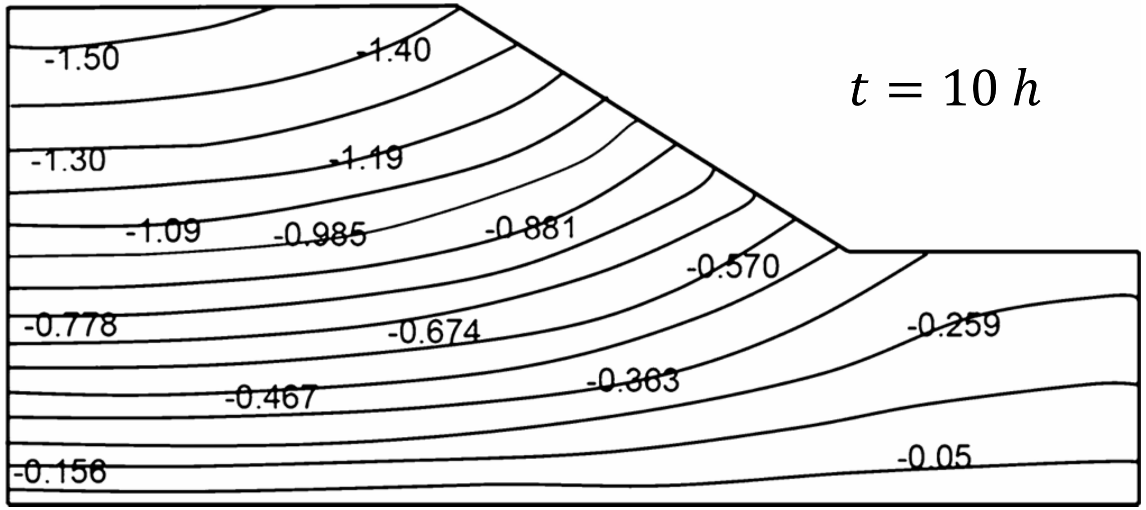
Initial pore-water pressure, $p_w = -\rho_w g y$

Initial pore-air pressure, $p_a = p_{atm}$

Results and Discussion

Soil transversely anisotropic – pore-water pressure head p_w in unsaturated soil slope





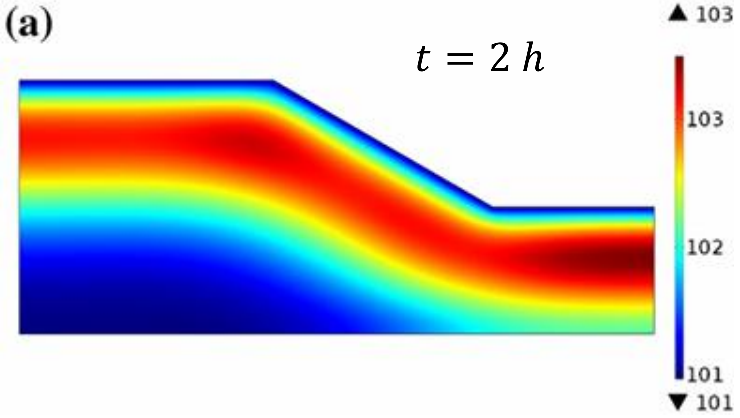
The influence of pore-water pressure head increases with rainfall time

1. Reduced shear strength of soil
2. Increased p_w promotes water movement within the soil → destabilize the slope
3. In unsaturated soils, p_w is typically negative (suction) and helps bind soil particles together. As p_w becomes less negative, suction is reduced, further weakening the soil structure.

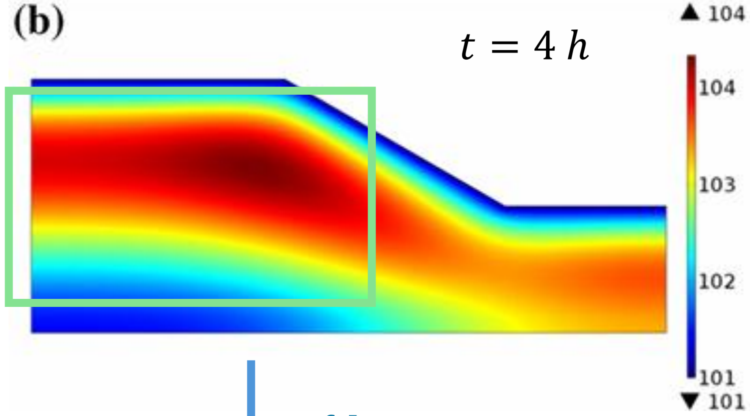
Soil transversely anisotropic – the pore-air pressure p_a profiles

The soil is not fully saturated + low vertical air permeability
→ p_a concentrated in the shallow slope

1. The air is discharged in horizontal direction
→ horizontal permeability > vertical permeability
2. p_a is gradually concentrated on the left side.
→ Not filled with water



infiltrate



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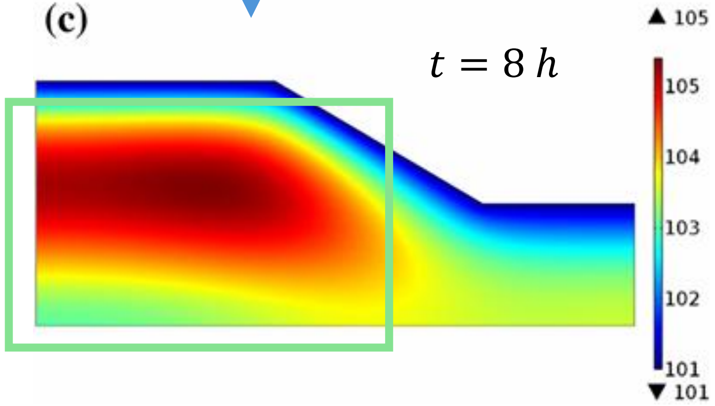
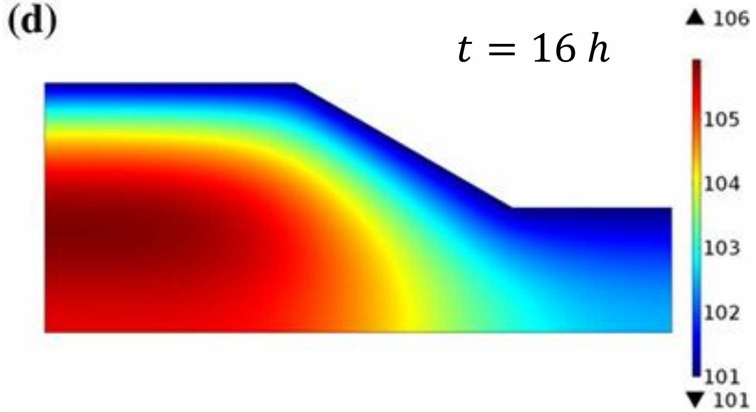


