Stochastic Inversion of Aquifer Properties Based on Integrated Kalman Filter and Adjoint State Methods

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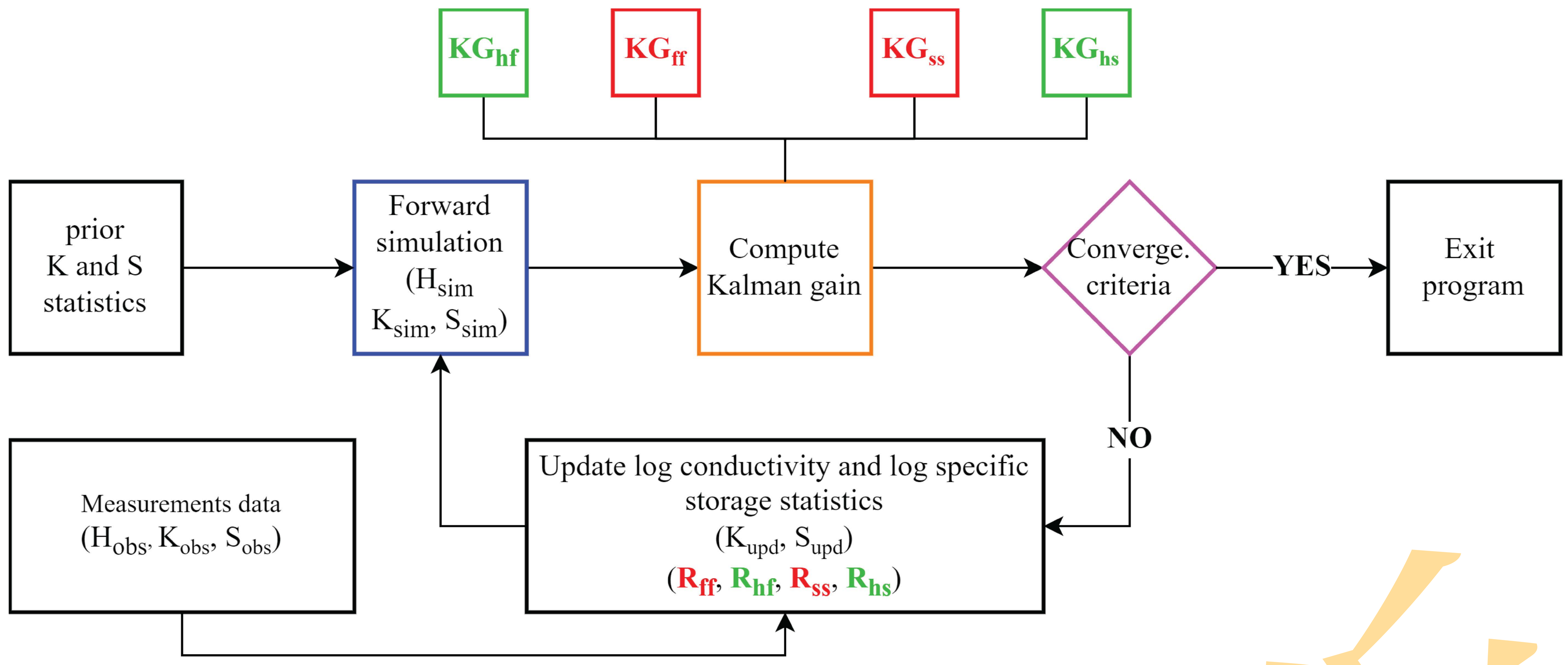
Introduction

A stochastic inversion model of aquifer properties predicts hydraulic conductivity and specific storage distributions with high resolution in heterogeneous saturated porous media. EnKF-AS (Ensemble Kalman Filter - Adjoint State) model accounts for transient flow sensitivities using hydraulic head measurements from cross-hole hydraulic pumping or injection tests. We test the model's ability to estimate hydraulic conductivity and specific storage in saturated porous media using two synthetic cases: an one-dimensional well-posed and ill-posed case and a two-dimensional vertical profile of aquifer. The results of the numerical experiments are promising. This model performed better than Monte Carlo Simulation (MCS) and traditional Ensemble Kalman Filter (EnKF) in two-dimensional synthetic aquifer cases with high correlation at 0.80 compared to 0.70 and 0.71, respectively. Determining the EnKF-AS model took 2 and 4 times less time than MCS and traditional EnKF approaches.

General Optimization of Linearization Algorithm



Governings Equation and Numerical Considerations

Governing equation

$$\nabla.[K(x)\nabla H] + Q(x_p) = S_s(x)\frac{\partial H}{\partial t}$$

subject to the initial condition and boundary conditions:

$$H|_{\Gamma 1} = H_1, [K(x)\nabla H].\mathrm{n}|_{\Gamma 2} = q, \mathrm{H}|_{t=0} = H_0$$

The general concept of Kalman Filter technique

$$K_{corrected} = K_{simulated} + KG_{ff} \times (K_{observed} - K_{simluated}) + F_K \times KG_{hf} \times (H_{observed} - H_{simluated}$$

$$S_{corrected} = S_{simulated} + KG_{ss} \times (S_{observed} - S_{simluated}) + F_S \times KG_{hs} \times (H_{observed} - H_{simluated})$$

$$KG_{ff} = \frac{R_{ff} \times (K_{map})^T}{K_{map} \times R_{ff} \times (K_{map})^T + M_{ff}}; \; KG_{hf} = \frac{R_{hf}}{H_{map} \times R_{hf} + M_{hf}}$$

$$KG_{ss} = \frac{R_{ss} \times (S_{map})^T}{S_{map} \times R_{ss} \times (S_{map})^T + M_{ss}}; \; KG_{hs} = \frac{R_{hs}}{H_{map} \times R_{hs} + M_{hs}}$$

Sensitivity analysis of transient flow by Adjoint State method

$$\frac{\partial H(x_k, t_l)}{\partial lnK(x_n)} = \int_T \int_\Omega \left(\frac{\partial K(x)}{\partial K(x_n)} \frac{\partial \Phi^*}{\partial x_i} \frac{\partial H}{\partial x_i}\right) dtd\Omega$$

$$\frac{\partial H(x_k, t_l)}{\partial lnS_s(x_n)} = \int_T \int_\Omega \left(\frac{\partial S(x)}{\partial S_s(x_n)} \frac{\partial \Phi^*}{\partial x_i} \frac{\partial H}{\partial t}\right) dtd\Omega$$

After an arbitrary function and mean head are calculated, sensitivities obtained above equation can be used to calculated cross-covariance with parameters using first-order approximation.