infiltrate

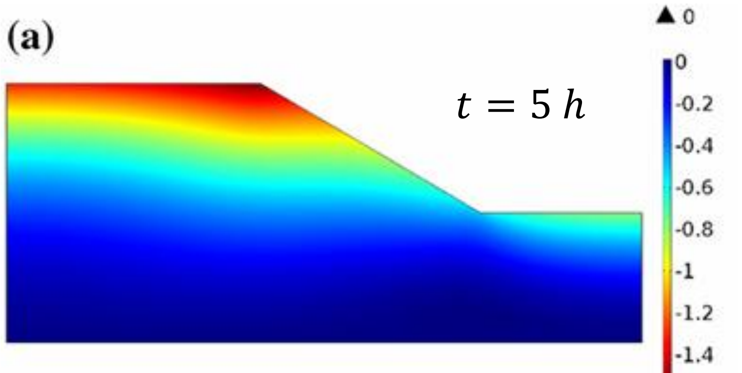


1. More air is compressed into the left.
2. Soil anisotropy dominates the air transport path.

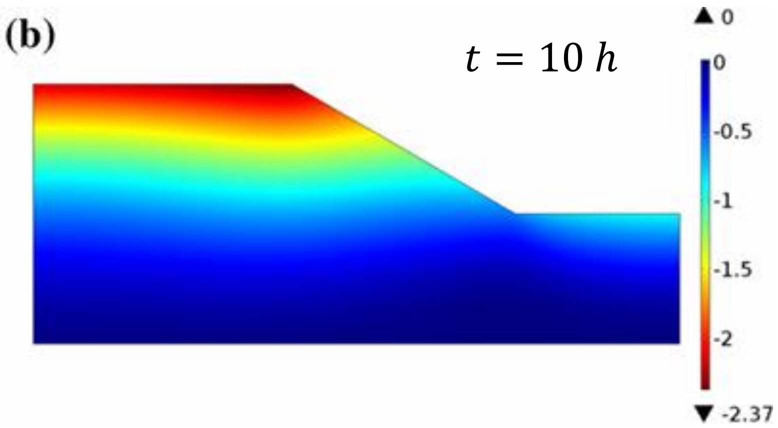
p_a is concentrated in the lower left
→ the left become the main exhaust path

Soil transversely anisotropic – the changes in vertical and horizontal displacements with time.

Vertical displacement, u_y

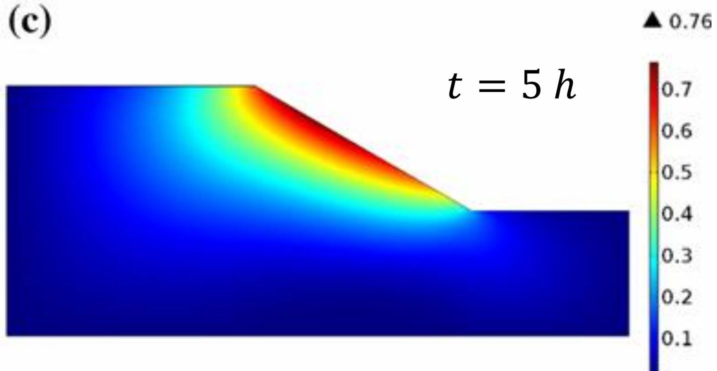


u_y increases clearly near the **top of the slope**

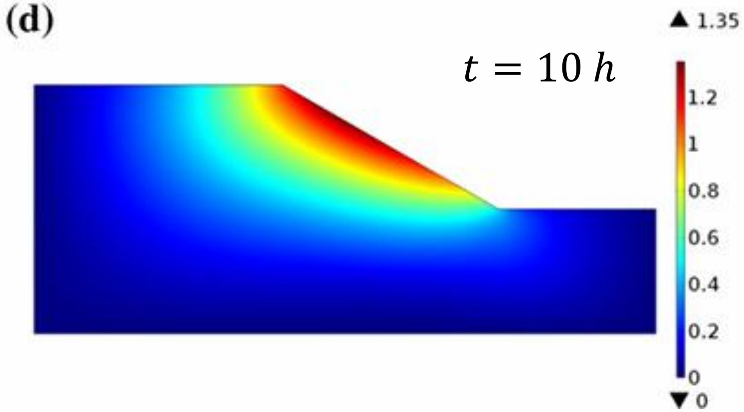


1. Rainfall infiltration reduce the effective stress
2. The permeability of the upper layer is higher, → p_w increase, deformation is faster.