A two-dimensional synthetic aquifer case

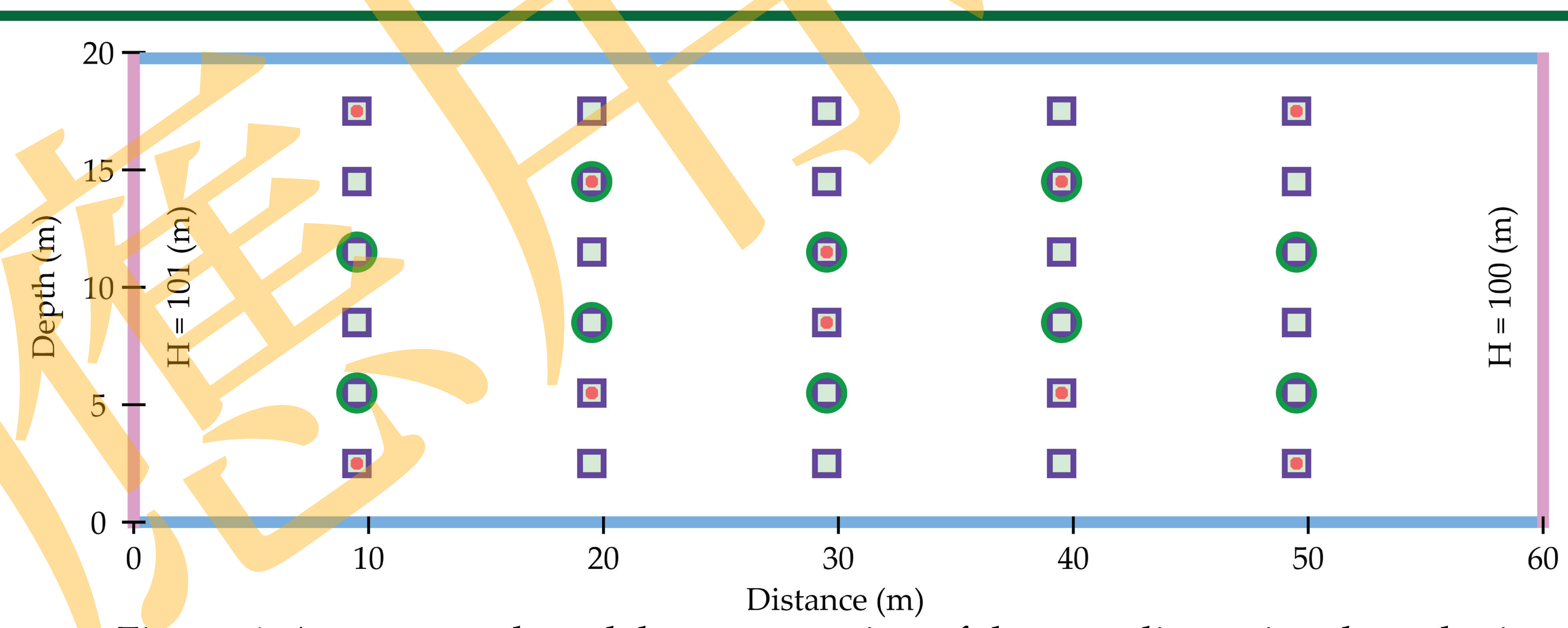
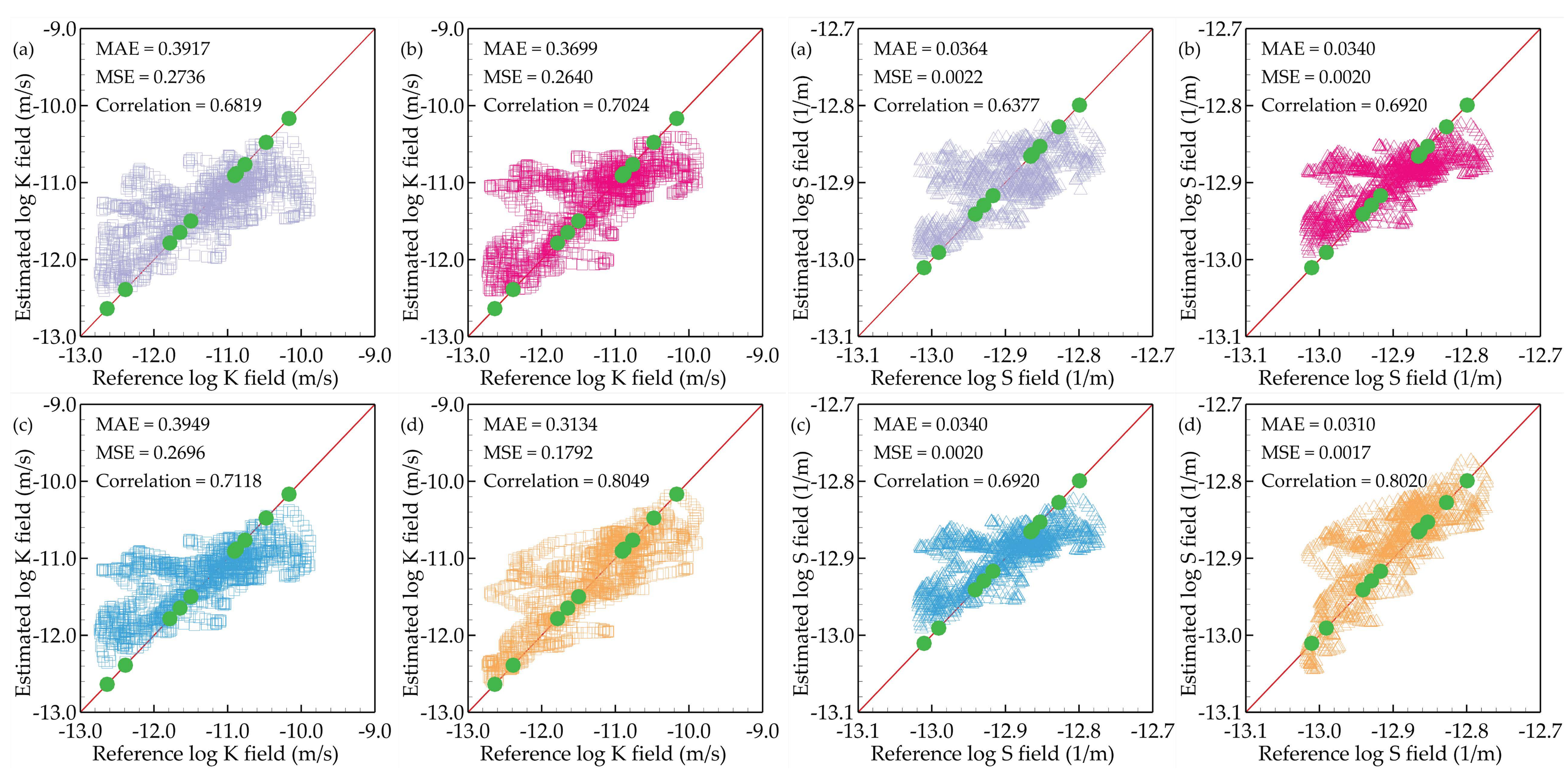