Horizontal displacement, u_x

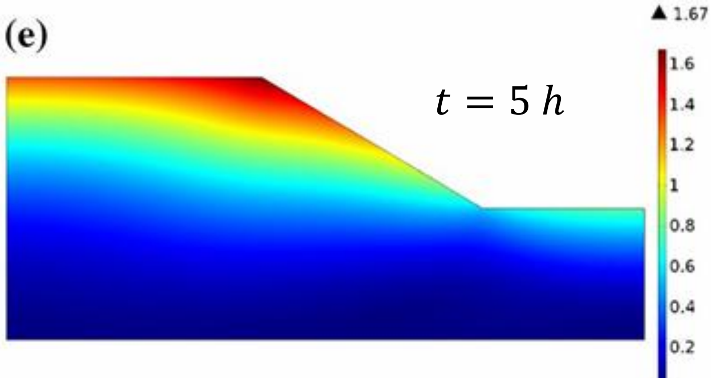


u_x increases clearly near the **slope face and crest**

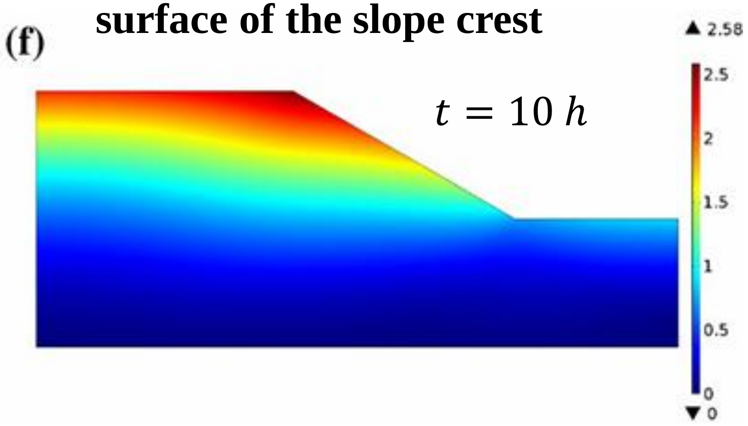


1. As water infiltrates deeper, slope loses stability → slope lateral movement
2. Steep gradient impact on horizontal sliding, particularly at the crest and shallow.

Total displacement, u_t



Max u_t occur at the **free surface of the slope crest**

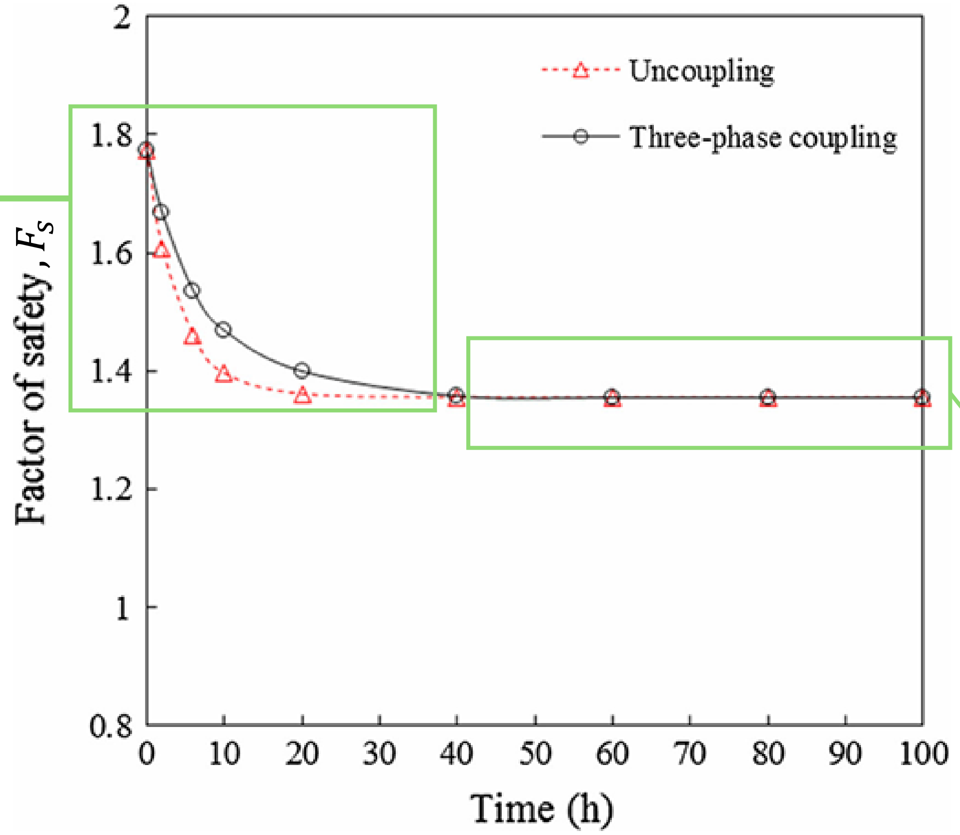


Long time rainfall infiltration reduce soil strength clearly.

→ Causing larger slope deformations, particularly in the slope crest

Soil transversely anisotropic – changes in the slope stability with rainfall duration.

Both scenarios rapid decrease, but the uncoupling case steeper drop



Uncoupling：
solid, liquid (water), and gas (air), three phases are independent.

Three-phase coupling (realistic)：
Considers the interactions among three phases (solid, liquid, and gas)

F_s stabilizes in both cases, but the coupled scenario maintains a slightly higher

Uncoupling (F_s decrease faster) :

Ignore the effect of pore-air and soil deformation
→ Overestimate slope instability

Coupling (F_s stabilizes better) :

Interactions between water infiltration, air migration, and soil deformation moderate the slope's response to rainfall.

→ More accurate and conservative estimate of slope stability

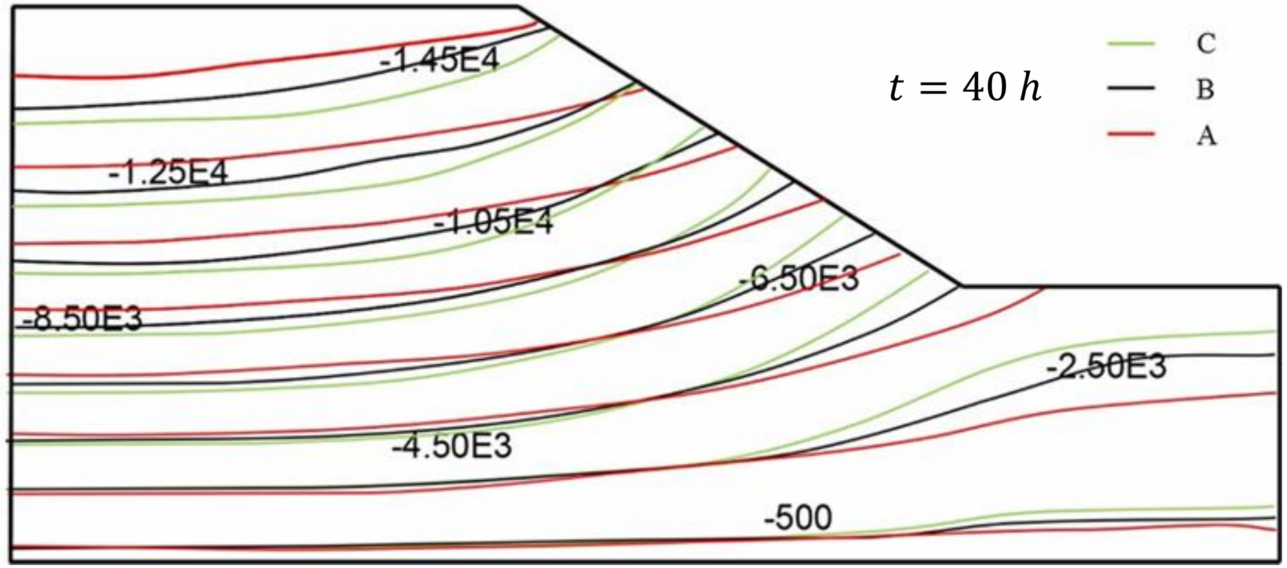
Influence of anisotropy on the distribution of pore-water pressure p_w in the unsaturated soil slope.