Figure 4. A conceptual model representation of the two-dimensional synthetic confine aquifer. The purple square represents to monitoring points. The green circle and red dot display for pumping locations and conditioning points, respectively.



An one-dimensional synthetic case

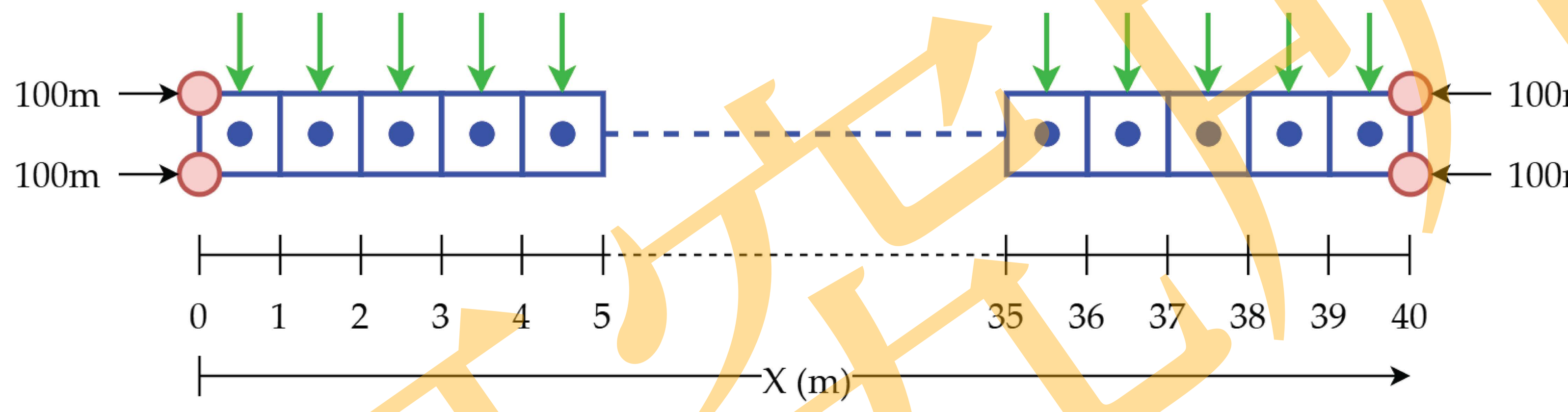


Figure 1. Conceptual model for one-dimensional synthetic case.