E_x/E_y: stiffness of the soil in the horizontal (x) and vertical (y)

k_{x0}/k_{y0}: how easily water moves horizontally (x) or vertically (y)

Transversely anisotropic elastic modulus E and permeability k :

No.	E_x/E_y	k_{x0}/k_{y0}	
A	0.6	1.4	→ deforms and flows horizontally
B	1	1	→ deforms and flows uniformly
C	1.4	0.6	→ deforms and flows vertically



Shallow zone

No. C

1. Water infiltrates fastest because the soil is stiffer horizontally and hydraulic properties promote vertical water movement.
2. p_w increases rapidly and deeper → faster infiltration into the slope

No. A

1. Water infiltration is slower because favors horizontal movement.

→ Water tends to gather near the surface

Deeper zone

No. A (horizontal permeability)

Water advances more in the horizontal direction

Water accumulation near the shallow zone and slower progress downward.

No. C (vertical permeability)

Water infiltrates deeper into the soil due to faster vertical movement.

No. B (isotropic)

Water infiltrates uniformly.

The effect of slope anisotropy on the factor of safety

E_x/E_y: how easily deform in the horizontal (x) and vertical (y)

k_{x0}/k_{y0}: how easily water moves horizontally (x) or vertically (y)

Transversely anisotropic elastic modulus E and permeability k :

No.	E_x/E_y	k_{x0}/k_{y0}
A	0.6	1.4
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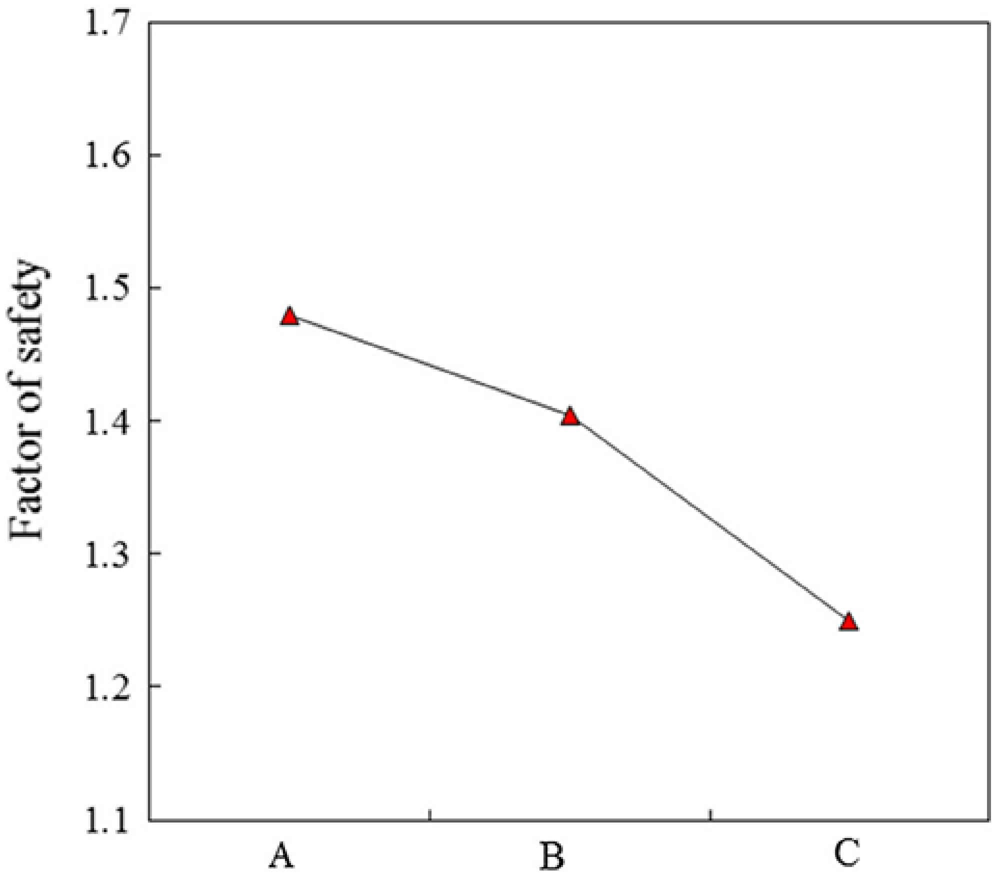
→ deforms and flows horizontally

→ deforms and flows uniformly

→ deforms and flows vertically



Factor of safety (F_s)

- **The highest F_s in No. A :**

  $E_x < E_y$: The soil is softer or more deformable horizontally, reduces stress concentration.

  $k_{x0} > k_{y0}$: Rainwater infiltration to dissipate quickly horizontally rather than accumulating vertically.

  The lowest F_s in No. C :

  $E_y < E_x$: The soil is more deformable vertically, localized stress concentration, which amplifies deformation in the vertical direction.

  $k_{x0} < k_{y0}$: Restricted horizontal permeability leads to pore pressure buildup, reducing effective stress and stability.

Conclusions

- The pore-water pressure increases over infiltration time. The pore-air pressure of the soil slope varies with depth. The maximum pore-air pressure occurs in the deep layer of the slope and is difficult to dissipate.

- The influence of soil anisotropy on the slope seepage varies not only with the depth of the soil slope, but also with changes in the total stresses in the slope.