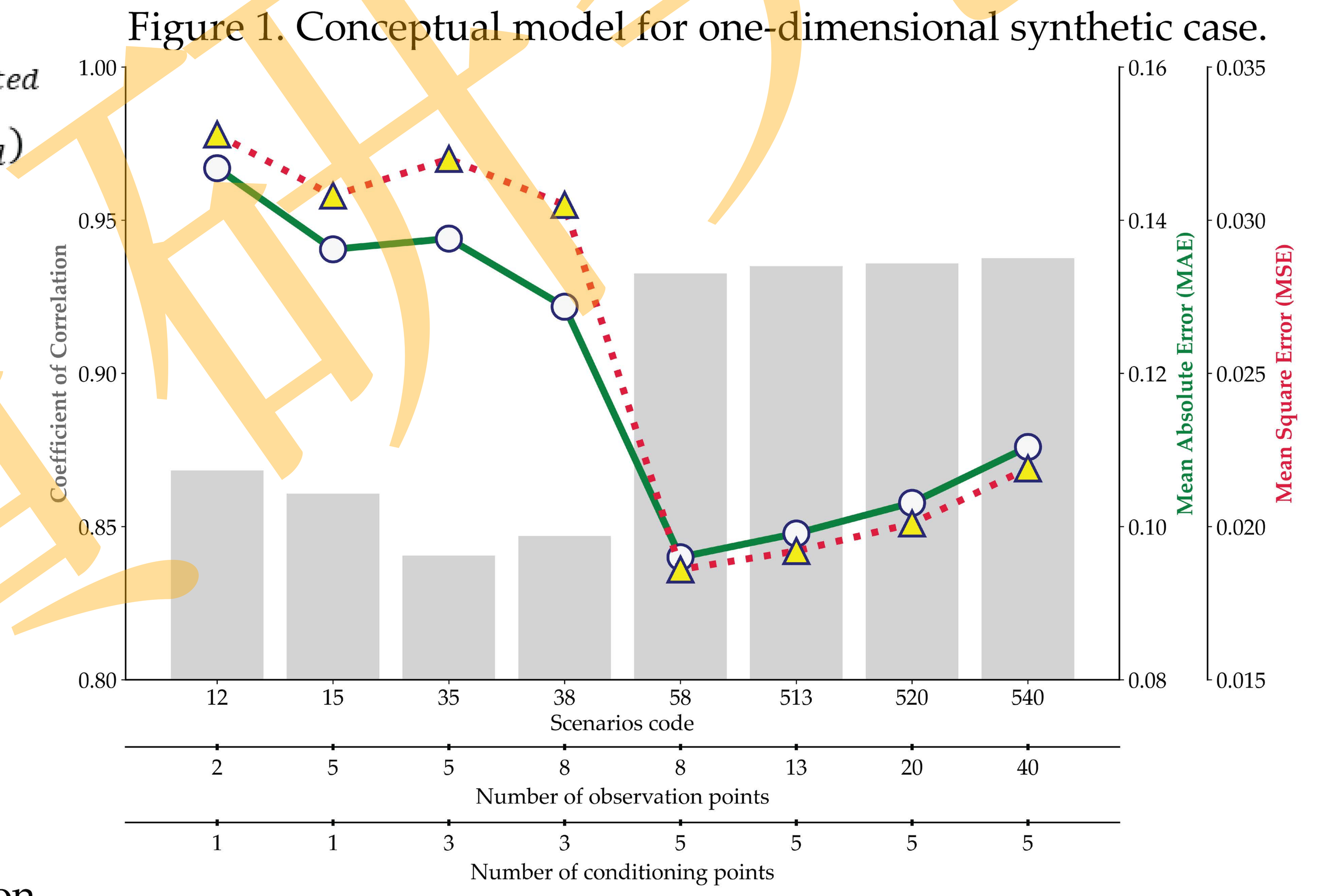


Figure 2. Evaluation performance metric of parameter estimation in ill-posed synthetic case for hydraulic conductivity in logarithm.