- The anisotropy of the soil properties such as elastic modulus and coefficient of permeability has a marked effect on the soil slope stability.

Thank you for your attention.

The coupled model

Volumetric moisture:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha h)^n]^m} \quad (1)$$

The relative hydraulic conductivities of liquid and air:

$$k_{rw} = \sqrt{S_e}[1-(1-S_e^{\frac{1}{m}})^m]^2 \quad (2) \quad \text{liquid}$$

$$k_{ra} = \sqrt{1-S_e}(1-S_e^{\frac{1}{m}})^{2m} \quad (3) \quad \text{air}$$

$$S_e = \frac{S_w - S_r}{S_s - S_r} \quad (4)$$

The relationship between the coefficient of permeability and porosity of the soil mass:

$$k_s = k_{s0}\frac{(1+\varepsilon_v/n_0)^3}{1+\varepsilon_v} \quad (5)$$

The relationship between porosity and strain:

$$n_t = 1-(1-n_0)\exp(-\varepsilon_v) \quad (6)$$

θ: the volumetric moisture

h: the pressure head (m)

α, m and n: empirical parameters

θ_r: the residual moisture

θ_s: the saturated moisture

k_{rw}: the relative hydraulic conductivity of fluid

k_{ra}: the relative hydraulic conductivity of air

S_e: the effective saturation

S_r: the residual saturation

S_w: the saturation

S_s: the saturated saturation

ε_v: the bulk strain of soil mass

k_s: the saturated coefficient of permeability

k_{s0}: the initial coefficient of permeability of saturated soils

n_t: the porosity

n_0: the initial porosity

The coupled model- Mass conservation equations

The flow of liquid and gas in a porous medium obeys Darcy's law:

$$\tilde{v}^{ws} = -\frac{kk_{rw}}{\mu_w}(\nabla p_w - \rho_w g) \qquad (11a)$$ liquid

$$\tilde{v}^{as} = nS_a(v^a - v^s) = -\frac{kk_{ra}}{\mu_a}(\nabla p_a - \rho_a g) \qquad (11b)$$ gas

The liquid pressure and deformation are related to the solid density in a porous medium:

$$\frac{1}{\rho_s}\frac{d\rho_s}{dt} = \frac{\alpha - n}{K_S}\frac{dp_s}{dt} - (1-\alpha)\nabla\frac{d\varepsilon_v}{dt} \qquad (12)$$

Under the isothermal condition, the relationship between water density and pressure:

$$\frac{1}{K_w}\frac{dp_w}{dt} = \frac{1}{\rho_w}\frac{d\rho_w}{dt} \qquad (13)$$

$\tilde{v}^{ws}$: the Darcian velocity of water

$\tilde{v}^{as}$: the Darcian velocity of air

p_w: pore-water pressure

p_a: pore-air pressure

g: the acceleration of gravity

k: the inherent hydraulic conductivity for a porous medium

μ_w: the coefficient of viscosity for water

μ_a: the coefficient of viscosity for air

k_{rw}: the relative hydraulic conductivity of water

k_{ra}: the relative hydraulic conductivity of air

α: the Biot coefficient, $\alpha = 1 - K_T - K_S$; K_T is the bulk modulus of soil skeleton; K_S is the bulk modulus of soil particles

p_s: the fluid pressure, $p_s = S_w p_w + S_a p_a$

p_c: matric suction, $p_c = p_a - p_w$

K_W: the compression modulus of liquid

The coupled model- Mass conservation equations

$$\frac{1}{K_W}\frac{dp_w}{dt}=\frac{1}{\rho_w}\frac{d\rho_w}{dt} \quad (13)$$

An ideal air state equation is $\rho_a = p_a M_a/RT$,
Then we can obtain from the equation of state

$$\frac{1}{\rho_a}\frac{d\rho_a}{dt}=\frac{1}{K_a}\frac{dp_a}{dt} \quad (14)$$

The soil–water characteristic curve is closely associated with soil suction $(p_a - p_w)$.

The derivative of soil suction with respect to time:

$$\frac{\partial S_w}{\partial t}=\frac{\partial S_w}{\partial p_c}\frac{\partial(p_a - p_w)}{\partial t} \quad (15)$$

R: air constant (8.31432 J/mol k)

M_a: the molecular weight of air (kg)

T: the absolute temperature (K)

Substituting Eqs. (11) - (15) into Eqs. (9) and (10), the equations of conservation of mass for water and air are written as:

$$\left[S_w\frac{\alpha-n}{K_s}\left(S_w+\frac{\partial S_w}{\partial p_c}p_c\right)+\frac{nS_w}{K_W}-n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t} \quad (16)$$
$$+\left[S_w\frac{\alpha-n}{K_s}\left(1-S_w-\frac{\partial S_w}{\partial p_c}p_c\right)+n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_a}{\partial t}+\alpha S_w\frac{\partial \varepsilon_v}{\partial t}$$
$$+\nabla\tilde{v}^{ws}=0$$

$$\left[(1-S_w)\frac{\alpha-n}{K_s}\left(1-S_w-\frac{\partial S_w}{\partial p_c}p_c\right)+\frac{n(1-S_w)}{K_W} \quad (17)\right.$$
$$\left.-n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_a}{\partial t}+\left[(1-S_w)\frac{\alpha-n}{K_s}\left(S_w+\frac{\partial S_w}{\partial p_c}p_c\right)+n\frac{\partial S_w}{\partial p_c}\right]\frac{\partial p_w}{\partial t}$$
$$+\alpha(1-S_w)\frac{\partial \varepsilon_v}{\partial t}+\nabla\tilde{v}^{as}=0$$

K_a: the compression modulus of gas and $\boldsymbol{K_a \equiv p_a}$

$\tilde{v}^{WS}$: the Darcian velocity of water

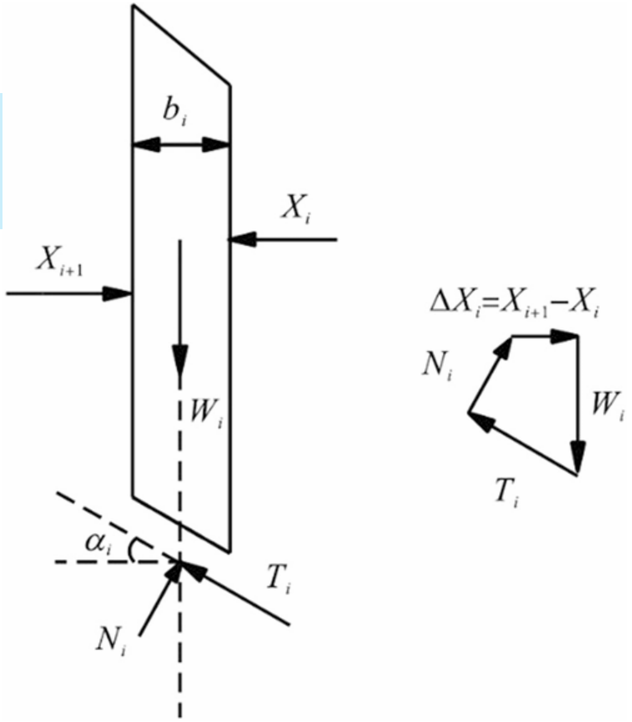
$\nabla\tilde{v}^{as}$: the Darcian velocity of air

Analysis of partially saturated soil slope stability

In partially saturated soil mass, the shear strength equation:

$$\tau = c' + (\sigma - p_a)\tan\varphi' + (p_a - p_w)\tan\varphi^b \quad (21)$$

Soil slice forces in the simplified Bishop method



φ': the effective angle of internal friction of unsaturated soils.
φ^b: the effective internal friction angle
T_i: the tangential force at the slice i at its bottom
α_i: the angle between the bottom plane and horizon
w_i: the weight of the soil
b_i: the width of the soil slice
N_i: the normal force of soil slice i at its bottom
N: the number of soil slices

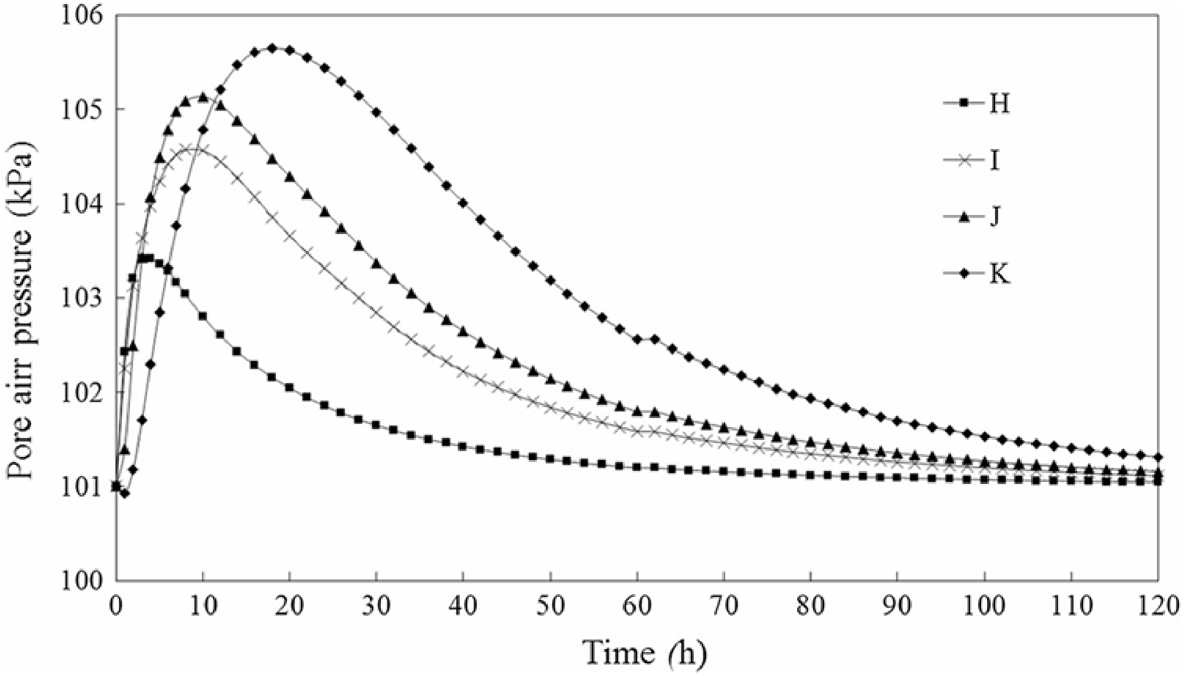
Based on moment equilibrium of the soil slices, we can obtain:

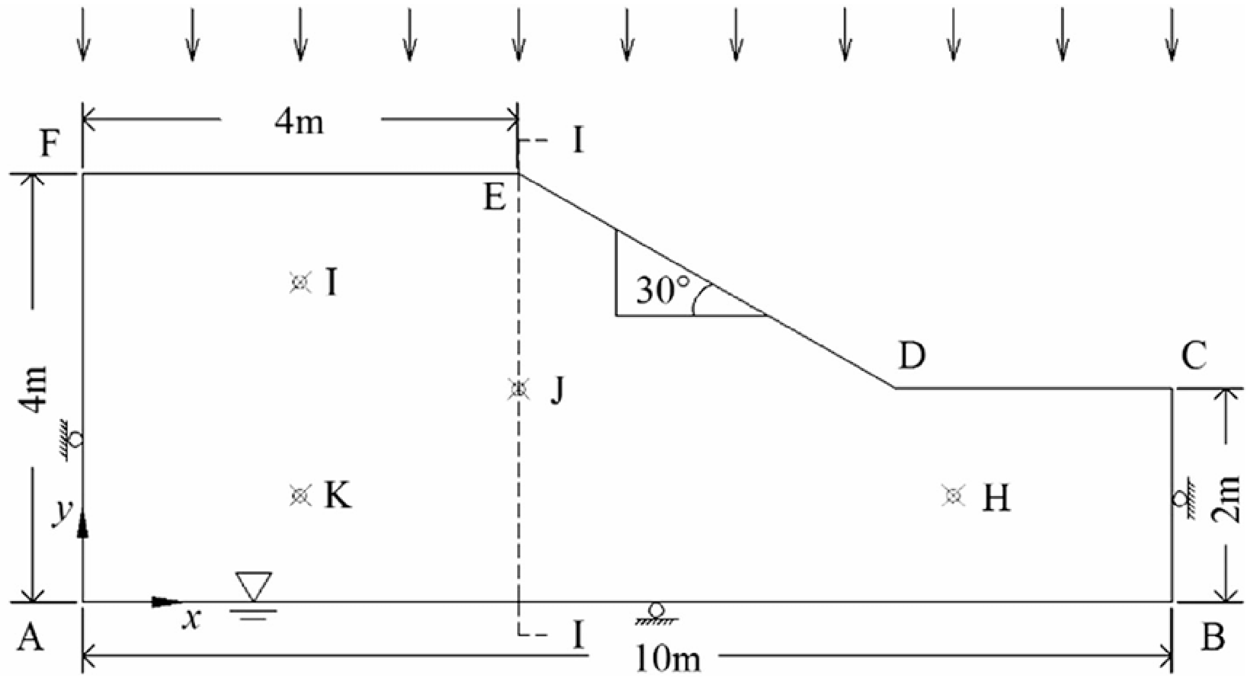
$$\sum_{i=1}^{N}(w_i \sin\alpha_i - T_i) = 0 \quad (22)$$