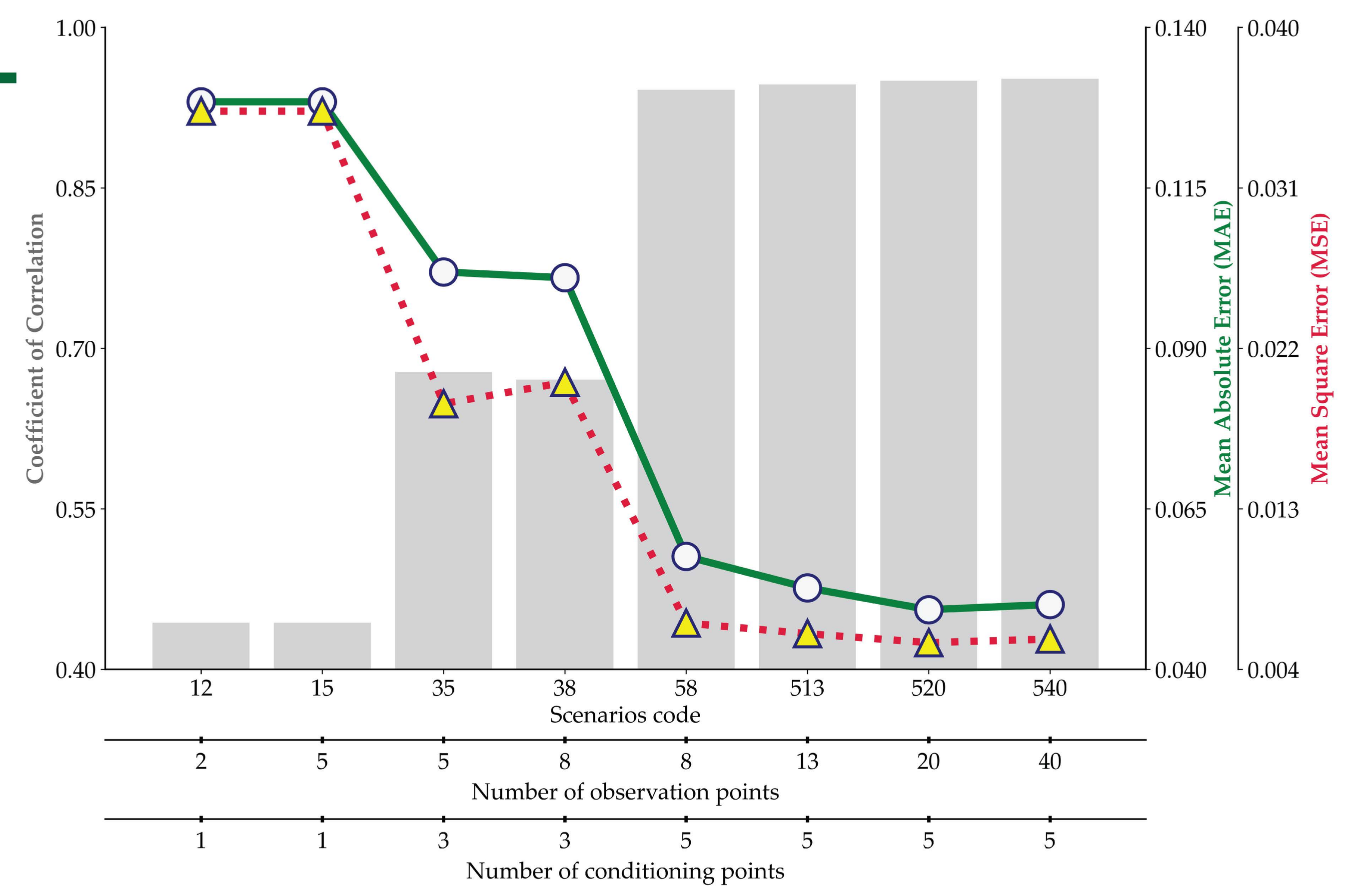


Figure 3. Evaluation performance metric of parameter estimation in ill-posed synthetic case for specific storage in logarithm.

Conclusion

- We developed successfully the EnKF-AS model that account for the sensitivities of transient flows using the hydraulic head measurmentes from cross-hole hydraulic pumping or injection test.
- EnKF-AS model is outperformed with high correlation of coefficient at 0.80 while MCS and EnKF are 0.70 and 0.71, respectively in parameter estimation of hydraulic conductivity.
- EnKF-AS model point outs the specific number of observations data and conditioning points that are required to understand an aquifer system.
- EnKF-AS demonstrated superior computational efficiency, executing calculations at speeds 2.0x and 4.0x faster then MCS and EnKF implementations, resepctively.
- EnKF-AS achieved optimal parameter estimation performance as measured by evaluation metrics while maintaining reduced memory requirements.

Acknowledgement

This study was financially supported by the Ministry of Science and Technolgy, Taiwan under grants NSTC 111-2621-M-008-003, NSTC 112-MOEA-M-008-001, and NSTC 112-2123-M-008-001