The factor of safety, F_s, considering the coupled effects is expressed as:

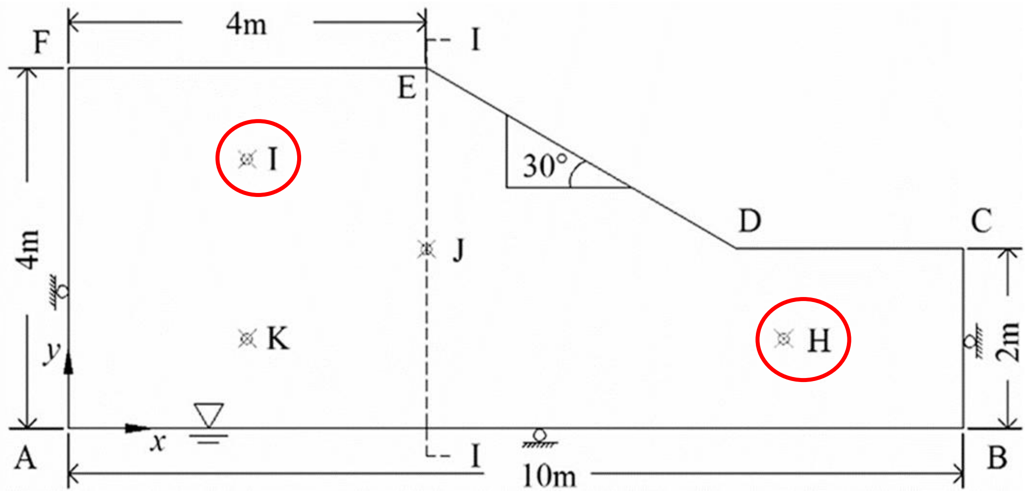
$$F_s = \frac{\sum_{i=1}^{N}\{[c'^{b_i} + (W_i - p_a b_i)\tan\varphi' + b_i(p_a - p_w)\tan\varphi_b]/[\cos\alpha_i(1 + \tan\alpha_i \tan\varphi'/F)]\}}{\sum_{I=1}^{N}(w_i \sin\alpha_i)} \quad (23)$$

Soil transversely anisotropic – the variation of pore-air pressure with time at points

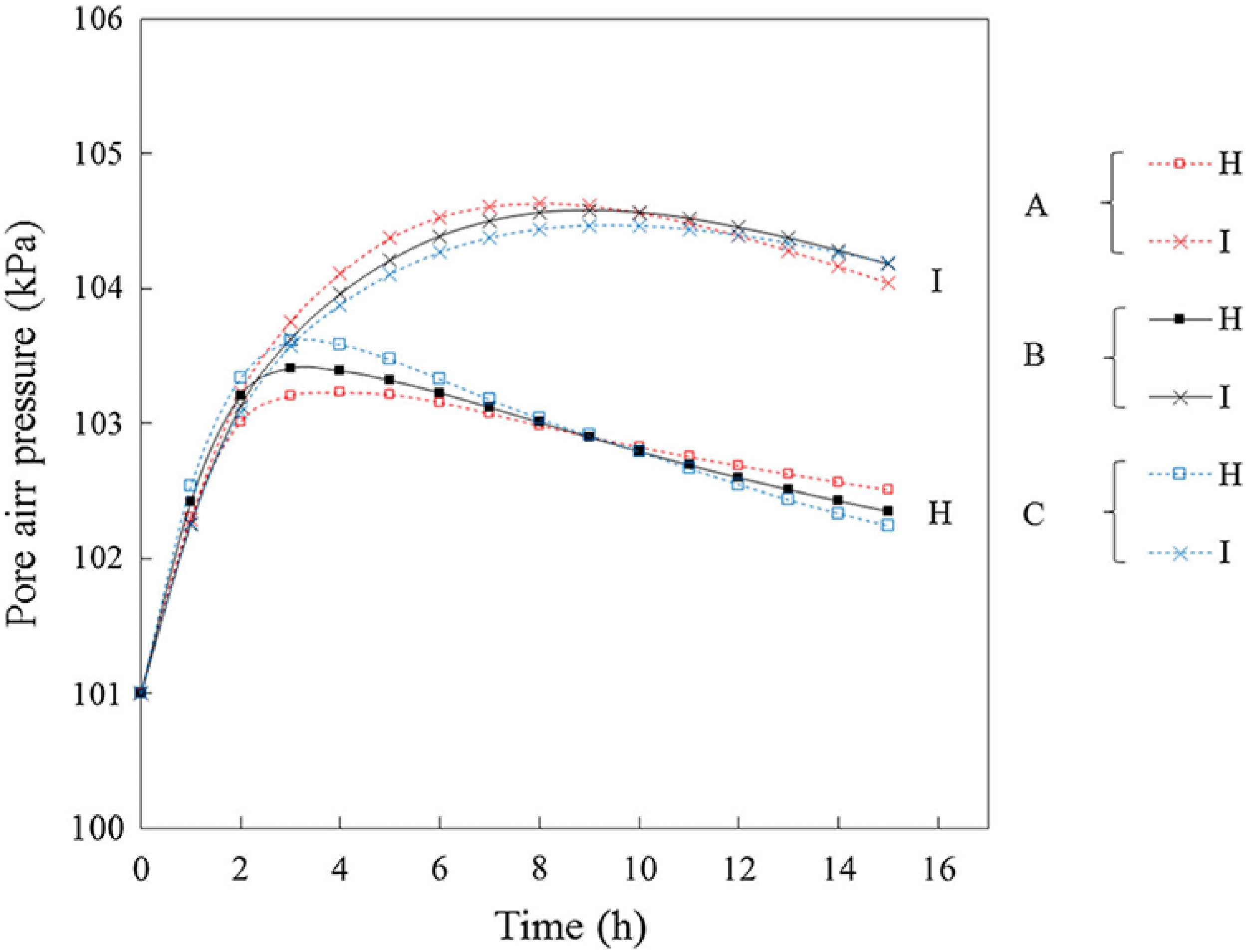




Influence of anisotropy on the pore-air pressure



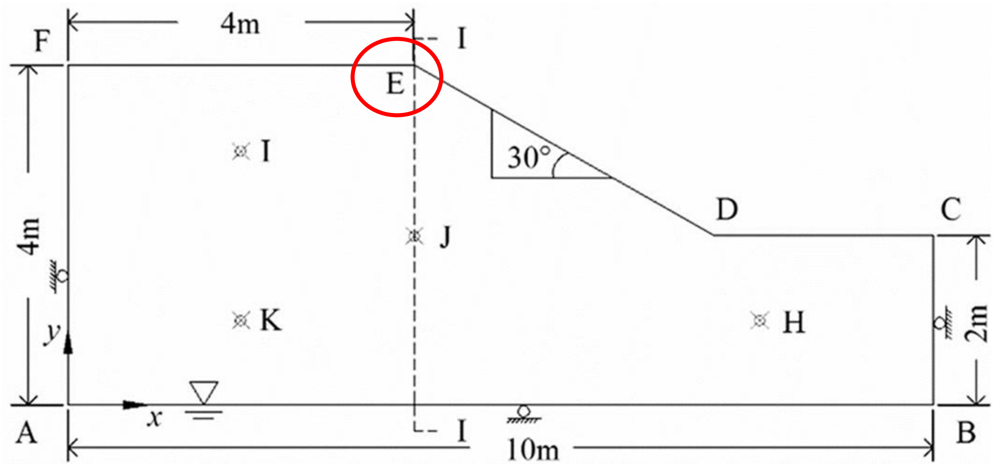
No.	E_x/E_y	K_{x0}/k_{y0}
A	0.6	1.4
B	1	1
C	1.4	0.6



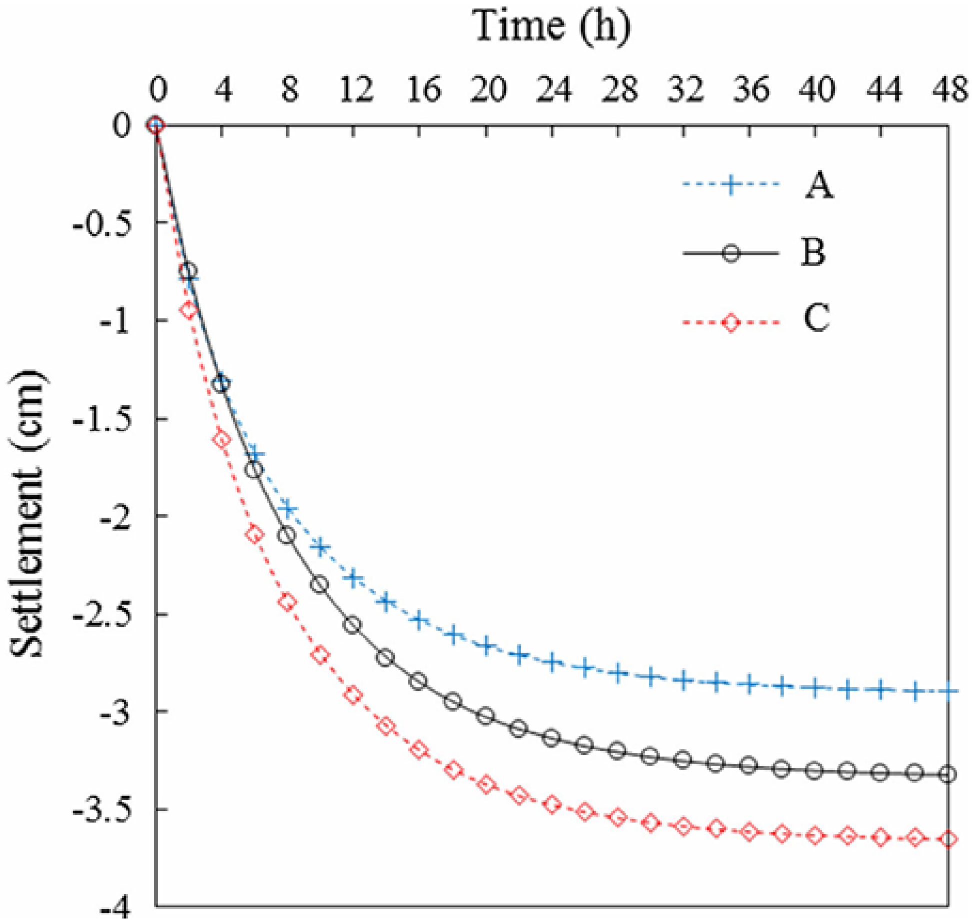
At point H, the pore-air pressure is the largest in case C. However, the pore-air pressure in case A and B moves slower than that in case C.

When $E_x > E_y$ and $k_{x0} < k_{y0}$, the pore-air pressure has a larger peak and dissipates more easily. The situation is on the contrary at point I.

The effect of anisotropy on the vertical displacement at point E



No.	E_x/E_y	K_{x0}/k_{y0}
A	0.6	1.4
B	1	1
C	1.4	0.6



The settlement at point E is the largest in case C.

When $E_x > E_y$ and $k_{x0} < k_{y0}$, the deformation of the unsaturated soil slope is the largest.